An Achievable Rate Region for Interfering Broadcast Channel and Multiple Access Channel with Cognitive Transmitters

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***Abstract*—We consider the inter-cluster cognitive behavior of two dissimilar interfering clusters, where the first cluster (i.e. primary cluster) consists of a broadcast channel (BC) which has a single sender and two receivers and the second cluster (i.e. cognitive cluster) is a single multiple access channel (MAC) with two senders and one receiver. The MAC is assumed to have cognitive transmitters, who know the message transmitted by the other non-cognitive broadcast transmitter, in a non-causal manner. A Gel'fand-Pinsker coding-like technique is used to mitigate inter-cluster interference. First, an achievable rate region for this model is derived based on the Marton with common message region for the general BC and using a result of Slepian-Wolf rate region for the MAC. Then by using the rate region of the second cluster, we obtain the achievable rate region for the two-user MAC with common message and common side information known non-causally at the transmitters and show that the obtained achievable rate region is optimal when common side information is known at the receiver as well.**

Keywords- Achievable rate region; broadcast channel; cognitive radio channel; multiple access channel, Gel'fand–Pinsker coding

I. INTRODUCTION

The cognitive radio technology in the wireless communications has made significant improvements in spectral efficiency. Cognitive radios could sense their wireless environment, obtain the messages of other already transmitting users and adapt themselves to their surroundings. In [1] and [2], Devroye *et al.* introduced the *cognitive radio channel* with 2 senders and 2 receivers, in which one user knows the other user's message non-causally and simultaneously transmits its own message. The authors also demonstrated an achievable region for this channel. Similar works are presented in [3]-[5], which are referred as "interference channel with unidirectional cooperation" or "interference channel with degraded message sets".

In [6], the authors considered an arbitrary wireless network consisting of cognitive and (possibly) non-cognitive radio devices. Moreover, the wireless network is partitioned into clusters with three different types of intra\inter-cluster behaviors: *competitive*, *cooperative*, and *cognitive*. Inter-cluster cognitive behavior refers to when some interfering clusters obtain the messages to be transmitted by other cluster(s) and use this knowledge to improve the overall rate region of wireless network. The inter-cluster cognitive behavior of two clusters that both of them are multiple access channels (MACs) is presented in [7]. In this case, each cluster consists of one receiver and at least two transmitters. Another related work is studied in [8], where the inter-cluster cognitive behavior of two broadcast channel (BC) clusters is considered. Each cluster assumed to have a single transmitter and two receivers, while the transmitter of one BC is non-causally aware of the message transmitted by the adjacent BC. An achievable rate region is derived for this model.

In this paper, we consider the inter-cluster cognitive behavior of two dissimilar interfering clusters, in which a MAC cluster (with two senders) is the cognitive cluster and is aware of the message transmitted by the adjacent BC cluster (with two receivers) transmitter, in a non-causal manner. We propose an achievable rate region for this model. Also, as a special case, the achievable rate region for the two-user MAC with common message and common side information known non-causally at the transmitters is obtained and show that the obtained achievable rate region is optimal when common side information is known at receiver as well. The achievable region is derived based on the superposition coding, binning scheme, simultaneous decoding and a Gel'fand-Pinsker coding-like technique [11] in order to mitigate inter-cluster interference. The obtained rate region subsumes the Marton region for the general BC [10][13][14][15], and the Slepian-Wolf region for the MAC with common message [9] as its special cases. For ease, we denote the proposed channel with $BC - MAC_G$. We depict $BC - MAC_G$ in Fig. 1. Now, we review the capacity region for the MAC with common message and the best achievable rate region for a two-user BC with common message.

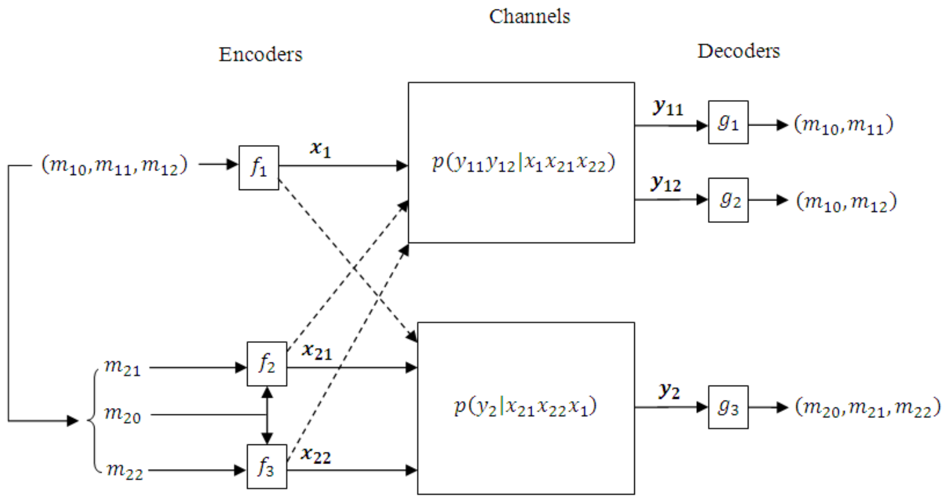

Fig. 1. Interfering broadcast channel and multiple access channel with cognitive transmitters ($BC - MAC_G$).

The capacity region for MAC with common message is derived as follows [9]:

$$R_1 \leq I(X_1; Y | X_2 W)$$
$$R_2 \leq I(X_2; Y | X_1 W)$$
$$R_1 + R_2 \leq I(X_1 X_2; Y | W)$$
$$R_0 + R_1 + R_2 \leq I(X_1 X_2; Y)$$

where the capacity region is computed over all joint distributions that factor as:

$$p(w, x_1, x_2, y) = p(w) p(x_1 | w) p(x_2 | w) p(y | x_1, x_2).$$

The largest achievable rate region for a two-user BC with common message is as follows[15]:

$$R_0 + R_1 \leq I(Y_1; U_1 W)$$
$$R_0 + R_2 \leq I(Y_2; U_2 W)$$
$$R_0 + R_1 + R_2 \leq \min\{I(Y_1; W), I(Y_2; W)\} - I(U_1; U_2 | W) + I(U_1; Y_1 | W) + I(U_2; Y_2 | W)$$
$$2R_0 + R_1 + R_2 \leq I(Y_1; U_1 W) + I(Y_2; U_2 W) - I(U_1; U_2 | W)$$

where the rate region is calculated over all joint distributions that factor as:

$$p(w, u_1, u_2, x, y_1, y_2) = p(w) p(u_1, u_2 | w) p(x | w, u_1, u_2) p(y_1, y_2 | x)$$

The rest of the paper is organized as follows. In Section II, we define channel model. All the main results are presented in Section III. A conclusion is prepared in Section V. Finally, the achievability of the proposed rate region is proved in the Appendix.

II. DEFINITIONS AND CHANNEL MODEL

We denote random variables by $X_1, X_2, Y_1, \cdots$ with values $x_1, x_2, y_1, \cdots$ in finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \cdots$ respectively; n-tuple vectors of $X_1, X_2, Y_1, \cdots$ are denoted with $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{y_1}, \cdots$.

Consider the proposed channel with finite input alphabets $\mathcal{X}_1, \mathcal{X}_{21}, \mathcal{X}_{22}$, finite output alphabets $\mathcal{Y}_{11}, \mathcal{Y}_{12}, \mathcal{Y}_2$, and transition probability function $p(y_{11}, y_{12}, y_2 | x_1, x_{21}, x_{22})$ with conditional marginal distributions $p(y_{11}, y_{12} | x_1, x_{21}, x_{22})$ and $p(y_2 | x_{21}, x_{22}, x_1)$, as in Fig. 1 . The channel is memoryless in the sense that for n channel uses, we have

$$p(\boldsymbol{y_{11}}, \boldsymbol{y_{12}} | \boldsymbol{x_1}, \boldsymbol{x_{21}}, \boldsymbol{x_{22}}) = \prod_{i=1}^{n} p(y_{11i}, y_{12i} | x_{1i}, x_{21i}, x_{22i})$$
$$p(\boldsymbol{y_2} | \boldsymbol{x_{21}}, \boldsymbol{x_{22}}, \boldsymbol{x_1}) = \prod_{i=1}^{n} p(y_{2i} | x_{21i}, x_{22i}, x_{1i})$$

where $\boldsymbol{x_1} = (x_{11}, \cdots, x_{1n}) \in \mathcal{X}_1^n$, $\boldsymbol{x_{2t}} = (x_{2t1}, \cdots, x_{2tn}) \in \mathcal{X}_{2t}^n$, $\boldsymbol{y_{1t}} = (y_{1t1}, \cdots, y_{1tn}) \in \mathcal{Y}_{1t}^n$ and $\boldsymbol{y_2} = (y_{21}, \cdots, y_{2n}) \in \mathcal{Y}_2^n$ for $t = 1,2$. Sender 1 sends two private messages $m_{1t} \in \mathcal{M}_{1t} = \{1, \cdots, M_{1t}\}, t = 1,2$; together with a common message $m_{10} \in \mathcal{M}_{10} = \{1, \cdots, M_{10}\}$ to the first and second receivers over the channel $p(y_{11}, y_{12} | x_1, x_{21}, x_{22})$. Senders 2 and 3 send their own private messages $m_{21} \in \mathcal{M}_{21} = \{1, \cdots, M_{21}\}$ and $m_{22} \in \mathcal{M}_{22} = \{1, \cdots, M_{22}\}$ to the third receiver, respectively. Moreover, both senders send a common message $m_{20} \in \mathcal{M}_{20} = \{1, \cdots, M_{20}\}$ to the third receiver over the channel $p(y_2 | x_{21}, x_{22}, x_1)$ .Assume that the transmitted message index $m_{ij}, i = 1,2$, $j = 0,1,2$ is selected uniformly from the message set $\mathcal{M}_{ij}$.

Let $\boldsymbol{C}$ denote the discrete memoryless $BC - MAC_G$ defined above. An $(n, M_{10} = \lfloor 2^{nR_{10}} \rfloor, M_{11} = \lfloor 2^{nR_{11}} \rfloor, M_{12} = \lfloor 2^{nR_{12}} \rfloor, M_{20} = \lfloor 2^{nR_{20}} \rfloor, M_{21} = \lfloor 2^{nR_{21}} \rfloor, M_{22} = \lfloor 2^{nR_{22}} \rfloor, \varepsilon)$ code exists for the channel $\boldsymbol{C}$ if and only if there exist three encoding functions

$f_1: \mathcal{M}_{10} \times \mathcal{M}_{11} \times \mathcal{M}_{12} \to \mathcal{X}_1^n,$
$f_2: \mathcal{M}_{10} \times \mathcal{M}_{11} \times \mathcal{M}_{12} \times \mathcal{M}_{20} \times \mathcal{M}_{21} \to \mathcal{X}_{21}^n,$
$f_3: \mathcal{M}_{10} \times \mathcal{M}_{11} \times \mathcal{M}_{12} \times \mathcal{M}_{20} \times \mathcal{M}_{22} \to \mathcal{X}_{22}^n,$

and three decoding functions

$g_1: \mathcal{Y}_{11}^n \to \mathcal{M}_{10} \times \mathcal{M}_{11},$
$g_2: \mathcal{Y}_{12}^n \to \mathcal{M}_{10} \times \mathcal{M}_{12},$
$g_3: \mathcal{Y}_2^n \to \mathcal{M}_{20} \times \mathcal{M}_{21} \times \mathcal{M}_{22},$

such that $P_e^n \leq \varepsilon$. P_e^n is the average error probability for a $(n, 2^{nR_{10}}, 2^{nR_{11}}, 2^{nR_{12}}, 2^{nR_{20}}, 2^{nR_{21}}, 2^{nR_{22}}, \varepsilon)$ code and defined as,

$$P_e^n = \frac{1}{2^{nR_*}} \sum_{m_* \in \mathcal{M}_*} \Pr\{(g_1(\boldsymbol{y_{11}}) \neq \overline{m_1}) \cup (g_2(\boldsymbol{y_{12}}) \neq \overline{m_2}) \cup (g_3(\boldsymbol{y_2}) \neq \overline{m_3}) \mid (\overline{m_1}, \overline{m_2}, \overline{m_3}) \text{sent}\}$$

where $R_* = R_{10} + R_{11} + R_{12} + R_{20} + R_{21} + R_{22}$, $m_* = (m_{10}, m_{11}, m_{12}, m_{20}, m_{21}, m_{22})$, $\mathcal{M}_* = (\mathcal{M}_{10}, \mathcal{M}_{11}, \mathcal{M}_{12}, \mathcal{M}_{20}, \mathcal{M}_{21}, \mathcal{M}_{22})$, $\overline{m_1} = (m_{10}, m_{11})$, $\overline{m_2} = (m_{10}, m_{12})$ and $\overline{m_3} = (m_{20}, m_{21}, m_{22})$.

A nonnegative rate tuple $(R_{10}, R_{11}, R_{12}, R_{20}, R_{21}, R_{22})$ is achievable for the channel $\boldsymbol{\mathcal{C}}$ if for any given $0 < \varepsilon < 1$, and sufficiently large n, there exists a code $(n, 2^{nR_{10}}, 2^{nR_{11}}, 2^{nR_{12}}, 2^{nR_{20}}, 2^{nR_{21}}, 2^{nR_{22}}, \varepsilon)$, such that $P_e^n \leq \varepsilon$.

The capacity region for the channel $\boldsymbol{\mathcal{C}}$ is defined as the closure of the set of all the achievable rate tuples, while an achievable rate region for the channel $\boldsymbol{\mathcal{C}}$ is a subset of the capacity region.

III. Main Result: An Achievable Rate Region for the Discrete Memoryless $BC - MAC_G$

A. Description of the rate region

Let us consider auxiliary random variables W_{10}, W_{20}, U_{11}, and U_{12} defined on arbitrary finite sets $\mathcal{W}_{10}, \mathcal{W}_{20}, \mathcal{U}_{11}$, and $\mathcal{U}_{12}$, respectively. The auxiliary random variables W_{10}, U_{11}, and U_{12} are used to encode messages m_{10}, m_{11}, and m_{12} in the broadcast cluster ,respectively. The auxiliary random variable W_{10} contains common message m_{10} and is decoded by both receivers 11 and 12. However, U_{1t} is just decoded by receiver $1t, t = 1,2$. In the MAC cluster, W_{20} contains common message m_{20} and is decoded by receiver 2. Two private messages m_{21} and m_{22} which are superimposed over m_{20}, are conveyed by X_{21} and X_{22}, respectively. For the $BC - MAC_G$ (Fig. 1), let $Z = (W_{10}, U_{11}, U_{12}, W_{20}, U_{21}, U_{22}, X_1, X_{21}, X_{22}, Y_{11}, Y_{12}, Y_2)$ and let $\mathcal{P}$ be the set of all distributions of the form (1):

$$P(w_{10}, u_{11}, u_{12}, w_{20}, u_{21}, u_{22}, x_1, x_{21}, x_{22}, y_{11}, y_{12}, y_2) = \quad (1)$$
$$P(w_{10}) P(u_{11}, u_{12}|w_{10}) P(x_1|w_{10}, u_{11}, u_{12}) \times$$
$$P(w_{20}|w_{10}, u_{11}, u_{12}) P(u_{21}|w_{20}, w_{10}, u_{11}, u_{12}) \times$$
$$P(u_{22}|w_{20}, w_{10}, u_{11}, u_{12}) \times$$
$$P(x_{21}|u_{21}, w_{20}, w_{10}, u_{11}, u_{12}) P(x_{22}|u_{22}, w_{20}, u_{10}, u_{11}, u_{12}) \times$$
$$P(y_{11}, y_{12}, y_2|x_1, x_{21}, x_{22})$$

The joint distribution (1) results in the following Markov chains,

$$U_{21} \leftrightarrow (W_{10}, U_{11}, U_{12}, W_{20}) \leftrightarrow U_{22} \quad (2\text{-}1)$$

$$(W_{10}, U_{11}, U_{12}, W_{20}, U_{21}, U_{22}) \leftrightarrow (X_1, X_{21}, X_{22}) \leftrightarrow (Y_{11}, Y_{12}, Y_2) \quad (2\text{-}2)$$

Theorem 1: For any $Z \in \mathcal{P}$ let $\mathcal{S}(Z)$ be the set of all rate tuples $(R_{10}, R_{11}, R_{12}, R_{20}, R_{21}, R_{22})$ of non-negative real numbers such that

$$R_{10} + R_{11} \leq I(Y_{11}; W_{10} U_{11}) \quad (3\text{-}1)$$
$$R_{10} + R_{12} \leq I(Y_{12}; W_{10} U_{12}) \quad (3\text{-}2)$$
$$R \quad (3\text{-}3)$$
$$+ \min\{I(Y_{11}; W_{10}), I(Y_{12}; W_{10})\} - I(U_{11}; U_{12}|W_{10})$$
$$2R_0 + R_{11} + R_{12} \leq I(Y_{11}; W_{10} U_{11}) + I(Y_{12}; W_{10} U_{12}) - I(U_{11}; U_{12}|W_{10}) \quad (3\text{-}4)$$
$$R_{21} \leq I(U_{21}; Y_2|W_{20}, U_{22}) - I(U_{21}; W_{10}, U_{11}, U_{12}|W_{20}, U_{22}) \quad (3\text{-}5)$$
$$R_{22} \leq I(U_{22}; Y_2|W_{20}, U_{21}) - I(U_{22}; W_{10}, U_{11}, U_{12}|W_{20}, U_{21}) \quad (3\text{-}6)$$
$$R_{21} + R_{22} \leq I(U_{21}, U_{22}; Y_2|W_{20}) - I(U_{21}, U_{22}; W_{10}, U_{11}, U_{12}|W_{20}) \quad (3\text{-}7)$$
$$R_{20} + R_{21} + R_{22} \leq I(W_{20}, U_{21}, U_{22}; Y_2) - I(W_{20}, U_{21}, U_{22}; W_{10}, U_{11}, U_{12}) \quad (3\text{-}8)$$

then any element of the closure of $\bigcup_{Z \in \mathcal{P}} \mathcal{S}(Z)$ is achievable for the $BC - MAC_G$.

Proof: Refer to APPENDIX.

Remark 1: By isolating the second cluster (MAC) from the first one (BC) and therefore, eliminating the cognitive behavior,

$$I(W_{20}; W_{10} U_{11} U_{12}) = 0,$$
$$I(X_{21}; W_{10} U_{11} U_{12}|W_{20}) = 0,$$
$$I(X_{22}; W_{10} U_{11} U_{12}|W_{20}) = 0,$$

we reach to the capacity region for MAC with common message [9].

Remark 2: By setting $S = (W_{10}, U_{11}, U_{12})$ in inequalities (3-4)-(3-7) we obtain the achievable rate region for the two-user MAC with common message and common side information S known non-causally at the transmitters [16], which is the set of all rate triples (R_{20}, R_{21}, R_{22}), satisfying

$$R_{21} \leq I(U_{21}; Y_2|W_{20}, U_{22}) - I(U_{21}; S|W_{20}, U_{22}) \quad (4\text{-}1)$$
$$R_{22} \leq I(U_{22}; Y_2|W_{20}, U_{21}) - I(U_{22}; S|W_{20}, U_{21}) \quad (4\text{-}2)$$
$$R_{21} + R_{22} \leq I(U_{21}, U_{22}; Y_2|W_{20}) - I(U_{21}, U_{22}; S|W_{20}) \quad (4\text{-}3)$$

$$R_{20}+R_{21}+R_{22} \leq I(W_{20},U_{21},U_{22};Y_2)-I(W_{20},U_{21},U_{22};S) \quad (4\text{-}4)$$

for some joint distribution of the form,

$$P(s,w_{20},u_{21},u_{22},x_{21},x_{22},y_2) = P(s)P(w_{20}|s)P(u_{21}|w_{20},s)P(u_{22}|w_{20},s) \times P(x_{21}|u_{21},w_{20},s)P(x_{22}|u_{22},w_{20},s)P(y_2|x_{21},x_{22}), \quad (5)$$

The joint distribution (5) results in the Markov chains below,

$$(U_{21},X_{21}) \leftrightarrow (S,W_{20}) \leftrightarrow (U_{22},X_{22}) \quad (6\text{-}1)$$

$$(W_{20},U_{21},U_{22}) \leftrightarrow (S,X_{21},X_{22}) \leftrightarrow Y_2 \quad (6\text{-}2)$$

Remark 3: For the case where the side information S_R is available at receiver Y_2, it is sufficient to replace Y_2 with (Y_2,S_R) in the corresponding expressions in *Remark 2*.

The following Theorem establishes the capacity region for the two-user MAC with common message and common side information S available non-causally at the transmitters and receiver ($S_R=S$).

Theorem 2: The capacity region of the two-user MAC with common message and common side information S available non-causally at the transmitters and receiver is the closure of the set of all rates that

$$R_{21} \leq I(U_{21};Y_2|S,W_{20},U_{22}) \quad (7\text{-}1)$$

$$R_{22} \leq I(U_{22};Y_2|S,W_{20},U_{21}) \quad (7\text{-}2)$$

$$R_{21}+R_{22} \leq I(U_{21},U_{22};Y_2|S,W_{20}) \quad (7\text{-}3)$$

$$R_{20}+R_{21}+R_{22} \leq I(W_{20},U_{21},U_{22};Y_2|S) \quad (7\text{-}4)$$

for some joint distribution of the form (5).

Proof: The achievability is obtained from *Theorem 1*, Markov chains (6), *Remark 3* and setting $S_R=$ S. For the converse part, we define auxiliary random variables $W_{20,t},U_{21,t},U_{22,t},t=1,\dots,n$ as follows:

$$W_{20,t} \triangleq (m_{20},S^n),\ U_{21,t} \triangleq (m_{21},m_{20},S^n),$$

$$U_{22,t} \triangleq (m_{22},m_{20},S^n).$$

Now, consider

$$nR_{21}=H(m_{21})=H(m_{21}|m_{20},m_{22},S^n) \quad (8\text{-}1)$$

$$=H(m_{21}|m_{20},m_{22},S^n,Y_{2,1}^n)+I(m_{21};Y_{2,1}^n|m_{20},m_{22},S^n)$$

$$\leq n\varepsilon_{1,n}+\sum_{t=1}^{n} I(m_{21};Y_{2,t}|m_{20},m_{22},S^n,Y_{2,1}^{t-1}) \quad (8\text{-}2)$$

$$=n\varepsilon_{1,n}+\sum_{t=1}^{n} H(Y_{2,t}|m_{20},m_{22},S^n,Y_{2,1}^{t-1}) -\sum_{t=1}^{n} H(Y_{2,t}|m_{20},m_{21},m_{22},S^n,Y_{2,1}^{t-1})$$

$$=n\varepsilon_{1,n}+\sum_{t=1}^{n} H(Y_{2,t}|m_{20},m_{22},S^n,Y_{2,1}^{t-1}) -\sum_{t=1}^{n} H(Y_{2,t}|m_{20},m_{21},m_{22},S^n) \quad (8\text{-}3)$$

$$=n\varepsilon_{1,n}+\sum_{t=1}^{n} H(Y_{2,t}|S_t,m_{20},m_{22},S^n,Y_{2,1}^{t-1}) -\sum_{t=1}^{n} H(Y_{2,t}|S_t,m_{20},m_{21},m_{22},S^n)$$

$$\leq n\varepsilon_{1,n}+\sum_{t=1}^{n} H(Y_{2,t}|S_t,W_{20,t},U_{22,t}) -\sum_{t=1}^{n} H(Y_{2,t}|S_t,W_{20,t},U_{21,t},U_{22,t}) \quad (8\text{-}4)$$

$$=n\varepsilon_{1,n}+\sum_{t=1}^{n} I(U_{21,t};Y_{2,t}|S_t,W_{20,t},U_{22,t}),$$

where $\varepsilon_{1,n}\to 0$ as n approaches infinity. Here, (8-1) because m_{21} and (m_{20},m_{22},S_1^n) are independent, (8-2) follows from Fano's inequality, (8-3) follows from the Markov chain $Y_{2,1}^{t-1} \leftrightarrow (m_{20},m_{21},m_{22},S_1^n) \leftrightarrow Y_{2,t}$ and (8-4) leads from the fact that conditioning reduces entropy. Similarly we can obtain:

$$nR_{22} \leq n\varepsilon_{2,n}+\sum_{t=1}^{n} I(U_{22,t};Y_{2,t}|S_t,W_{20,t},U_{21,t}),$$

where $\varepsilon_{2,n}\to 0$ as $n\to\infty$. Now, we bound the sum rate $R_{21}+R_{22}$:

$$n(R_{21}+R_{22})=H(m_{21},m_{22})=H(m_{21},m_{22}|m_{20},S^n)$$

$$=H(m_{21},m_{22}|m_{20},S^n,Y_{2,1}^n)+I(m_{21},m_{22};Y_{2,1}^n|m_{20},S^n)$$

$$\leq n\varepsilon_{3,n}+\sum_{t=1}^{n} I(m_{21},m_{22};Y_{2,t}|m_{20},S^n,Y_{2,1}^{t-1})$$

$$=n\varepsilon_{3,n}+\sum_{t=1}^{n} H(Y_{2,t}|m_{20},S^n,Y_{2,1}^{t-1}) -\sum_{t=1}^{n} H(Y_{2,t}|m_{20},m_{21},m_{22},S^n,Y_{2,1}^{t-1})$$

$$=n\varepsilon_{3,n}+\sum_{t=1}^{n} H(Y_{2,t}|S_t,m_{20},S^n,Y_{2,1}^{t-1}) -\sum_{t=1}^{n} H(Y_{2,t}|S_t,m_{20},S^n,m_{21},m_{22})$$

$$\leq n\varepsilon_{3,n}+\sum_{t=1}^{n} H(Y_{2,t}|S_t,W_{20,t})$$

$$-\sum_{t=1}^{n} H\left(Y_{2,t} \middle| S_t, W_{20,t}, U_{21,t}, U_{22,t}\right)$$

$$= n\varepsilon_{3,n} + \sum_{t=1}^{n} I\left(U_{21,t}, U_{22,t}; Y_{2,t} \middle| S_t, W_{20,t}\right),$$

where $\varepsilon_{3,n} \to 0$ as $n \to \infty$. Finally, we bound the sum rate $R_{20} + R_{21} + R_{22}$:

$$n(R_{20} + R_{21} + R_{22}) = H(m_{20}, m_{21}, m_{22})$$
$$= H(m_{20}, m_{21}, m_{22}|S^n)$$
$$= H\left(m_{20}, m_{21}, m_{22}|S^n, Y_{2,1}^n\right) + I\left(m_{20}, m_{21}, m_{22}; Y_{2,1}^n \middle| S^n\right)$$
$$\leq n\varepsilon_{4,n} + \sum_{t=1}^{n} I\left(m_{20}, m_{21}, m_{22}; Y_{2,t} \middle| S^n, Y_{2,1}^{t-1}\right)$$
$$= n\varepsilon_{4,n} + \sum_{t=1}^{n} H\left(Y_{2,t} \middle| S^n, Y_{2,1}^{t-1}\right) - \sum_{t=1}^{n} H\left(Y_{2,t} \middle| m_{20}, m_{21}, m_{22}, S^n, Y_{2,1}^{t-1}\right)$$
$$= n\varepsilon_{4,n} + \sum_{t=1}^{n} H\left(Y_{2,t} \middle| S_t, S^n, Y_{2,1}^{t-1}\right)$$
$$- \sum_{t=1}^{n} H\left(Y_{2,t} \middle| S_t, m_{20}, m_{21}, m_{22}, S^n\right)$$
$$\leq n\varepsilon_{4,n} + \sum_{t=1}^{n} H\left(Y_{2,t} \middle| S_t\right) - \sum_{t=1}^{n} H\left(Y_{2,t} \middle| S_t, W_{20,t}, U_{21,t}, U_{22,t}\right)$$
$$= n\varepsilon_{4,n} + \sum_{t=1}^{n} I\left(W_{20,t}, U_{21,t}, U_{22,t}; Y_{2,t} \middle| S_t\right),$$

where $\varepsilon_{4,n} \to 0$ as $n \to \infty$. This concludes the proof of the converse part.

IV. CONCLUSION

A wireless network model including interfering broadcast channel and a multiple access channel with cognitive transmitters is considered. The MAC is supposed to have two cognitive transmitters, which know the message transmitted by broadcast transmitter, in a non-causal manner. Using a result of Slepian-Wolf rate region for the multiple access channel and the BC with common message region along with random binning technique, an achievable rate region for this model is derived. The cognition for the second cluster is also assumed to be common side information known non-causally at the MAC transmitters while it is proved that the related achievable region for the MAC cluster is optimal.

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APPENDIX
PROOF OF THEOREM 1

It is sufficient to show that any element of $\mathcal{S}(Z)$ for each $Z \in \mathcal{P}$ is achievable. So, fix $Z = (W_{10}W_{20}U_{11}U_{12}X_1X_{21}X_{22}Y_{11}Y_{12}Y_2)$ and take any $(R_{10}, R_{11}, R_{12}, R_{20}, R_{21}, R_{22})$ satisfying the constraints of the *Theorem 1*.

Codebook generation: Consider $n > 0$ and some distribution of the form (1). We construct the codebook as follows:

1. generate $\lfloor 2^{nR_{10}} \rfloor$ independent codewords $\boldsymbol{w_{10}}(j)$, $j \in \{1, \cdots, \lfloor 2^{nR_{10}} \rfloor\}$ according to $\prod_{i=1}^{n} p(w_{10i})$.
2. For each codeword $\boldsymbol{w_{10}}(j)$, at encoder 1:
   - 2.1 generate $\lfloor 2^{n(R_{11}+R'_{11})} \rfloor$ independent codewords $\boldsymbol{u_{11}}$, according to $\prod_{i=1}^{n} p(u_{11i}|w_{10i})$ and throw them randomly into $\lfloor 2^{nR_{11}} \rfloor$ bins such that the *k-th* sequence $\boldsymbol{u_{11}}, k \in \{1, \cdots, \lfloor 2^{nR'_{11}} \rfloor\}$, in bin b_1 is denoted as $\boldsymbol{u_{11}}(j, b_1, k)$, $b_1 \in \{1, \cdots, \lfloor 2^{nR_{11}} \rfloor\}$.
   - 2.2 generate $\lfloor 2^{n(R_{12}+R'_{12})} \rfloor$ independent codewords $\boldsymbol{u_{12}}$,

according to $\prod_{i=1}^{n} p(u_{12i}|w_{10i})$ and throw them randomly into $\lfloor 2^{nR_{12}} \rfloor$ bins such that the *l-th* sequence $\boldsymbol{u_{12}}, l \in \{1, \cdots, \lfloor 2^{nR'_{12}} \rfloor\}$, in bin b_2 is denoted as $\boldsymbol{u_{12}}(j, b_2, l)$, $b_2 \in \{1, \cdots, \lfloor 2^{nR_{12}} \rfloor\}$..

3. generate $\lfloor 2^{n(R_{20}+R'_{20})} \rfloor$ independent codewords $\boldsymbol{w_{20}}$, according to $\prod_{i=1}^{n} p(w_{20i})$ and throw them randomly into $\lfloor 2^{nR_{20}} \rfloor$ bins such that the *m-th* sequence $\boldsymbol{w_{20}}$ in bin b_3, $m \in \{1, \cdots, 2^{nR'_{20}}\}$, is denoted as $\boldsymbol{w_{20}}(b_3, m)$, $b_3 \in \{1, \cdots, \lfloor 2^{nR_{20}} \rfloor\}$.
4. For each codeword $\boldsymbol{w_{20}}(m)$:
   4.1 at encoder 2, generate $\lfloor 2^{n(R_{21}+R'_{21})} \rfloor$ independent codewords $\boldsymbol{u_{21}}$, according to $\prod_{i=1}^{n} p(x_{21i}|w_{20i})$ and throw them randomly into $\lfloor 2^{nR_{21}} \rfloor$ bins such that the *n-th* sequence $\boldsymbol{u_{21}}, n \in \{1, \cdots, \lfloor 2^{nR'_{21}} \rfloor\}$, in bin b_4 is denoted as $\boldsymbol{u_{21}}(m, b_4, n)$, $b_4 \in \{1, \cdots, \lfloor 2^{nR_{21}} \rfloor\}$.
   4.2 at encoder 3, generate $\lfloor 2^{n(R_{22}+R'_{22})} \rfloor$ independent codewords $\boldsymbol{u_{22}}$, according to $\prod_{i=1}^{n} p(x_{22i}|w_{20i})$ and throw them randomly into $\lfloor 2^{nR_{22}} \rfloor$ bins such that the *o-th* sequence $\boldsymbol{u_{22}}$, $o \in \{1, \cdots, \lfloor 2^{nR'_{22}} \rfloor\}$, in bin b_5 is denoted as $\boldsymbol{u_{22}}(m, b_5, o)$, $b_5 \in \{1, \cdots, \lfloor 2^{nR_{22}} \rfloor\}$.

Encoding & transmission: The aim is to send a six dimensional message $(j, b_1, b_2, b_3, b_4, b_5)$ whose first component j is a message index and last five components b_1, b_2, b_3, b_4 and b_5 are bin indices. The messages actually sent over the $BC - MAC_G$ channel are $\boldsymbol{x_1}, \boldsymbol{x_{21}}$ and $\boldsymbol{x_{22}}$**.** The message and bin indices are mapped into $\boldsymbol{x_1}, \boldsymbol{x_{21}}$ and $\boldsymbol{x_{22}}$, as follows.

The sender TX_1 to send (j, b_1, b_2) first looks for $\boldsymbol{w_{10}}(j)$ and then finds sequences $\boldsymbol{u_{11}}(j,k)$ and $\boldsymbol{u_{12}}(j,l)$ in bin b_1 and bin b_2 respectively, such that $(\boldsymbol{w_{10}}(j), \boldsymbol{u_{11}}(j, b_1, k), \boldsymbol{u_{12}}(j, b_2, l)) \in A_\varepsilon^n$; finally TX_1 generates $\boldsymbol{x_1}$ i.i.d. according to $\prod_{i=1}^{n} p(x_{1i}|w_{10i}u_{11i}u_{12i})$ and sends it.

The cognitive sender TX_{21} knowing $\boldsymbol{w_{10}}(j), \boldsymbol{u_{11}}(j, b_1, k), \boldsymbol{u_{12}}(j, b_2, l)$ non-causally, to send (b_3, b_4) finds a sequence $\boldsymbol{w_{20}}(m)$ in bin b_3 such that $(\boldsymbol{w_{10}}(j), \boldsymbol{u_{11}}(j, b_1, k), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{w_{20}}(b_3, m)) \in A_\varepsilon^n$; and then finds a sequence $\boldsymbol{u_{21}}(m,n)$ in bin b_4 , such that $(\boldsymbol{w_{10}}(j), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{w_{20}}(b_3, m), \boldsymbol{u_{21}}(m, b_4, n)) \in A_\varepsilon^n$; finally TX_{21} generates $\boldsymbol{x_{21}}(m, b_4, n)$ i.i.d. according to $\prod_{i=1}^{n} p(x_{21i}|w_{20i}u_{21i})$ and sends it.

Similarly, the cognitive sender TX_{22} knowing $\boldsymbol{w_{10}}(j), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{u_{12}}(j, b_2, l)$ non-causally, to send (b_3, b_5) finds a sequence $\boldsymbol{w_{20}}(m)$ in bin b_3 such that $(\boldsymbol{w_{10}}(j), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{w_{20}}(b_3, m)) \in A_\varepsilon^n$; and then finds a sequence $\boldsymbol{u_{22}}(m,o)$ in bin b_5 , such that $(\boldsymbol{w_{10}}(j), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{u_{12}}(j, b_2, l), \boldsymbol{w_{20}}(b_3, m), \boldsymbol{u_{22}}(m, b_5, o)) \in A_\varepsilon^n$; finally TX_{22} generates $\boldsymbol{x_{22}}(m, b_4, n)$ i.i.d. according to $\prod_{i=1}^{n} p(x_{22i}|w_{20i}u_{22i})$ and sends it.

Error probability analysis: The messages are decoded based on strong joint typicality as in [12]. Assuming all messages to be equiprobable, we consider the situation where $(j = 1, b_1 = 1, b_2 = 1, b_3 = 1, b_4 = 1, b_5 = 1)$ was sent. First, we consider encoding errors. The probability of encoding error will be negligible if the below binning conditions are held:

$$R'_{11} + R'_{12} \geq I(U_{11}; U_{12}|W_{10}) \quad (9\text{-}1)$$
$$R'_{20} \geq I(W_{20}; W_{10}U_{11}U_{12}) \quad (9\text{-}2)$$
$$R'_{21} \geq I(X_{21}; W_{10}U_{11}U_{12}|W_{20}) \quad (9\text{-}3)$$
$$R'_{22} \geq I(X_{22}; W_{10}U_{11}U_{12}|W_{20}) \quad (9\text{-}4)$$

Finally, consider decoding errors at the decoders. If we assume no encoding errors, with standard techniques of information theory it can be shown that the decoding errors can be avoided if

$$R_{10} \leq I(Y_{11}; W_{10}, U_{11}) \quad (10\text{-}1)$$
$$R_{10} \leq I(Y_{12}; W_{10}, U_{12}) \quad (10\text{-}2)$$
$$R_{11} + R'_{11} \leq I(Y_{11}; U_{11}|W_{10}) \quad (10\text{-}3)$$
$$R_{12} + R'_{12} \leq I(Y_{12}; U_{12}|W_{10}) \quad (10\text{-}4)$$
$$R \quad (10\text{-}5)$$
$$R \quad (10\text{-}6)$$
$$R_{21} + R'_{21} \leq I(Y_2; U_{21}|W_{20}U_{22}) + I(U_{21}; U_{22}|W_{20}) \quad (10\text{-}7)$$
$$R_{22} + R'_{22} \leq I(Y_2; U_{22}|W_{20}U_{21}) + I(U_{22}; U_{21}|W_{20}) \quad (10\text{-}8)$$
$$R_{21} + R'_{21} + R_{22} + R'_{22} \leq I(Y_2; U_{21}U_{22}|W_{20}) + I(U_{21}; U_{22}|W_{20}) \quad (10\text{-}9)$$
$$R_{20} + R'_{20} + R_{21} + R'_{21} + R_{22} + R'_{22} \leq I(Y_2; U_{21}U_{22}W_{20}) + I(U_{21}; U_{22}|W_{20}) \quad (10\text{-}10)$$

First, consider the BC cluster expressions, i.e. (10-1, 6) .The expressions (10-1) and (10-2) are redundant due to (10-5) and (10-6), respectively. Eliminating R'_{11} and R'_{12} with the help of binning conditions and also applying Lemma 1 in [15], we reach to (3-1, 4). About the MAC cluster, replacing the binning conditions, (9-2, 3 and 4) is the all work. However, a little simplification is needed.