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Isogeometrical analysis of functionally graded materials in plane elasticity problems

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ABSTRACT: A unified modeling and analysis approach to address the functionally graded plane problems is suggested which is making use of B-Splines and NURBS for the definition of geometry and material properties as well as the analysis. The recently developed isogeometric analysis numerical method is concisely explained and the functionally graded materials are briefly introduced. It is shown that the difficulties encountered in the Finite Element analysis of the FGMs are to a large degree alleviated by employing the mentioned method. Finally, a few examples are presented to demonstrate the efficiency of the method.

1 INTRODUCTION

The theory of functionally graded materials (FGM) was introduced in the early 1980's by a group of material scientists in Japan [1–3]. These materials are a new generation of advanced inhomogeneous composite materials that were originally proposed for thermal barriers of modern engineering structures in extremely high temperature environments and have been increasingly applied since then [4]. They are now being regarded as one of the most promising candidates for future smart composites in many engineering disciplines [5].

Functionally graded materials, which are kind of heterogeneous materials, often possess better mechanical, thermal or electrical performances in comparison with the traditional homogeneous objects [6–10]. FGMs may exhibit isotropic or anisotropic material properties depending on the processing technique and the practical engineering requirements. The material properties of FGMs demonstrate a continuous and smooth change from one side of the object to another by gradually varying the volume fraction of constituent materials that leads to elimination of the interface and layering problems. However, beside the problem of proper manufacturing of the material, the design of the parameters and analysis of such heterogeneous materials is a challenge in front of the engineering community [1]. Due to the high cost and difficulties of experimental tests for material design, the mathematical theory of homogenization may be employed for determination of the microscopic properties of the composite constituents [11–14].

Assuming that we are able to manufacture FGMs, still serious difficulties may arise when the conventional computational methods, such as finite elements, are employed for their stress analysis. To overcome these problems, in this research work, the applicability of the B-spline based analysis, e.g. the isogeometrical method [15], is investigated.

On the other hand, Isogeometric Analysis (IA) is a new approach, as a logical extension of the finite element method, for the analysis of problems governed by partial differential equations. It is a generalization of the classical finite element analysis and has many features in common with it. However, it is more geometrically based and takes inspiration from Computer Aided Design (CAD). A primary goal of B-Spline analysis is to be geometrically precise no matter how coarse the descretization. Another goal is to simplify mesh refinement by eliminating the needs for communication with the CAD geometry once the initial mesh is constructed [15–23].

Due to the possibility of simultaneously modeling of the structural shape, material distribution, imposition of the exact boundary conditions and structural analysis, the B-Spline analysis constitutes an important tool for studying integrated modeling and analysis of FGMs. In addition, this method, beside being independent of a mesh generation tool and a considerable ease of refinement, has some extra advantages such as ending up with smaller system of equations and consuming less computational time for interpolation of functions. Furthermore, refinement is carried out with a considerable ease.

The B-splines have already been employed by some researchers for specifying the variations of the mechanical properties of FGMs. References [1] and [24–28] can be consulted for a thorough review of the literature on the subject. For analysis, either analytical techniques or numerical methods such as finite elements have already been employed for certain problems [29].

In this study, a unified approach is presented which employs the isogeometrical method for both modeling and analysis of the FGM plane problems.

The constitutive material matrix is considered to be isotropic at each point where the elastic modulus is assumed to vary continuously throughout the domain according to an assumed law of distribution.

In Section 2 the functionally graded materials are very briefly introduced. In section 3, fundamentals of the isogeometric analysis method is concisely presented. Since in this method the variations of the displacement coordinates as well as the mechanical properties of the problems are imagined as imaginary surfaces, Section 4 is devoted to the preliminary introduction to the surface generation. The derivation of the numerical formulation is discussed in section 5. A few examples are solved in section 6 to demonstrate the efficiency of the method. Finally, Conclusions and proposed further research is the subject of Section 7.

2 ISOGEOMETRICAL ANALYSIS

Inspired by the fact that by using proper versions of splines one is able to define complex geometries, any component of a field variable, which satisfies a governing partial differential equation, might be imagined as a surface. From this comes the idea of discretizing the domain of interest by the defining points of these splines instead of, for instance, using the finite element meshes, finite difference grids, or the collection of points in the meshfree methods. Applicability and usefulness of this idea has already been verified in the case of ordinary differential equations [10] and very promising results are reported by Hughes and his coworkers [1–6] where the idea is employed in several advanced problems in the area of computational mechanics.

The main advantages of the isogeometrical analysis method are flexibility and accuracy in the definition of the geometry and its boundaries, a considerable reduction in the size of system of equations, accuracy in satisfaction of the essential boundary conditions and considerable ease in implementing adaptivity and mesh refinement.

3 FUNCTIONALLY GRADED MATERIALS

FGMs are relatively complicated composites in which the volume fraction of constituent materials varies gradually, giving a non-uniform microstructure with continuously graded macro properties. These materials are multifunctional and the main idea of their introduction was taking advantage of ideal behavior of its constituents, e.g. heat and corrosion resistance of ceramics together with mechanical strength and toughness of metals. By functional gradation new properties can be obtained that can improve both material and component structures to achieve higher performance and material efficiency.

FGMs are composite materials that are microscopically homogeneous but at macro level the mechanical properties vary continuously from one point to another by smoothly varying the volume fractions of the material constituents. A considerable part of studies on FGMs is devoted to thermal stress and fracture analysis in plates and shells [31].

A key feature that distinguishes FGMs from homogeneous materials is that the mechanical, electrical and thermal properties of the material vary spatially. Thus, the analysis of functionally graded materials is significantly more complex than the homogeneous case.

Due to the complexity of the analysis, depending on the material's anisotropy and inhomogeneity, only limited cases are solved by the analytical methods. Most of the closed form solutions for nonhomogeneous materials are carried out under the simplifying assumption of material isotropy. The most common numerical approach for the FGM analysis problems is the FE method. Usually, the domain of the problem is divided into different regions with constant mechanical properties. In other words, multiple dissimilar materials are assigned to different elements which involve some extra approximation and to obtain acceptable results, fine meshes are required that itself leads to larger system of equations.

In this article, an isogeometric analysis method is utilized to simultaneous modeling and analysis of the isotropic FGM plane problems.

4 NUMERICAL EXPERIMENTS

To demonstrate the performance of the method, in this section, two plane stress linear elasticity problems are presented. The first is a cantilever beam subjected to a load at the free end. The second is an infinite plate with a circular hole subjected to a unit tensile traction in the horizontal direction. The problems have analytical solutions and therefore are useful for verification.

4.1 *Deep Cantilever beam*

A square plate with unit thickness subjected to two equal point loads at the free end corners, as illustrated in Figure 1, under plane stress conditions is considered. Due to the complexity involved in the analysis of FGM problems, for the sake of verification and to demonstrate the accuracy of the results, a rational procedure is here followed and the problem is solved in four different cases which are indicated at the right hand side of Figure 1.

In the first case, a constant modulus of elasticity equal to 10,000 was considered and the problem was solved. For the FE analysis, a mesh of 20*20 nonconforming quad elements was employed that resulted in a linear system of 882 equations. The IA was carried out by using a net of 5*5 control points and a linear system of 50 equations was resulted. The obtained results for the vertical displacement U_y by the FE and IA methods is illustrated in Figures 2-a and 2-b, respectively. As can be noted from Figure 3, the maximum relative difference between the values obtained by these two methods for the vertical deflection of the centerline of the plate is less than one percent. The fact that the resulted system of equations in the IA

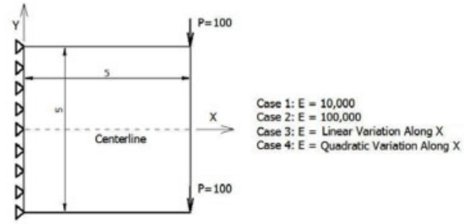

Figure 1. Problem definition.

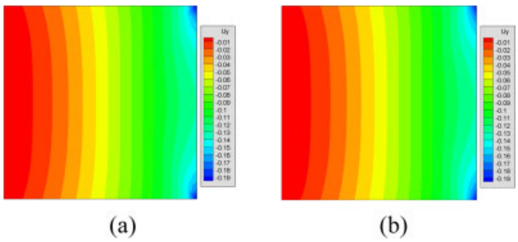

Figure 2. (a) Finite Element Results, (b) Isogeometric results.

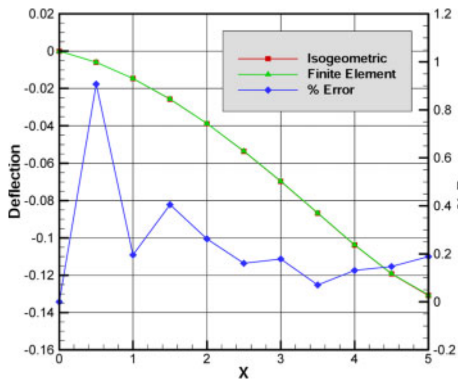

Figure 3. Comparison between Isogeometric and FE results for E = 10,000.

method is far smaller than the FE method indicates the accuracy and efficiency of the IA method.

Then, the modulus of elasticity was increased to 100,000 and the problem was solved again. In the third stage, the modulus of elasticity was assumed to vary linearly from 100,000 at the left side to 10,000 at the right edge; See Figure 4. Finally, a quadratic variation for the modulus of elasticity, with the values as above and its minimum value equal to 10,000 at the right hand side (Figure 4), was considered. Obviously, it is plausible to expect that the obtained results for the vertical deflections of the linear and quadratic variations for the elasticity modulus fall within the values of the first and second problems which is the case as can be observed from Figure 5. Also, as it is expected, since the stiffness of the problem with a linear variation of the modulus of elasticity is relatively higher than that with quadratic variation, the vertical displacements of the horizontal centerline of the plate with the linear variation of E is less than the same problem with the quadratic variation.

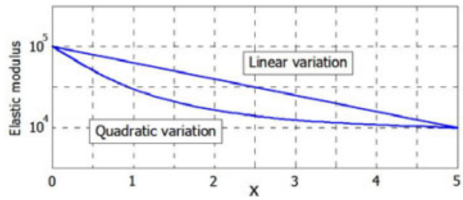

Figure 4. Linear and quadratic variations of the elasticity modulus.

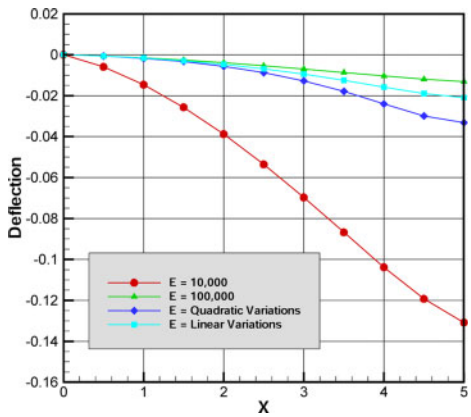

Figure 5. Variations of the centerline vertical displacements obtained by IA for different cases.

5 CONCLUSION

As it is expected, the IA is a suitable method for solving partial differential equations with non-constant coefficients and the analysis of the functionally graded materials are good examples of these equations. From the examples presented in this paper, regarding the much fewer variables employed, it is seen that the quality of the solution by the presented method is comparatively better than the finite element method, especially due to the fact that the material properties within each conventional finite element needs to be considered as constant. One of the main advantages of the isogeometrical analysis method, beside its accuracy in defining the boundary conditions and material properties, is the considerable reduction in the resulted system of equations which is cost effective and makes it a potential and promised substitute to the other numerical methods.

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