Proceedings of the ASME 2010 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference
IDETC/CIE 2010
August 15-18, 2010, Montreal, Quebec, Canada

DETC2010-29177

BEYOND PULL-IN STABILIZATION OF DUAL AXIS MICROMIRRORS USING FUZZY CONTROLLERS

Hamid Moeenfard
School of Mechanical Engineering
Sharif University of Technology, Tehran, Iran

Mohammad Taghi Ahmadian
Center of Excellence in Design, Robotics, and Automation (CEDRA), School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

Ali Soroush
School of Mechanical Engineering
Sharif University of Technology, Tehran, Iran

Aria Alasty
Center of Excellence in Design, Robotics, and Automation (CEDRA), School of Mechanical Engineering, Sharif University of Technology, Tehran, Iran

ABSTRACT
Dual axis micromirrors are actuated using strongly nonlinear electrostatic actuation and their operating range suffers from the pull-in problem. So investigation of their dynamics and control issues has become a challenge for the researchers. The current paper makes use of fuzzy controllers for the purpose of stabilizing the dual axis micromirror at the desired tilt angles beyond pull-in. At first the dynamic model of the micromirror is presented. Then for the purpose of finding the linguistic laws governing the system behavior, several step voltages are introduced to the system. The proposed fuzzy controller consists of singleton fuzzifier, product inference engine and center average defuzzifier. It was observed from the simulation results that the presented controller can effectively and immediately stabilize and control the micromirror tilt angles beyond pull-in, with a short rise time and also a short overshoot.

KEYWORDS: MEMS, Dual axis micromirror, Electrostatic actuation, Fuzzy controller, Nonlinear control.

INTRODUCTION
Technology can touch our daily lives in so many different ways, but the role of miniature devices and systems is not immediately apparent [1]. Technology of micro electro mechanical systems has experienced a lot of progress in testing and fabricating new devices recently. Their low manufacturing cost, batch production, light weight, small size, durability, low energy consumption and compatibility with integrated circuits, makes them even more attractive [2, 3]. Successful MEMS devices rely not only on well developed fabrication technologies, but also on the knowledge of device behavior, based on which a favorable structure of the device can be forged [3]. So simulation of micromachined systems and sensors is becoming increasingly important. Before prototyping a device, one wishes to virtually build the device and predict its behavior. This allows the optimization of various design parameters according to the specifications [4].
Among micro devices which are being fabricated recently, micromirrors have received much attention. The dual-axis micromirror for instance, has promising applications, such as free-space fiber optic switch [5,6], miniaturized projection display [7] and endoscopic optical coherence tomography [8].
There are variety of actuation methods in micro dimension, such as thermal actuation [9], optical actuation [10] and electrostatic actuation [11]. Because of lower power consumption, higher efficiency, simple driving electronics and ease of fabrication and integration, electrostatic actuation is the most popular actuation scheme for the micro device [11]. Electrostatic actuation on the other hand suffers from the well-known pull-in problem. In the pull-in phenomenon, the applied voltage to the micro system is such that the mechanical restoring force is no more capable of restoring the system and as a result, the system collapses.
So far variety of nonlinear control schemes have been applied for the control of MEMS devices. Chu and pister [12], stabilized a microgripper theoretically. Lu and Fedder [13] present a controller design for servoing the position of a parallel-plate electrostatic microactuator beyond its open-loop instability point. Their designed controller, considers nonlinearities from both the parallel-plate actuator and the parallel-plate position sensor, to ensure robust stability within the feedback loop. Nadal et al [14] introduced two different approaches to control charge in the actuator by means of current driving. Their theoretical equations derived for each method showed that full range of travel can be achieved without voltage penalty. Both of their approaches were based on the use of current pulses injecting the required amount of charge to fix the position of the movable plate. Juneau et al [15] describes tilt angle control of a dual-axis optical mirror from the perspective of a fully integrated solution. They present a control solution conducive to integration, followed by

experimental proof of concept using a microprocessor control to emulate on-chip circuits. Yazdi et al [16] present high-resolution control of torsional electrostatic micromirrors beyond their inherent pull-in instability using robust sliding-mode control (SMC). Their presented SMC enables compact realization of a robust controller tolerant of device characteristic variations, non-linearities, and many types of inherent instabilities. They demonstrated robustness of their control loop through extensive simulations and measurements on MEMS with a wide range in their characteristics. They experimentally confirmed control of two-axis gimbaled micromirrors beyond their pull-in instability with overall 14 milli-degrees pointing accuracy. Zhao et al [11] applied a feedback control method, called integral sliding mode control (ISMC) to stabilize the dual-axis micromirror beyond the pull-in point. They formed ISMC by augmenting standard sliding mode control (SMC) [17] with an integrator in the input channel. Their augmented controller could achieve zero steady-state error.

As it can be seen most of control methods used for stabilizing MEMS devices requires accurate knowledge of the system dynamics and the precise effect of the control signal to the overall system output. Fuzzy control on the other hand uses simple linguistic laws to control a dynamic system, no matter simple or complicated.

Fuzzy logic was first proposed by Lotfi A. Zadeh in a 1965 paper [18]. He elaborated on his ideas in a 1973 paper [19] that introduced the concept of linguistic variables.

Fuzzy controllers are very simple conceptually. They consist of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs, such as switches, thumbwheels, and so on, to the appropriate membership functions and truth values. The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value [20]. Because of this simple logical structure, fuzzy controllers have attracted much attention among researchers. For example Itoh[21] Automaticly controlled the motion of protozoa by using the image processing technology and fuzzy control method. Sinha and Lyshevski [22] examined a class of microelectromechanical systems described by nonlinear differential equations with random delays. Then they designed robust fuzzy controllers to control the energy conversion processes with the ultimate objective to guarantee optimal achievable performance.

In this paper we have proposed a fuzzy controller to stabilize the dual axis micromirror. The mentioned fuzzy controller consists of singleton fuzzifier, product inference engine and center average defuzzifier. First of all some step voltages are introduced to the system in order to identify the linguistic laws governing the system behavior. These linguistic laws are used for constructing the required if-then rules which will be used later for constructing the controller. It is observed that the established fuzzy controller is able to control the tilt angles of the micromirror beyond pull-in effectively and immediately. In some specific cases, the accuracy of the proposed model and the presented controller were checked using the published literature.

PROBLEM FORMULATION

Figure (1) shows a schematic view of the dual axis micromirror. Table (1) shows the geometrical parameters shown in figure (1). The dual axis micromirror is a two input, two output (TITO) system. The inputs are two control voltages and the outputs are the tilt angles along the x and y axis, θ_x and θ_y respectively. It should be noted that for moderate bias voltage, the displacement along z axis can be neglected without making any significant error.

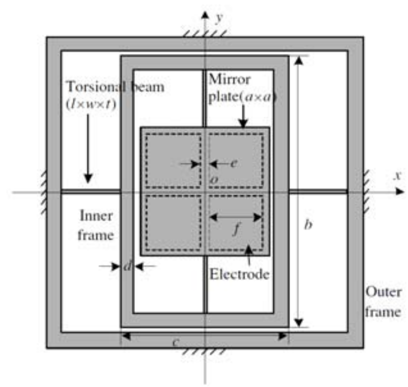

Figure (1): Schematic view of the dual axis micromirror [11].

Table (1): Geometrical parameters of the micromirror shown in figure (1) [11].

Parameter	Symbol	Value (μm)
Torsional beam length	l	180
Torsional beam width	w	2
Torsional beam thickness	t	1.5
Mirror plate size	a	418
Inner frame length	b	882
Inner frame width	c	542
Frame offset	d	52
Distance between two electrodes	e	20
Width of electrodes	f	166
Gap between electrode and mirror	g	68

The dynamic equations of the micromirror are as follows [11].

$$\ddot{\theta}_x + 2\varsigma_x \omega_{nx} \dot{\theta}_x + \omega_{nx}^2 \theta_x = G_x T_x \tag{1}$$

$$\ddot{\theta}_y + 2\varsigma_y \omega_{ny} \dot{\theta}_y + \omega_{ny}^2 \theta_y = G_y T_y \quad (2)$$

In these equations ς_x and ς_y are the damping ratios, ω_{nx} and ω_{ny} are the natural frequencies, G_x and G_y are the gains and T_x and T_y are the electrostatic torques applied in x and y directions respectively. Table (2) shows the numerical values of the parameters of equations (1) and (2).

The electrostatic torques have the following relations with the applied voltages and tilt angles [23].

Table (2): Numerical values of parameters given in equations (1) and (2) [11].

Parameter	Quantity
ς_x	0.19
ς_y	0.04
ω_{nx}	$1633(rad/s)$
ω_{ny}	$3198(rad/s)$
G_x	4.91×10^{15}
G_y	1.52×10^{16}

$$T_x = \frac{1}{2}\varepsilon_0 \sum_{i=1}^{4} (V_b + V_i)^2 \times \iint_{E_i} -y \left(\frac{\sin\alpha}{\alpha} \frac{1}{g - x\cos\theta_x \sin\theta_y + y\sin\theta_x} \right)^2 dxdy \quad (3)$$

$$T_y = \frac{1}{2}\varepsilon_0 \sum_{i=1}^{4} (V_b + V_i)^2 \times \iint_{E_i} x \left(\frac{\sin\alpha}{\alpha} \frac{1}{g - x\cos\theta_x \sin\theta_y + y\sin\theta_x} \right)^2 dxdy \quad (4)$$

In equations (3) and (4), ε_0 is the vacuum permittivity, which is equal to $8.85\times10^{-12}\,F/m$, V_b is the bias voltage, α is the slope between the mirror plate and the substrate which can be stated as equation (5), E_i and V_i are the range of the *i*th electrode on the xy plane shown in figure (1) and the voltage applied to them, respectively [23].

$$\alpha = \cos^{-1}\left(\cos\theta_x \cos\theta_y\right) \quad (5)$$

The applied voltages V_i $(i = 1, 2, 3, 4)$ are generated using a combination of logic gates and amplifier circuits and can be expressed simply as equations (6) to (9) [11].

$$V_1 = 10\left(-V_x + V_y\right) \quad (6)$$

$$V_2 = 10\left(-V_x - V_y\right) \quad (7)$$

$$V_3 = 10\left(V_x - V_y\right) \quad (8)$$

$$V_4 = 10\left(V_x + V_y\right) \quad (9)$$

In these equations, V_x and V_y are the control voltages. In practice, the rotation angles θ_x and θ_y are not measured directly, in turn, a laser beam is radiated to the micromirror and its reflection is detected by an position sensitive detector (PSD) and the tilt angles are calculated using the position of the reflected laser spot on the PSD.

The relation between the position of the reflected laser spot on the PSD and the tilt angles can be computed using equation (10) and (11).

$$x_{PSD} = \left(\cos^2\theta_x \sin^2\theta_y + \sin 2\theta_x \cos\theta_y\right)|d| \quad (10)$$

$$Y_{PSD} = \sqrt{2}\cos\theta_x \sin\theta_y \left(\cos\theta_x \cos\theta_y - \sin\theta_x\right)|d| \quad (11)$$

In these equations $|d|$ is the distance between central points of the micromirror and the PSD which is set equals to $18mm$ [11].

FUZZY CONTROLLER DESIGN

Fuzzy systems are knowledge-based or rule-based systems. The heart of a fuzzy system is a knowledge based consisting of the so-called fuzzy IF-THEN rules. A fuzzy IF-THEN rule is an IF-THEN statement in which some words are characterized by continuous membership functions [24]. In fact fuzzy systems are directly linked to the aspect of representing, handling and utilizing the non-numeric character of information available to the fuzzy controller [25]. These IF-THEN rules should be combined into a single system. Different fuzzy systems use different principles for this combination. Inputs and outputs of fuzzy systems are fuzzy sets which are expressed in natural languages. Since in engineering systems one deals with real valued numbers, fuzzy systems are usually equipped with fuzzifier and defuzzifier. The former transforms the input real valued variable into a fuzzy set and the latter transforms a fuzzy set into the output real valued variable.

In order to control a system using fuzzy controllers, one should know the linguistic laws governing the system behavior. Unfortunately for the system under investigation these rules are not immediately apparent. So in order to identify these rules for this system, one has to introduce several step voltages to the system. Figure (2) shows a typical response of the dual axis micromirror to the inputs $V_x = V_y = 1V$ with a bias voltage of $V_b = 50V$ and characteristics given in table (2).

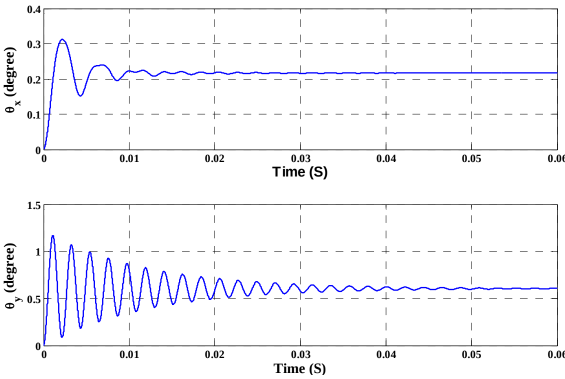

Figure (2): Step response of the dual axis micromirror in x and y directions.

It can be seen that with application of the step input voltages, tilt angles start to vary and will finally stop at some position. This rest angle depends on the applied input voltages. This is how we have derived the rules we were looking for. First the final rest position of tilt angles for different inputs is obtained. Figure (3) shows a typical steady state response of the system to the different step input voltages.

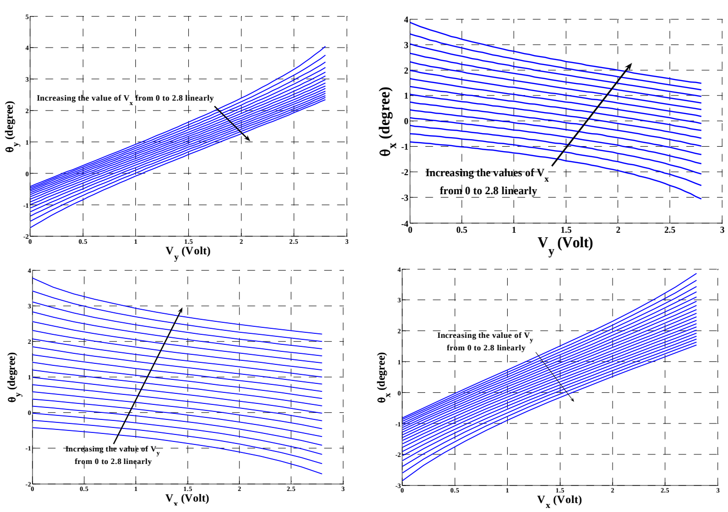

Figure (3): Typical steady state response of the system to different input step voltages.

Figure (3) can be used for generating the required linguistic laws. For example one can conclude that in order to decrees the values of the θ_x and θ_y he or she should decrease both V_x and V_y. Tables (3) and (4) shows the complete set of rules which can be extracted from figure (3).

In these tables, S, M and L denote some fuzzy sets for applied voltages, which indicate small, medium and large respectively. Similarly N, Z and P denote some fuzzy sets which indicates negative, zero and positive respectively with membership function shown in figure (4). In practice these rules are not enough to control such a strongly nonlinear system. The reason is that these rules does not use the state $\dot{\theta}_x$ and $\dot{\theta}_y$. For example when $e_{\theta_x} < 0$ and $e_{\theta_x} < 0$, but both $\dot{\theta}_x$ and $\dot{\theta}_y$ are positive, the system is correcting itself and there is no need to use the voltage to control it. So a more sophisticated set of rules can be obtained using $\dot{\theta}_x$ and $\dot{\theta}_y$. These rules have been shown in appendix A.

Table (3): Set of rules which can be extracted from figure (3) For V_x

V_x		e_{θ_y}		
		N	**Z**	**P**
e_{θ_x}	**N**	S	S	S
	Z	S	M	L
	P	L	L	L

Table (4): Set of rules which can be extracted from figure (3) For V_y

V_y		e_{θ_y}		
		N	**Z**	**P**
e_{θ_x}	**N**	S	S	L
	Z	S	M	L
	P	S	L	L

It should be noted that by choosing the value of D in figure (4) as small as possible, would help the controller to have better control performance, lower steady state error, and fast settling time, however choosing this value very small would produce some kind of chatter in the steady state response . So in the following simulations, the value of D is chosen 0.005° and $1r/s$ for θ and $\dot{\theta}$ respectively.

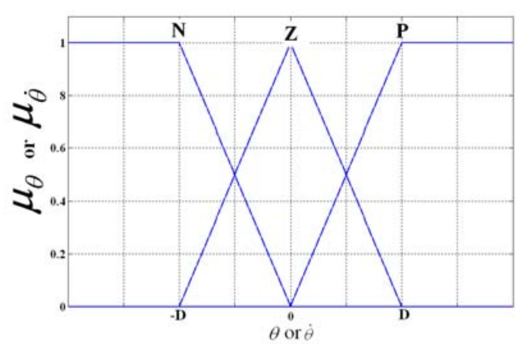

Figure (4): membership functions for fuzzy sets N, Z and P.

Figure (5) shows the membership functions for fuzzy sets A_i $(1 \le i \le 8)$. A_i's are some normal, consistent and complete fuzzy sets with triangular membership functions with the condition $A_i < A_j$ if $i < j$.

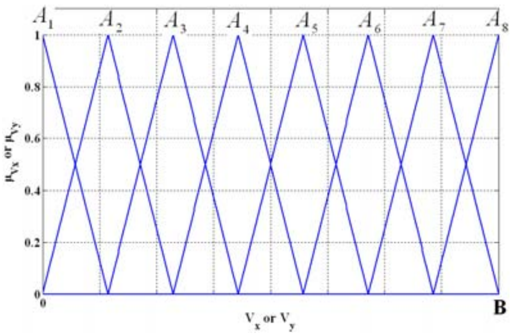

Figure (5): membership functions for fuzzy sets $A_i \quad (1 \le i \le 8)$.

When the desired tilt angles are relatively large, the value of B (i.e. the maximum applicable applied voltage) in figure (5) should be increased.

Using the fuzzy rule base presented in appendix A and membership functions shown in figures (4) and (5), a fuzzy controller is developed using singleton fuzzifier, product inference engine and center average defuzzifier. The control signals of this controller is as:

$$V_x = \frac{\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \mu_{\theta_{x_i}} \mu_{\theta_{y_j}} \mu_{\dot{\theta}_{x_k}} \mu_{\dot{\theta}_{y_l}} \bar{y}_{V_x}^{i\,j\,k\,l}}{\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \mu_{\theta_{x_i}} \mu_{\theta_{y_j}} \mu_{\dot{\theta}_{x_k}} \mu_{\dot{\theta}_{y_l}}} \qquad (10)$$

$$V_y = \frac{\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \mu_{\theta_{x_i}} \mu_{\theta_{y_j}} \mu_{\dot{\theta}_{x_k}} \mu_{\dot{\theta}_{y_l}} \bar{y}_{V_x}^{i\,j\,k\,l}}{\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \mu_{\theta_{x_i}} \mu_{\theta_{y_j}} \mu_{\dot{\theta}_{x_k}} \mu_{\dot{\theta}_{y_l}}} \qquad (11)$$

Where $\bar{y}_V^{i\,j\,k\,l}$ denotes the values of the center of the fuzzy set $A_{27(i-1)+9(j-1)+3(k-1)+l}$. Using this controller, several simulations have been made. In all simulations the initial conditions have been set to zero. Figure (6) shows how the system follows controller command in order to achieve tilt angles $\left(3^\circ, 2^\circ\right)$. Figure (7) shows the corresponding control signals (i.e. V_x and V_y).

Figure (7) shows that controller signals vary at first but finally would achieve a rest value. This is due to the fact that the point $\left(3^\circ, 2^\circ\right)$ is below the pull-in limits of the micromirror. However the outstanding feature of the developed controller is that is can extend the operating range of the micromirror tilt angles beyond pull-in instability of the system. In this case the control voltage won't reach a constant value and would change all the time in order to prevent pull-in and at the same time keep the desired tilt angles. For example figure (8) shows how the controller can stabilize the system at the tilt angles $\left(5^\circ, 5^\circ\right)$ which is beyond the safe operation limits of the micromirror. Figure (9) shows the corresponding control voltages. As it was mentioned, it is observed that although the tilt angles have reached a final relatively constant value, but V_x and V_y varies rapidly in order to keep the desired dynamics.

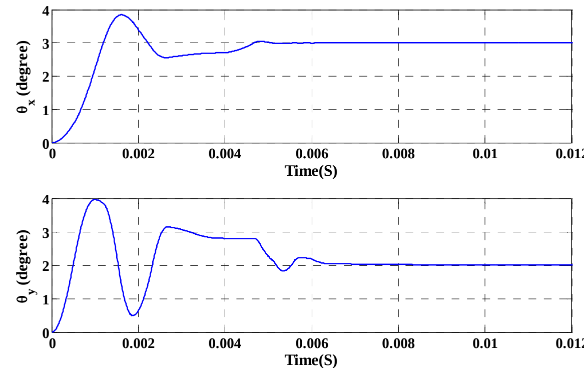

Figure (6): Time history of the controlled system in following the command $\left(3^\circ, 2^\circ\right)$.

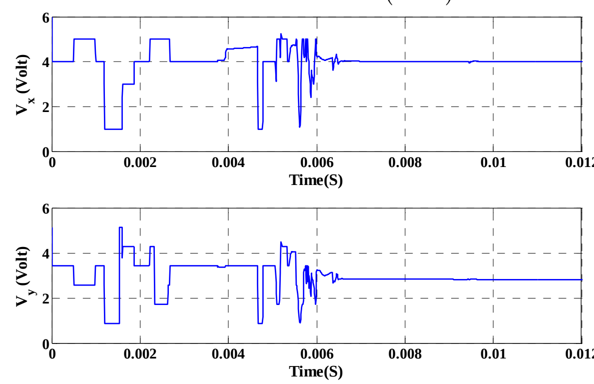

Figure (7): Control signals for following the command $\left(3^\circ, 2^\circ\right)$.

CONCLUSION

Electrostatically actuated dual axis micromirrors have variety of industrial and biomedical applications. But unfortunately their operational range is greatly reduced by the well-known pull-in phenomenon. In this paper, for the first time, a fuzzy controller has been proposed in order to increase the operating range of the micromirror and control tilt angles beyond pull-in. for this purpose, several step voltages were introduced to the micromirror and the response of the system was investigated precisely in order to find the linguistic laws which can be used to control tilt angles. Then using these laws a fuzzy controller was developed using singleton fuzzifier, product inference engine and center average defuzzifier. The performance of the proposed controller was checked at tilt angles below pull-in. It was observed that micromirror equipped with the presented controller can effectively follow the desired commands with a short rise time and settle time and an acceptable overshoot. Then the performance of the controller was checked beyond pull-in. Again it was observed that the controller can full fill the character of a perfect controller and the system would follow the desired command. Results of this paper can be used to optimize the operating range of such systems.

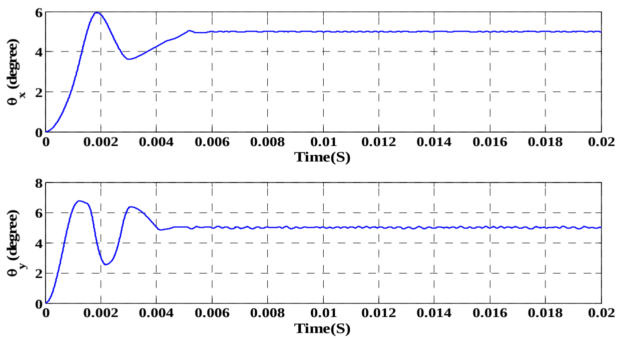

Figure (8): Time history of the controlled system in following the command $(5^{\circ},5^{\circ})$.

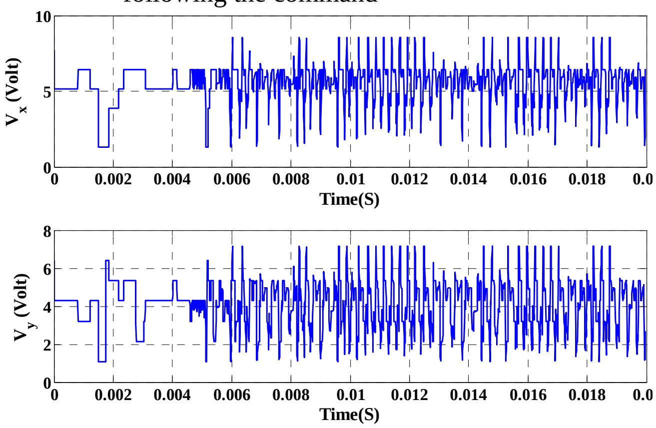

Figure (9): Control signals for following the command $(5^{\circ},5^{\circ})$.

REFERENCES

[1] Maluf, N., Williams, K., 1999, “An Introduction to Microelectromechanical Systems Engineering”, Second Edition, MicroelectromechanicalSystems (MEMS) Series, Boston, London, Artech House Inc.

[2] Younis, M.I, “Modeling and Simulation of Microelectromechanical Systems in Multi-Physics Fields”, Dissertation submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering Mechanics.

[3] Chao, P.C.P, Chiu, C.W. and Tsai, C.Y., 2006, “A novel method to predict the pull-in voltage in a closed form for micro-plates actuated by a distributed electrostatic force”, J. Micromech.Microeng. 16, 986–998.

[4] Beeby, S. Ensell, G., Kraft, M., White, N., 2004, “MEMS Mechanical Sensors”, Microelectromechanical Systems (MEMS) Series, Boston, London, Artech House Inc.

[5] Ford, J.E., Aksyuk, V.A., Bishop, D.J. and Walker, J.A., 1999, “Wavelength add-drop switching using tilt micromirrors”, J. Lightwave Technol.

[6] Lee S.S., Huang L.S., Kim, C.J. and Wu, M.C., 1999, “Free-space fiber optic switches based on MEMS vertical torsion mirrors”, J. Lightwave Technol.

[7] Kiang, M.H., Solgaard, O., Lau, K.Y. and Muller, R., 1998, “Polysilicon optical microscanners for laser scanning displays”, Sensors Actuators A: physical, Volume 70, Issues 1-2, 1 October 1998, Pages 195-199.

[8] Pan Y T, Xie H K and Fedder G K 2001 Endoscopic optical coherence tomography based on a microelectromechanical mirror Opt. Lett. **26** 1966–8.

[9] Henneken, V.A., Tichem, M. and Sarro, P.M.,2006, “In-package MEMS-based thermal actuators for micro assembly”, J. Micromech. Microeng. 16 , S107–S115.

[10] Domachuk, P., Mägi, E., Eggleton, B.J., Cronin-Golomb, M., 2006, “Actuation of cantilevers by optical trapping”, Appl. Phys. Lett. **89**, 071106, doi:10.1063/1.2335983.

[11] Zhao, Y., Tay, F.E.H., Chau, F.S. and Zhou, G., 2006, ” Stabilization of dual-axis micromirrors beyond the pull-in point by integral sliding mode control”, J. Micromech. Microeng. 16, 1242–1250.

[12] Chu, P.B. and Pister, K.S.J., 1994, “Analysis of closed-loop control of parallelplate electrostatic microgrippers “, Proc. IEEE Int. Conf. Robotics and Automation pp 820–5.

[13] Lu, M.S.C. and Fedder, G.K., 2002, “Closed-loop control of parallel-plate microactuator beyond the pull-in limit”, Tech. Dig. IEEE Solid-State Sensor Actuator Workshop (Hilton Head Island, SC) pp 255–8.

[14] Nadal-Guardia R., Dehe, A., Aigner, R. and Castaner, L.M., 2002, “Current drive methods to extend the range of travel of electrostatic microactuators beyond the voltage pull-in point”, J. Microelectromech. Systems, VOL. 11, NO. 3.

[15] Juneau, T., Unterkofler, K., Seliverstov, T., Zhang, S. and Judy, M., 2003, “Dual-axis optical mirror positioning using a nonlinear closed-loop controller”, Proc. 12th Int. Conf. on Solid State Sensors, Actuators and Microsystems (Boston, MA) pp 560–3.

[16] Yazdi, N., Sane, H., Kudrle, T.D. and Mastrangelo, C.H., 2003, “Robust slidingmode control of electrostatic Torsional micromirrors beyond the pull-in limit”, Proc. 12th Int. Conf. on Solid State Sensors, Actuators and Microsystems (Boston, MA)

pp 1450–3.
[17] Ertugrul, M., Kaynak, O., Sabanovic, A. and Ohnishi, K., 1996, "A generalized approach for Lyapunov design of sliding mode. controllers for motion control applications", Proc. Advanced Motion Control Conf. pp 407–12.
[18] Zadeh, L.A., 1965, "Fuzzy Sets", Informat. Control, 8, pp. 338-353.
[19] Zadeh, L.A., 1973, "Outline of a new approach to the Analysis of Complex Systems and Decission Processes", IEEE Trans. On System, Man, and Cybern.,3, 1, pp. 28-44.
[20] http://en.wikipedia.org/wiki/Fuzzy_control_system.
[21] Itoh, A., 1999, "Motion Control of Protozoa for Bio MEMS", Proceedings of the 1999 IEEE/ASME International Conference on Advanced Intelligent Mechatronics, September 19-23, 1999 Atlanta, USA.
[22] Sinha, A.S.C., Lyshevski, S., 2005, "Fuzzy control with random delays using invariant cones and its application to control of energy processes in microelectromechanical motion devices", Energy Conversion and Management , 1305–1318.
[23] Toshiyoshi,H., Piyawattanametha, W., Chan, C.T. and Wu, M.C., 2001, "Linearization of Electrostatically Actuated Surface Micromachined 2-D Optical Scanner", JOURNAL OF MICROELECTROMECHANICAL SYSTEMS, VOL. 10, NO. 2.
[24] Wang, L.X., 1997, "A Course in Fuzzy Systems and Control", Prentice-Hall International, Inc.
[25] Farinwata, S.S., Filev, D., Langari, R., 2000, "Fuzzy Control, Synthesis and Analysis", John Wiley and Sons, LTD.