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PREVENTION OF UNDESIRABLE STRUCTURAL ELEMENTS IN THREE DIMENSIONAL TOPOLOGY OPTIMIZATION

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ABSTRACT

In this paper, the topological optimization of three-dimensional structures based on optimality criteria and artificial material model is briefly reviewed and prevention of undesirable structural elements is discussed. Prevention of formation of checkerboard patterns and the finite element mesh dependency, which are the two well-known instabilities in the process of structural topology optimization, is also considered. The idea of employing the noise cleaning techniques borrowed from image processing techniques, suggested by Sigmund for plane problems [1], is here extended to three-dimensional structures. To demonstrate the efficiency and practicality of the method, especially, when employed together with the continuation method [2] and non-conforming finite elements, some illustrative examples are presented.

Keywords: topology optimization, noise cleaning, non-conforming elements.

1. INTRODUCTION

The result of the structural topology optimization procedure is, indeed, an array of density of the material in the finite elements used for discretization of the problem [2]. From a practical design point of view, the obtained layout is not often completely satisfactory. Apart from having jagged boundaries, some elements may have intermediate densities which, for instance, will be depicted as gray elements in a black and white density contour graph. Also, in some problems, especially when lower order finite elements are used, a checkerboarding pattern occurs. These phenomena are not desirable for a design engineer. These problems, which together with the problem of mesh dependency are usually referred to as solution instabilities, can be regarded as undesirable noises. The problem of instabilities in topology optimization solution, is addressed by a few researchers [1,3,4].

The remedy suggested in this paper is based on borrowing some ideas from the relatively well-developed image processing and noise cleaning techniques and implementation of them within the optimization algorithm. This idea suggested by Sigmund in 1994 [1] for two dimensional plane problems and here it is extended to three-dimensional structures.

The optimization algorithm, due to nature of the problems, i.e. having a large number of design variables and only one global volume constraint, is constructed based on the optimality criteria methods [2] which have proven to be very efficient. Also, the artificial material models are adopted, mainly due to their simplicity and the fact that, when appropriate parameters are considered, they result in layouts of a more practical nature.

Furthermore, the effect of making use of non-conforming incompatible elements on the resulting layout is studied.

2. TOPOLOGY OPTIMIZATION PROBLEM

The problem at hand is defined as finding the stiffest possible structure when a certain amount of material is given. A structure with maximum global stiffness provides a minimum for the external work with the real displacement field or minimum mean compliance. Since, minimization of mean compliance is equivalent to the maximization of the total potential energy [2], the topology optimization problem can be constructed as below

$$\begin{array}{ll} \max & \min\ \Pi(u) \\ \text{subject to} & V_s \le \overline{V}_s \\ \text{design restrictions} & \end{array} \tag{1}$$

where $\mathbf{u}$ is displacement field, Π is total potential energy and $\overline{V}_s$ is the amount of material available. V_s is the volume of solid material in each design. One should note that minimization of $\Pi(\mathbf{u})$ in (1) is equivalent to satisfying the state equations or equilibrium.

In structural topology optimization, the problem is how to distribute the material in order to minimize the objective function. In other words, the goal can be thought of as determination of the optimal spatial material distribution. Using a cellular body with a periodic microstructure is proven to be quite useful and efficient. This relaxes the governing variational problem and changes the on-off (material-no material) nature of the problem and therefore does not require use of discrete optimization algorithms.

One of the most commonly used material models is a unit cubic cell with a rectangular hole located in the center of it as illustrated in Figure 1.

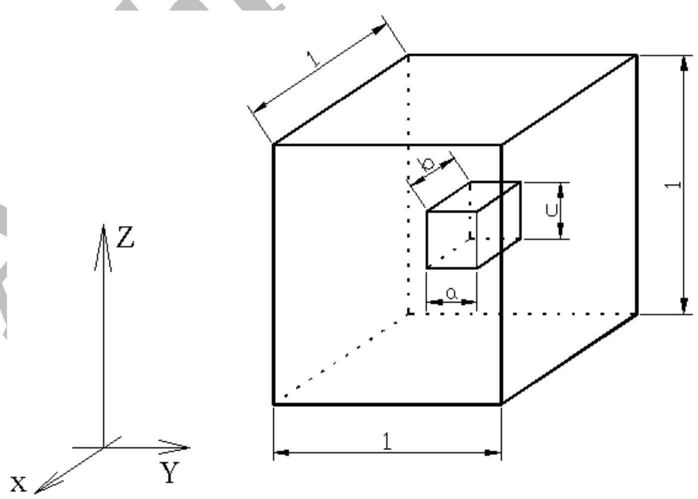


Figure 1 Unit cubic cell with a rectangular hole

The homogenization method can be use to find the elements of the elasticity matrix for such a material model. If the artificial material model is adopted, which is the case in this paper, the artificial elasticity matrix can be written as

$$C=\frac{E(1-abc)^{\mu}}{(1+\nu)(1-2\nu)}\begin{bmatrix}1-\nu & \nu & \nu & 0 & 0 & 0\\ & 1-\nu & \nu & 0 & 0 & 0\\ & & 1-\nu & 0 & 0 & 0\\ & & & (1-2\nu)/2 & 0 & 0\\ & \text{sym.} & & & (1-2\nu)/2 & 0\\ & & & & & (1-2\nu)/2\end{bmatrix} \tag{2}$$

where E is the young's modulus for the solid isotropic material, ν is the poisson's ratio and a, b and c are the dimensions of the void. μ is a penalty exponent to suppress the gray areas in the resulted layout. It is assumed that each finite element is comprised of such cellular material and the dimensions of voids (a, b and c) within the unit cubic cells of different finite elements are the design variables of optimization problem. Therefore, in its discretized sense, the optimization problem (1) can be written as

$$\begin{array}{lll}\max\limits_{a^e,b^e,c^e} & \min\limits_{(e=1,\dots,N)} \Pi(u) & \\ \text{subject to} & \sum\limits_{e=1}^{N}(1-a^eb^ec^e)V^e-\bar{V}_s\le 0 & \\ \text{and} & a^e-1\le 0\;,\quad -a^e\le 0 & e=1,\dots,N\\ & b^e-1\le 0\;,\quad -b^e\le 0 & e=1,\dots,N\\ & c^e-1\le 0\;,\quad -c^e\le 0 & e=1,\dots,N\end{array} \tag{3}$$

where N denotes the number of elements in the finite element mesh.

The total potential energy function $\Pi(\mathbf{u})$, after discretization of the domain of problem can be written as

$$\Pi(u)=\frac{1}{2}\sum_{e=1}^{N}\int_{V^e}\varepsilon^T(u)C^e\varepsilon(u)\,dV-\sum_{e=1}^{N}\int_{V^e}u^Tf\,dV-\sum_{e=1}^{N}\int_{\Omega_t^e}u^Tt\,d\Omega \tag{4}$$

where $\boldsymbol{\varepsilon}$ denotes strains, $\mathbf{f}$ are body forces, $\mathbf{t}$ are traction forces and V^e is the volume of element e.

Since the number of design variables in the optimization problem (3) is proportional to the number of elements in the finite element mesh, using the mathematical programming methods due to having a large number of design variables is somewhat impractical. Therefore an optimality criteria method is adopted. In this method by using the Lagrange multipliers and the Kuhn-Tucker conditions, the necessary conditions for optimality are obtained. The interested reader is suggested to consult references [1, 2] for more details.

A proper updating scheme for this problem can be obtained by extending the procedure suggested by Hassani [2] into the three-dimensional case. Considering one of the void parameters of a typical element e at step k, for example a_k^e, the following resizing scheme will be obtained.

$$a_{k+1}^e = \begin{cases} \min\left\{(1+\dfrac{\zeta}{\left|a_k^e-\zeta\right|})a_k^e, 1)\right\} & \text{if} \quad a_k^e(E_a^e)_k \le \max\{(1-\zeta)a_k^e, 0\} \\ a_k^e[(E_a^e)_k]^{-1/a_k^e} & \text{if} \quad \max\{(1-\zeta)a_k^e, 0\} < a_k^e(E_a^e)_k < \min\{(1+\zeta)a_k^e, 1\} \\ \text{if} \quad \min\{(1+\zeta)a_k^e, 1\} \le a_k^e(E_a^e)_k & \end{cases} \tag{5}$$

where ζ is the move limit and E_a^e is defined as

$$E_a^e = \frac{\frac{1}{2}\int_{V^e} \varepsilon^T(u)(\frac{\partial C}{\partial a^e})\varepsilon(u)\,dV - \int_{V^e} u^T(\frac{\partial f}{\partial a^e})\,dV}{-\Lambda b^e c^e V^e} \tag{6}$$

In (6), Λ denotes the Lagrange multiplier for the volume constraint. To determine Λ in each iteration the conventional bisection method can be used [2]. Alternatively, one may use the Newton-Raphson iterative procedure for this purpose.

The other void parameters b and c can be update in a similar fashion.

3. NOISE CLEANING

As mentioned before, the checkerboarding and mesh-dependency problems are not desirable from a practical point of view and are generally referred to as instabilities of the procedure. Furthermore, sometimes creation of relatively tiny members in the optimum layout, especially when relatively fine meshes are used to solve real life problems, may be considered undesirable, and hence, needs to be treated as noises. This problem is illustrated in Figure 2 where, for example, the small diagonal elements near the supports may be regarded as unwanted.

To prevent the formation of such undesirable structural members making use of noise cleaning techniques was suggested by Sigmund for two-dimensional plane problems [1]. In image processing techniques an image can be defined as a light intensity function. In the structural topology optimization problem, the discretized design domain may also be looked at as an image where each finite element resembles a pixel in an ordinary image. The density of material comprising any finite element may be regarded as the light intensity in a gray scale image, where white is equivalent to a complete void and black is solid material. In this respect, the formation of undesirable elements can be controlled by implementing the noise filtering techniques within the optimization process.

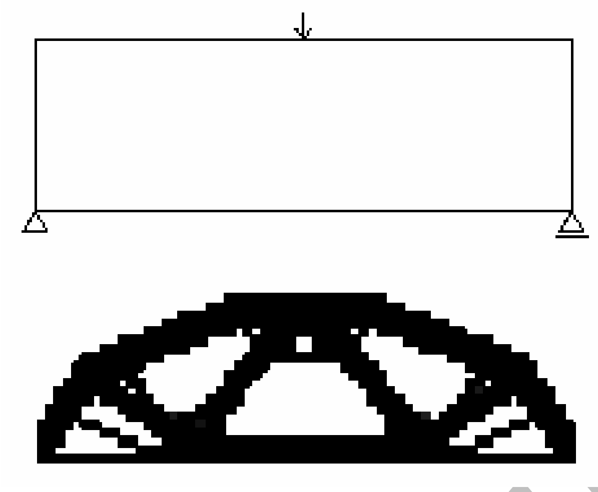


Figure 2 Design domain and optimal layout of a 2D beam

The noise cleaning techniques can be divided into linear and non-linear. In general, both of these techniques may be used for topological structural optimization. However, since these techniques should not destroy the smoothness of design problem [2], employing linear techniques are preferred. Linear noise cleaning techniques can be divided into two main approaches: Fourier transformation based techniques and convolution techniques. Fourier transformation based methods have the disadvantage that they can only be applied to regular rectangular finite element meshes [1,2]. Hence, the convolution technique is here employed.

In the convolution based methods the density of each element is changed according to information from its neighborhood. Various convolution methods differ from each other based on the employed convolution filter which is chosen according to the purpose of filter.

To solve the problem of mesh dependency and in order to obtain more practical results, Sigmund has extended the convolution method to prevent the creation of elements with dimensions less than a given size r_{min} in the optimum layout. In three dimensional structural topology optimization the convolution filter is defined as

$$H_i = V_i\left[r_{min} - r(e,i)\right] \qquad i \in \{1, 2, ..., n_H^e\} \tag{7}$$

where V_i is the volume of the element i and r(e,i) is the distance between the centers of elements e and i. n_H^e is the number of elements that satisfy $r(e,i) \le r_{min}$.

In this case when the artificial material is used, the strain energy density of an element e ω_e may be modified to:

$$\omega_e = \frac{\sum_{i=1}^{n_H^e} H_i \omega_i}{\sum_{i=1}^{n_H^e} H_i} \tag{8}$$

By employing the convolution noise cleaning technique, especially together with the continuation method [2], within the topology optimization algorithm very interesting results in two dimensional problems are reported by Sigmund [1,3]. The results were totally mesh independent and checkerboard free. By using this method, the undesirable elements that have been shown in Figure 2 are removed. The results are shown in Figure 3.

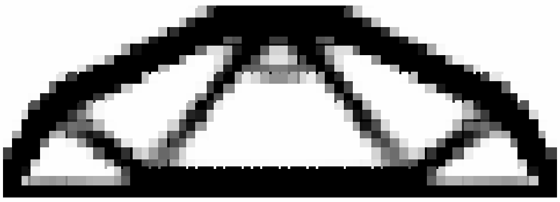


Figure 3 Elimination of undesirable elements in optimal layout of the 2D beam of Figure 2, by employing the noise cleaning method

4. EFFECT OF NON-CONFORMING FINITE ELEMENTS

In order to increase the accuracy of a finite element analysis usually elements of higher order, either standard or hierarchical, are used. However, this will affect the computational cost of the analysis quite dramatically. The non-conforming finite elements are lower-order elements which relieve the problem of the over stiffness of the linear elements, especially in pure bending [5,6]. To formulate these elements some appropriate incompatible displacement modes, which may be chosen by hierarchical shape functions [7], can be added to the linear approximation polynomials over each element [6].

In the structural topology optimization problem, in order to have an appropriate and accurate optimum layout, usually several finite elements are used for discretizing the design domain. On the other hand, using higher order finite elements, especially in three dimensional problems, is very time consuming and costly. Therefore using the non-conforming elements is plausible and quite common. Although, by employing the non-conforming elements the checkerboarding is not completely removed, however, by using these elements together with the noise cleaning techniques, good results, in comparison with the linear elements, are obtained. This will be demonstrated in the third example of section 5.

5. EXAMPLES

In this section, by using the developed two and three dimensional topology optimization code, given the name TOPS (Topology OPtimization of Structures) [8], three examples are peresented. TOPS is a structural topology optimization program based on the optimality criteria methods which allows the user to choose different element types, noise cleaning technique and the continuation method.

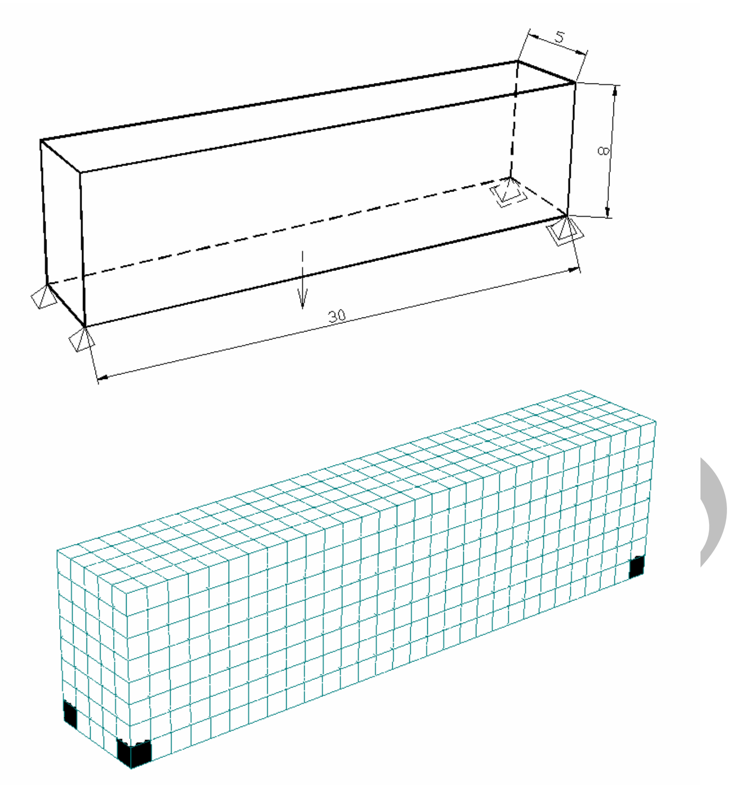


Figure 4 Design domain and finite elemet mesh of Example 1

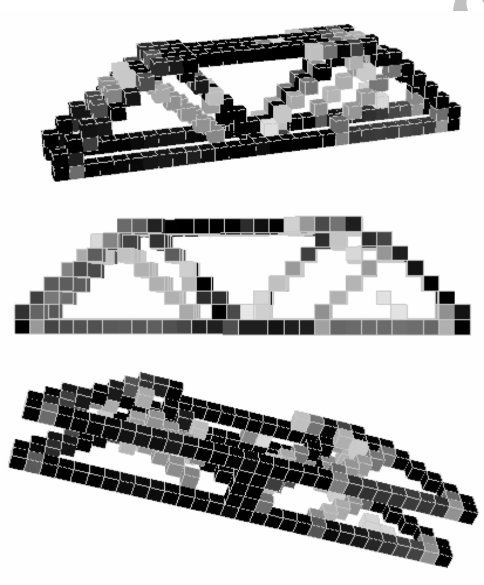


Figure 5 Optimum layout without noise cleaning

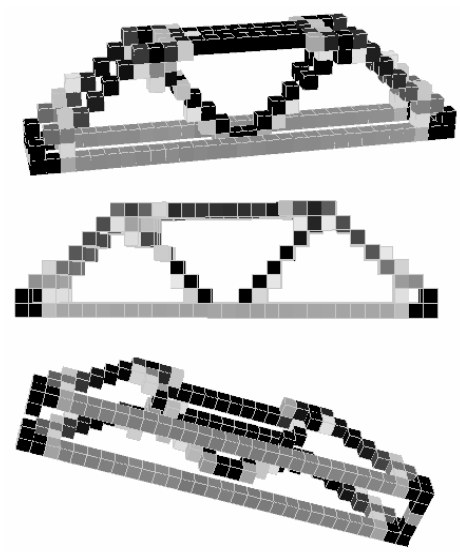


Figure 6 Optimum layout with noise cleaning

Example 1. The problem definition is illustrated in Figure 4. To descretize the reference domain 1200 eight-node finite elements are used. The volume fraction of solid isotropic material is assumed as $V_{solid} / V = 10\%$. The modulus of elesticity and the poisson's ratio are chosen to be 2.1e6 and 0.25, respectively.

By using the isotropic material model with the penalty exponent and without employing noise cleaning technique the layout of Figure 5, is obtained which from practical point of view is not quite desirable. When the noise cleaning method, considering, is used, the layout of Figure 6 is obtained. As can be observed, in this layout the undesirable elements are eliminated. The graph of varations of the strain energy throughout optimization iterations is illustrated in Figure 7.

It is noted that when the noise cleaning method is used, as it is expected, the final value obtained for the minimum strain energy is a little bit higher. Fortunately, the difference is quite negligible and employing the noise cleaning methods does not drastically affect the optimum value of the objective function.

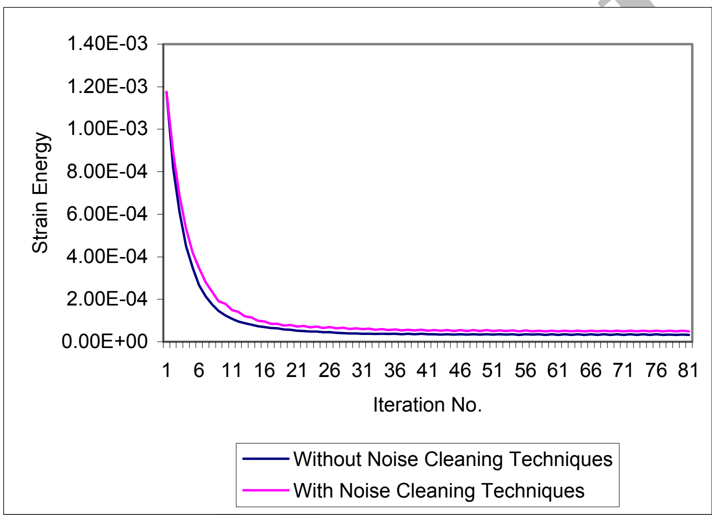
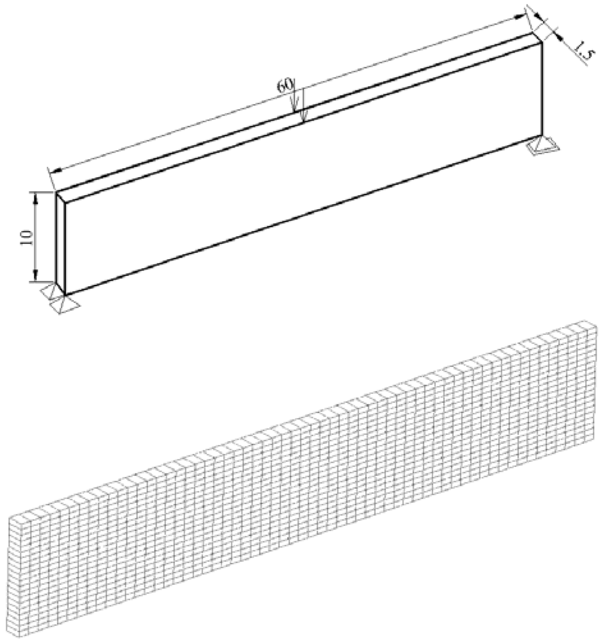


Figure 7 Iteration history of the 3D beam of Example 1

Example 2. In the second example, topological optimization of the MBB beam as a three dimensional problem. The design domain and the finite element mesh is illustrated in Figure 8. The design domain has been discretized into 1200 three dimensional eight-node elements. The solid material fraction is $V_{solid} / V = 60\%$**.** It is assumed that initially, the material is distributed uniformly.

The optimum layouts obtained without using the noise cleaning techniqe as well as by using it with r_{min}=0.048, are illustrated in Figures 9 and 10, respectively. As it is shown in Figure 10, the undesirable elements in the optimum toplogy of the MBB beam, have been removed.

Example 3. To demonstrate the effect of the use of non-conforming finite elements on the optimal result, a three dimensional plate model which is discretized by 400 eight-node elements, is considered. The geometry and finite element mesh are shown in Figure 11. It is assumed that a point load is applied at the center of plate. The volume fraction is set to $V_{solid}/V=30\%$.



Figurre 8 Problem definition of MBB beam of Exmple 2

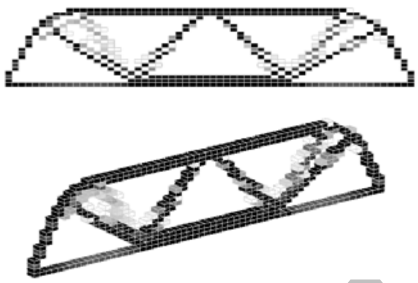


Figure 9 Optimum layout without noise cleaning

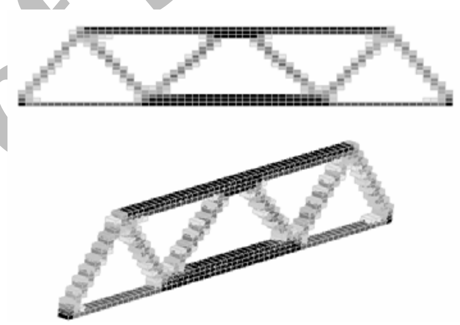


Figure 10 Optimum Layout with noise cleaning

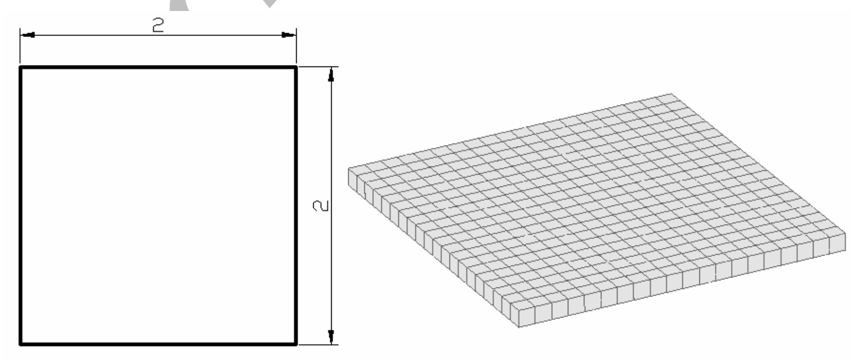


Figure 11. Design domain and the finite element mesh of Example 3

Without using the noise cleaning option, when either standard or non-conforming lower order elements are employed, the checkerboard pattern as shown in Figure 12 is obtained. When the noise cleaning method is employed, by using standard eight-node brick element the optimum result as illustrated in Figure 13 is obtained. The result of using non-conforming finite elements in this case is demonstrated in Figure 14.

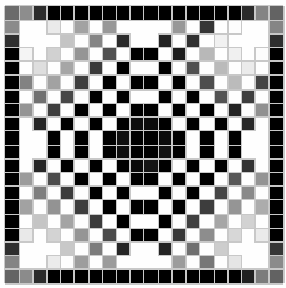


Figure 12. Checkerboard pattern obtained by using 8-node standard brick element

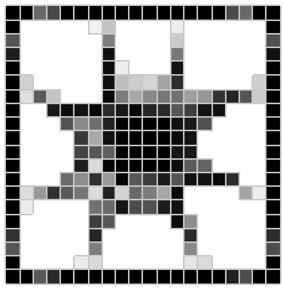


Figure 13. Optimal layout using 8-node standard element with noise cleaning

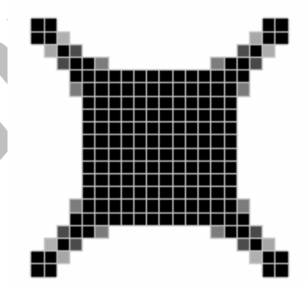


Figure 14. Optimal layout using 8-node non-conforming element with noise cleaning

6. CONCLUSIONS

Three-dimensional structural topology optimization is a very useful tool with several applications in industries real life problems. The checkerboarding and forming of undesirable impractical structural members in the optimal layout can be overcome by implementing the convolution based noise cleaning techniques inside the optimization algorithm. A very small increase in the objective function value pays off by obtaining a practical structural layout. By using the three dimensional noise cleaning method together with the non-conforming finite elements and the continuation method very good results are expected.

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