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A MULTI-OBJECTIVE STRUCTURAL OPTIMIZATION USING OPTIMALITY CRITERIA AND CELLULAR AUTOMATA

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ABSTRACT

This paper is devoted to the simultaneous weight and stiffness optimization of two dimensional structures. The necessary optimality conditions are derived and the obtained optimality criterion is briefly explained. Based on the paradigm of cellular automata, a local rule is constructed which alleviates the well known problems of mesh dependency and checker-boarding in topological structural optimization. It is shown that implementation of this algorithm is useful in prevention of the formation of undesirable members in the resulting layouts. In this approach, contrary to the conventional topological structural optimization methods, the shape and boundaries of the two dimensional continuum are not fixed and can undergo considerable changes during the optimization process. Hence, This approach may be considered as a generalized structural optimization method. To demonstrate the advantages of the method a couple of examples are presented.

Keywords: shape optimization, topology optimization, multi-objective, optimality criteria, cellular automata

1. INTRODUCTION

The goal of the topological structural optimization has conventionally been finding the stiffest structure with a certain given volume of materials [1-3] and the amount of material to be used in the optimization problem is usually chosen by the engineer's intuition. On the other hand, it is obvious that a bigger stiffness is obtained by a larger amount of material. Hence, due to the importance of the weight of the structural material from an economical point of view, it would be more advantageous if one defines an optimization problem which aims minimizing the weight of the structure simultaneously with maximizing the stiffness. This will obviously be a multi-objective optimization problem, where the stiffness and weight are kind of conflicting global functions. In this paper, following Tovar *et al* [4] the multi-objective optimization problem is constructed which combines the weight and

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stiffness of structure in a weighted maner.

Amongst the several optimization methods there are a few which are more suitable for solving a multi-objective optimization problem with a large number of design variables and a relatively few number of constraints, which is the case in the structural topology optimization problem. As some of the most common approaches, the so called approximation methods [5,6], the methods based on moving asymptotes such as CONLIN and MMA [7], and the optimality criteria methods [3,8,9,10,11] can be mentioned. Here, an optimality criteria approach is adopted and a resizing scheme similar to Hassani's [3] is employed.

From the out set, one of the well recognized issues amongst the structural optimization community, due to the inherent interaction between the shape and topology, has been the simultaneous optimal design of the both [2,3,12]. Beside the conventional shape optimization by the boundary variation method, in the last couple of decades, some other methods based on simulation of natural processes have been developed for continuum structural shape optimization. Amongst these methods, the so called simulation of the adaptive growth of trees [3,13] and the cellular automata can be mentioned. Due to its capability to be easily combined with the topology optimization process as a local rule, in this work the latter is employed.

The main idea of the cellular automata approach is mimicking the fact that the evolution of complex biological systems is based on the adaptation to the locally surrounded conditions in an attempt to improve its functionality. This approach has recently been employed for optimal structural design [14-15]. Apart from the possibility of gradual optimization of the shape of a structure together with its topology, due to similarity of its function to the noise cleaning techniques via the convolution method [16], this approach has the capability to circumvent the common instabilities of structural layout optimization such as mesh dependency and checkerboarding [17-19].

2. OPTIMALITY CRITERIA IN MULTI-OBJECTIVE CASE

Most of the real life engineering design optimization problems are multi-criteria. For multi-objective optimization several methods have been developed in the last few years. Amongst them, the utility function method, the inverted utility function method, the hierarchical method, the lexicographic method and fuzzy methods can be mentioned.

A well known intuitive approach to deal with the multi-objective optimization problems is to use a composite objective function, which in its simplest form can be a weighted summation of the objectives. In this work, our goal has been the simultaneous minimization of the strain energy and weight of a structure. The optimization problem can be constructed as [4,20,24]

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \alpha \frac{U}{U_0} + (1-\alpha)\frac{W}{W_0}$$

$$\textit{subject to} \quad \textit{equilibrium}, \qquad (1)$$

$$\textit{design restrictions},$$

where $\mathbf{x}$ is the vector of design variables, f is the composite objective function and U and W are the strain energy and weight of the structure, respectively. U_0 and W_0 are the correspondent strain energy and weight of the solid design domain. The role of the parameter $0 \le \alpha \le 1$ is to balance between the relative strain energy and the relative weight in the compound objective function.

The design variables of the structural topology optimization are geometrical parameters of the assumed material model. For example, a and b in the material model comprising square cells with rectangular holes. See Figure 1.

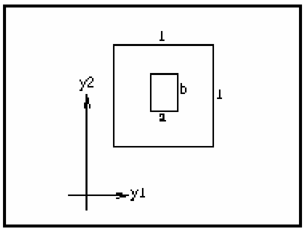


Figure 1. Unit cell with rectangular hole in microscopic coordinates

By using the Lagrange multipliers together with the Kuhn-Tucker conditions, the necessary conditions for optimality in the problem (1) are obtained. Adopting the finite element method for the analysis, we assume that the design variables are constant parameters inside each finite element. Therefore, (1) for a discretized problem can be written as

$$\min_{a^e, b^e \ (e=1,\dots,N)} \quad f(\mathbf{x}) = \alpha \frac{U}{U_0} + (1-\alpha)\frac{W}{W_0}$$

$$\textit{subject to} \quad \textit{equilibrium}, \qquad (2)$$

$$\textit{and} \quad a^e - 1 \le 0 \ , \quad -a^e \le 0 \quad e = 1,\dots,N$$

$$b^e - 1 \le 0 \ , \quad -b^e \le 0 \quad e = 1,\dots,N$$

where N is the number of finite elements in the discretized domain.

The Lagrangian function ℓ associated with the problem above can be constructed as

$$\ell = f(\mathbf{x}) + \lambda_{a0}^{e}(-a^{e} + s_{1}^{e}) + \lambda_{a1}^{e}(a^{e} - 1 + s_{2}^{e}) + \lambda_{b0}^{e}(-b^{e} + s_{3}^{e}) + \lambda_{b1}^{e}(b^{e} - 1 + s_{4}^{e}) \tag{3}$$

where λ_{a0}^{e}, λ_{a1}^{e}, λ_{b0}^{e} and λ_{b1}^{e} are the Lagrange multipliers and s_{i}^{e} are the slack variables.

Stationary of ℓ with respect to a requires that

$$\frac{\partial \ell}{\partial a^{e}} = \frac{\partial f(\mathbf{x})}{\partial a^{e}} - \lambda_{a0}^{e} + \lambda_{a1}^{e} = 0 \tag{4}$$

where

$$\frac{\partial f(\mathbf{x})}{\partial a^{e}} = \frac{\alpha}{U_{0}} \times \frac{\partial U^{e}}{\partial a^{e}} + \frac{(1-\alpha)}{W_{0}} \times \frac{\partial W^{e}}{\partial a^{e}} \tag{5}$$

Since a^{e}, b^{e} are constant through each finite element,

$$W^{e} = (1 - a^{e}b^{e})w_{0}^{e} \tag{6}$$

Therefore

$$\frac{\partial W^{e}}{\partial a^{e}} = -b^{e}w_{0}^{e} \tag{7}$$

By substituting $\frac{\partial f(\mathbf{x})}{\partial a^{e}}$ from (5) into (4), it follows that

$$\frac{\partial \ell}{\partial a^{e}} = \frac{\alpha}{U_{0}} \times \frac{\partial U^{e}}{\partial a^{e}} + \frac{1-\alpha}{W_{0}}(-bw_{0}^{e}) - \lambda_{a0}^{e} + \lambda_{a1}^{e} = 0 \tag{8}$$

By some extra manipulation, the above optimal condition can be written as

$$\frac{\partial U^{e}}{\partial a^{e}} + \frac{(1-\alpha)U_{0}}{\alpha W_{0}}(-bw_{0}^{e}) - \frac{U_{0}}{\alpha}\lambda_{a0}^{e} + \frac{U_{0}}{\alpha}\lambda_{a1}^{e} = 0\,. \tag{9}$$

We assume that E_{a}^{e} is defined as

$$\mathrm{E}_{\mathrm{a}}^{\mathrm{e}} = \partial \mathrm{U}^{\mathrm{e}} / \partial \mathrm{a}^{\mathrm{e}} = \frac{1}{2}\int_{\mathrm{V}^{\mathrm{e}}} \varepsilon^{\mathrm{T}}(\mathrm{u})\left(\frac{\partial \mathrm{C}}{\partial \mathrm{a}^{\mathrm{e}}}\right)\varepsilon(\mathrm{u})\,\mathrm{dV} \tag{10}$$

where $\boldsymbol{\varepsilon}$ is the strain tensor and $\mathbf{C}$ is the constitutive matrix. Also, we define H as

$$H = \frac{(1-\alpha)U_0}{\alpha W_0}(bw_0^e) \tag{11}$$

From equations (9), (10) and (11) it can be followed that

$$E_a^e = H + \frac{U_0}{\alpha}\lambda_{a0}^e - \frac{U_0}{\alpha}\lambda_{a1}^e \tag{12}$$

Let us imagine that in an iteration k, the design variable a_k^e has been decreased in order to move towards optimum point. Therefore $a_k^e < 1$ and the upper side limit is not active, which yields $\lambda_{a1}^e = 0$. Thus noticing that H is a positive real number and $\lambda_{a0}^e \geq 0$, from above equation it follows that $E_a^e \geq H$. On the other hand for increasing a_k^e we will get $E_a^e \leq H$. So inspired by this argument, we calculate the value of E_a^e and compare it with H. If $E_a^e \geq H$ then we let a_k^e decrease by the move limit ζ (i.e. $a_{k+1}^e = a_k^e(1-\zeta)$) and if $E_a^e < H$, then $a_{k+1}^e = a_k^e(1+\zeta)$. Therefore

$$\begin{aligned} &\text{if} \quad E_a^e < H \;\rightarrow\; a_{k+1}^e = a_k^e(1+\zeta) \\ &\text{if} \quad E_a^e > H \;\rightarrow\; a_{k+1}^e = a_k^e(1-\zeta) \end{aligned} \tag{13}$$

In order to have a smoothly convergent algorithm, the move limit needs to be adjusted according to the values of the design variables in each iteration. For this purpose, the Hassani's resizing scheme [3] was employed. This scheme allows having larger values for the move limit when the design variables are close to their constraint boundaries. For the problem at hand, considering one of the void parameters of a typical element *e* at step *k*, for example a_k^e, the resizing scheme can be constructed as follows

$$a_{k+1}^e = \begin{cases} \min\left\{(1+\frac{\zeta}{\left|a_k^e-\zeta\right|})a_k^e, 1)\right\} & if \quad a_k^e(E_a^e)_k \leq \max\{(H-\zeta)a_k^e, 0\} \\ a_k^e[(E_a^e)_k]^{-1/a_k^e} & if \quad \max\{(H-\zeta)a_k^e, 0\} < a_k^e(E_a^e)_k < \min\{(H+\zeta)a_k^e, 1\} \\ \max\left\{(1-\frac{\zeta}{\left|a_k^e-\zeta\right|})a_k^e, 0)\right\} & if \quad \min\{(H+\zeta)a_k^e, 1\} \leq a_k^e(E_a^e)_k \end{cases}$$

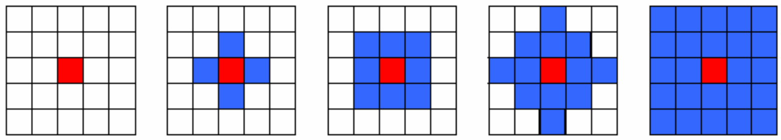
(14)

The other void parameter b can be updated in a similar fashion.

3. CELLULAR AUTOMATA

Inspired by the function of biological phenomena, the behavior of a part of a discrete space is only dependent on the behavior of its neighborhoods. Regarding this phenomenon, cellular automata (CA) was introduced as a method for describing as well as simulation of complex systems. This method was used as early as 1946 by Weiner and Rosenblunth to describe the operation of the heart muscle [22] and John Von Neumann made it formal at the end of 1940s [23].

Cellular automata models are composed of a regular lattice of cells. The neighborhoods of a cell can be specified by a radius $r_{\min}$ of action of a local rule. Some common neighborhood layouts are illustrated in Figure 2. The Von-Neumann layout and the Moore layout are used more than the others [21].



(a) (b) (c) (d) (e)

Figure 2. Cellular automata neighborhoods: (a) Empty, (b) Von Neumann, (c) Moore, (d) MvonN, (e) Extended.

As a general rule, in the cellular automata method, only the boundary cells can be modified. In this paper, the elements that violate the inequality (15) are regarded as boundary elements or cells. This condition is indeed an index of the difference between the average density of the neighbourhoods of a cell and the cell itself.

$$\left| \frac{\sum_{j=1}^{N}(1-a_j b_j)}{N} - (1-a_i b_i) \right| \leq \varepsilon \tag{15}$$

where N is the number of neighborhoods, a_j, b_j are design variables related to neighborhoods of the cell i and ε is a small number.

In this method, the density of each element is changed according to information from its neighborhood. To achieve this, and in order to keep the stability of the solution, the relative strain energies of elements are modified during the optimization process [16,21]. The modified strain energy of element e can be defined as follows

$$\hat{U}_e = \frac{\sum_{i=1}^{N+1} \hat{H}_i U_i}{\sum_{i=1}^{N+1} \hat{H}_i} \tag{16}$$

Where $\hat{H}_i$ is

$$\hat{H}_i = V_i [r_{\min} - r(e,i)] \quad i \in \{1,2,...,N+1\} \tag{17}$$

and V_i is the volume of the element i and $r(e,i)$ is the distance between the centers of elements e and i. N is the number of elements that satisfy $r(e,i) \le r_{\min}$.

As it was mentioned, the cellular automata technique, similar to the convolution based methods proposed by Sigmund [16], would be useful to alleviate the problem of mesh dependency and checker-boarding and to obtain more practical results [19, 25].

4. EXAMPLES

In this section, by using the developed topology optimization research code, given the name TOPS (Topology Optimization of Structures) [26], two examples are here presented. TOPS is a structural topology optimization program based on the optimaliy criteria methods which allows the user to have different options for using finite element types, noise cleaning technique and the continuation method. The TOPS code has now been further improved to suit for the case of multi-objective function together with the cellular automata technique which is the subject of this paper.

Example 1. The problem definition is illustrated in Figure 3. To discretize the design domain 3200 linear finite elements with sizes of 0.01×0.01, are used. The modulus of elesticity and the poisson's ratio are chosen to be 7.9E10 and 0.30, respectively. The initial solid material fraction is $V_{solid}/V = 70\%$. It is assumed that initially the material is distributed uniformly. The weight parameter α is considered to be 0.05.

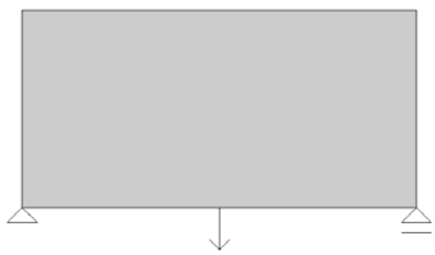


Figure 3. Design domain, natural and essential boundary conditions of example 1 (Michel-Type structure)

By using the isotropic material model and the contiuation method with the penalty exponent varying from $\mu = 3 \ to\ 5$, without using the cellular automata local rule, the obtained optimum result is illustrated in Figure 4.a. The parameter determining the minimum allowable size was taken $r_{\min} = 0.021$. For the sake of comparison, this problem was also solved by using the algorithm based on cellular automata. The obtained result is depicted in Figure 4.b. In this example, the MvonN neighborhoods were employed, and for the recognition of the boundary elements Equation (15) was used with $\varepsilon = 1e^{-4}$. The other optimization parameters are exactly similar to those of Figure 4.a.

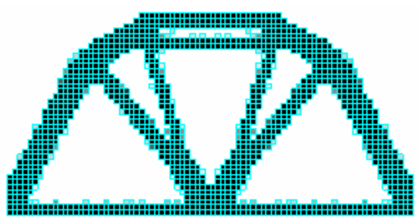
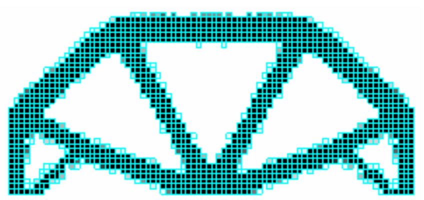
	
(a)	(b)

Figure 4. Optimal topologies by considering $\alpha = 0.05$ (a) without using cellular automata, (b) by using cellular automata with MvonN neighborhoods

The amount of the consumed volume of material in Figures 4.a and 4.b are 35 and 34 percent of total volume of design domain, respectively. As it is noticed, the weigths of the obtained structures are almost the same and by using the cellular automata approach only about 3% weight reduction is gained. However, the effect of the implementaion of the CA method on the material distribution, i.e. the topology as well as the shape of the resulted layout, can be easily noticed. The variation of the strain energies during the optimization process is shown in Figure 5. The fluctuation of the strain enrgies in the CA method, is related to balance between the relative strain energy and the relative weight in the compound objective function during the iterations of optimization algorithm.

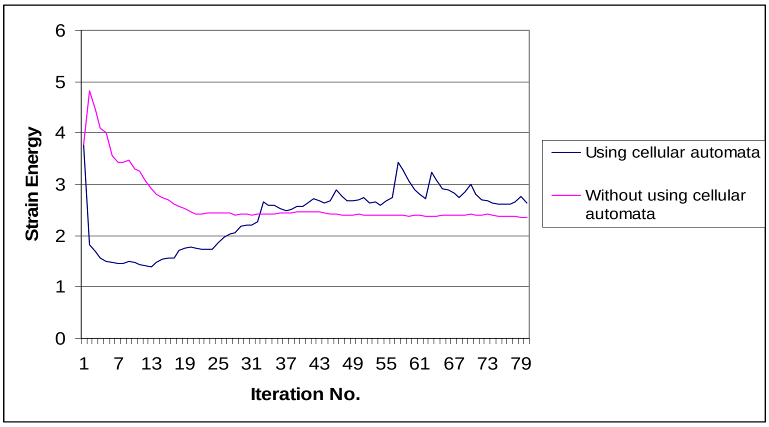


Figure 5. Variation of strain energy during the optimization process

It seems that, in this example, the optimal layout obtained by employing the cellular automata technique is of a more practical nature. The resulted topology and shape may be interpreted as Figure 6 for a CAD model.

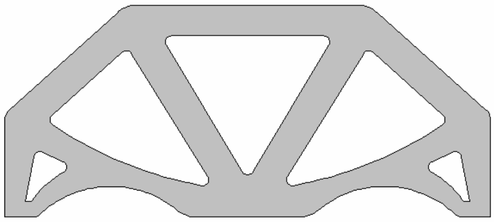
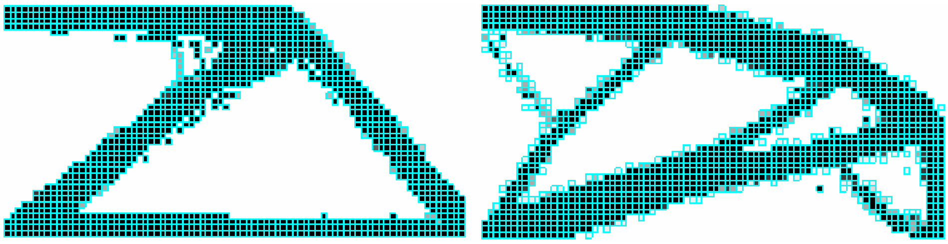


Figure 6. CAD model of optimum layout by using the cellular automata technique

Example 2. In the second example, topological optimization of a cantilever beam, illustrated in Figure 7, is considered. The design domain dimensions, the finite element mesh and the material properties are assumed similar to example 1. The initial solid material fraction is $V_{solid}/V = 80\%$. It is assumed that initially the material is distributed uniformly. The weight parameter α is considered to be 0.1.

Figure 7. Problem definition of cantilever beam of Example 2.

The optimum layouts obtained without and with using cellular automata, both with $r_{\min} = 0.021$, are shown in Figures 8.a and 8.b, respectively.



(a) (b)

Figure 8. Optimum layouts (a) without using cellular automata but with noise cleaning, (b) by using cellular automata with MvonN neighborhoods

In this example, the weight of the structure of Figure 8.b is about 14% more than that of the structure obtained without using cellular automata. It is also noticed that the topology and shape of the resulted layouts are totally different. The resulting layout of the integrated topology and shape optimization approach may be simulated by CAD software as illustrated in Figure 9.

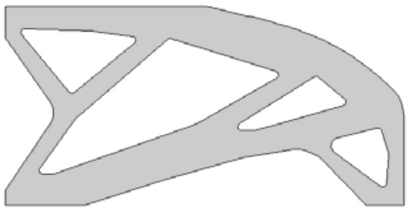


Figure 9. CAD model of the obtained optimum layout by using cellular automata technique

5. CONCLUSIONS

The effect of the implementation of the cellular automata technique together with the optimality criteria, which was derived from a generalized formulation of a multi-objective function, on the optimum layout was investigated. Regarding the fact that by employing the cellular automata as a local rule, alongside with the topology, shape of the structure can also vary during the optimization process, a relatively good improvement in the obtained layouts is expected. In the carried out experiences, in spite of using an improved resizing scheme, this didn't occur all the time. However, in most of the cases, the current method resulted in layouts of a more practical nature which obviously has its importance and attraction in the industrial community. From a theoretical point of view, this problem returns to the question of the relationship between the optimum layouts obtained from the minimization of the strain energy and weight and the reason for their similarities which is open to further research.

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