NUMERICAL DETERMINATION OF TENTION MODE STRESS INTENSITY FACTOR IN JOINTED ROCK MEDIUM USING STRESS EXTRAPOLATION METHOD

PhD Student, Hossein Mirzaei[1], Assistant Prof. Dr. Reza Kakaie[1], Associated Prof. Dr. Behroz Hassani[2]
[1]Faculty of Mining Engineering and Geophysics,
Shahrood University of Technology, Shahrood, **Iran**
[2]Faculty of Civil Engineering, Shahrood University of Technology, Shahrood, **Iran**

ABSTRACT

Stress intensity factor is the one of most important parameters in rock fracture mechanics. In practice, this factor is commonly calculated by various numerical techniques such as finite element and boundary element methods. These methods encounter mesh-related difficulties in dealing with fracture mechanics problems. In this paper the Element Free Galerkin method based on the linear elastic fracture mechanics was used to determine tension-mode stress intensity factor in jointed rock medium using stress extrapolation method. The visibility criterion was applied to model the rock fractures. In addition, Lagrange multipliers method was employed to enforce the boundary conditions. To verify the computational capability and accuracy of the method, an example of jointed sample including horizontal joint was considered under tension load and the stress intensity factor was determined and evaluated. The stress extrapolation method with %7 difference in comparison with analytical method has high precision and it also shows good agreement with the displacement extrapolation method based upon finite element method.

Keywords: Element Free Galerkin method, Stress intensity factor, stress extrapolation method, Rock fracture mechanics, Tension mode.

1 – INTRODUCTION

Fracture mechanics of rock materials is of essential significance for the design and reliable operation of structures constructed in rock medium. The mechanical behavior of jointed rock masses is mostly controlled by fractures or joints. Depending on the loading levels and the geometry of the fractures in rock medium, these may be propagated and cause to rupture of rock masses. As is well-known, the stress intensity factor (SIF) in linear elastic fracture mechanics is the main parameter capable to characterize the stress field in the vicinity of the crack tip. Its determination is a crucial task: it can be obtained from the stress field, the displacement field or from energy quantities [1]. In practice, commonly a numerical method such as finite element or boundary element is employed to calculate SIF [2,3]. Finite element and boundary element methods encounter mesh-related difficulties in dealing with fracture mechanics problems. To remove these difficulties, various mesh free methods such as element-free Galerkin method (EFGM) was developed [4].

Based on the geometry and loading condition, a crack propagates under the three basic failure modes or the mixed-mode condition (Fig. 1). Mode *I* is the tensile opening mode, mode *II* is the in-plane sliding or shear and mode *III* is the tearing or out of plane mode [5].

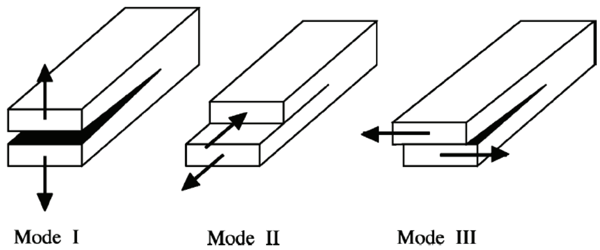


Figure 1. Three basic modes of crack propagation [5].

In numerical studies, the stress intensity factor is calculated by methods such as displacement extrapolation method, stress extrapolation method, J-integral [2]. In this paper, the element-free Galerkin method was used based on linear elastic fracture mechanics to determine the tension mode (mode I) Stress intensity factor in fractured rock medium. The method was evaluated by an example and the results were compared with the results of analytical and finite element methods.

2. THE EFG METHOD

The EFG method is one of the most promising meshless methods. It essentially consists of two aspects construction of meshless approximation using the moving least-square (MLS) technique and formulation of Galerkin weak form to govern the numerical approximation [6].

2.1. The MLS approximation

According to Lancaster et al. [7], the local approximation u^h of a field variable u(x) defined in the solution domain, Ω, is expressed as the inner product of a vector of the polynomial basis, $p(x)$, and a vector of the coefficients, $a(x)$

$$u^h = P^T(x).a(x) = \sum_{i=1}^{m} p_j(x)a_j(x) \tag{1}$$

where m is the number of monomials in the polynomial basis. In 2-D problems, a linear basis, i.e., $p^T = (1, x, y)$; corresponding to $m = 3$, is used. If the values at a set of nodes, $x_i, i = 1, 2, ..., n$, are known, the vector $a(x)$ can be determined by minimizing a weighted, discrete L_2 error norm defined as

$$J = \sum_{i=1}^{n} w(x, x_i)[u^h(x_i) - u_i]^2 \tag{2}$$

where $w(x, x_i)$ is a weight function defined over a the domain of influence of node i, u_i the nodal value at x_i, and n the number of nodes whose domain of influence contains the evaluation point, x. The present study employs the cubic spline weight function. The stationarity of *J* with respect to $a(x)$ leads to the solution of $a(x)$

$$\mathrm{a(x)} = \mathrm{A}^{-1}(\mathrm{x})\mathrm{B}(\mathrm{x})\mathrm{u} \quad (3)$$

$$[A(x)]_{IJ} = \sum_{i=1}^{n} w(x, x_i) p_I(x_i) p_J(x_i), \quad I, J = 1,2,...,m, \quad (4)$$

$$[B(x)]_{IJ} = w(x, x_J) p_I(x_J), \quad J = 1,2,...,n, \quad I = 1,2,...,m. \quad (5)$$

$$u^T = (u_1, u_2, ..., u_n) \quad (6)$$

Substitution of $\mathrm{a(x)}$ into Eq. (1) gives

$$u^h(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} p_j(x)(A^{-1}(x)B(x))_{ji} u_i = \sum_{i=1}^{n} \phi_i(x)\; u_i \quad (7)$$

2.2. The Galerkin weak form for elastostatics

As is well-known the equilibrium of a body that occupies the region Ω bounded by Γ can be stated mathematically as follows [6]:

$\nabla.\sigma + \mathrm{b} = 0$ in Ω (8)

$\mathrm{u} = \bar{\mathrm{u}}$ on Γ_u, $\sigma.\mathrm{n} = \bar{\mathrm{t}}$ on Γ_t (9)

where σ, b , u, n, are the stress tensor, the body forces, the displacement field and the unit outward normal to the boundary Γ,respectively. While $\bar{t}$ and $\bar{u}$ represent the given traction and displacements on the portion Γ_t and Γ_u of the boundary, respectively. The variational or weak form of the Eqs. (8) and (9) is

$$\int_\Omega \sigma^T \delta\varepsilon d\Omega - \int_\Omega b^T \delta u d\Omega - \int_{\Gamma_t} t^T \delta u d\Gamma - \delta W_u = 0, \quad (10)$$

where δ is the variational operator, ε the strain tensor, and δW_u represents a term that is introduced to enforce the essential boundary conditions. By the use of Lagrange multipliers to apply the essential boundary conditions, δW_u is defined as [8]

$$\delta W_u = \int_{\Gamma_u} \delta\lambda.(u - \bar{u}) d\Gamma + \int_{\Gamma_u} \delta u^T.\lambda d\Gamma, \quad (11)$$

where λ is the Lagrange multiplier. Substituting Eq. (7) and (11) into Eq. (10), we have

$$\begin{bmatrix} \boldsymbol{K} & \boldsymbol{G} \\ \boldsymbol{G}^T & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f} \\ \boldsymbol{q} \end{Bmatrix} \quad (12)$$

where

$$[K]_{ij} = \int_\Omega B_i^T D B_j d\Omega \quad (13)$$

$$G_{ik} = -\int_{\Gamma_u} \phi_i\; N_k d\Gamma \quad (14)$$

$$\boldsymbol{f} = \int_\Omega \Phi_\mathrm{I}\, \boldsymbol{b}\; \mathrm{d}\Omega + \int_{\Gamma_\mathrm{t}} \Phi_\mathrm{I}\, \bar{\boldsymbol{t}}\; \mathrm{d}\Gamma \quad (15)$$

$$q = -\int_{\Gamma_u} N_k\; \bar{u}\; d\Gamma \quad (16)$$

$$[B]_i = \begin{bmatrix} \phi_{i,x} & 0 \\ 0 & \phi_{i,y} \\ \phi_{i,y} & \phi_{i,x} \end{bmatrix} \tag{17}$$

$$N_k = \begin{bmatrix} N_k & 0 \\ 0 & N_k \end{bmatrix} \tag{18}$$

$$\mathrm{D} = \frac{\mathrm{E}}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1-\upsilon}{2} \end{bmatrix} \quad \text{for plain stress} \tag{19}$$

where E and υ are Young's modulus and Poisson's ratio of the material.

Geometric discontinuities such as cracks and joints can be modeled by the EFG method in different ways such as: *visibility criterion*, *diffraction method* and *transparency method* [4]. The *visibility criterion* is employed in the present paper.

4 - STRESS EXTRAPOLATION METHOD

We explain the stress extrapolation method by a straight forward example. Consider center-cracked tension plate (Fig. 2), based on the Westergaard stress function, the stress field in the vicinity of this crack can be written as Tailor expansion [9]:

$$\sigma_{ij} = c_1 \left(\frac{r}{a}\right)^{-1/2} f_{1ij}(\theta) + c_2 \left(\frac{r}{a}\right)^{0} f_{2ij}(\theta) + c_3 \left(\frac{r}{a}\right)^{1/2} f_{3ij}(\theta) + \ldots \tag{20}$$

In the vicinity of crack tip, we can dispense the higher order terms, hence we have:

$$\sigma_{ij} = \frac{c_1}{\sqrt{r}} f_{ij}(\theta) \tag{21}$$

Where

$$c_1 = \frac{K_I}{\sqrt{2\pi}} \tag{22}$$

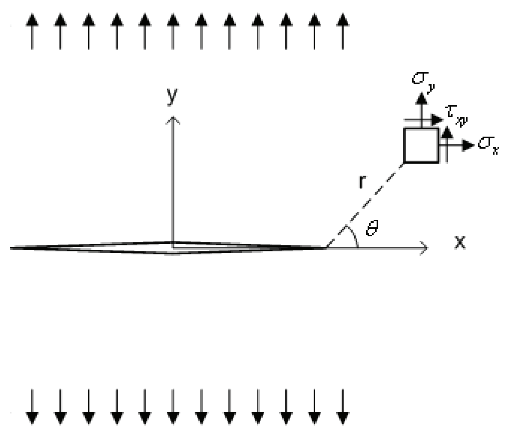


Figure 2. Geometry of the Center-Cracked Tension plate [9].

Consequently, the stress components in the vicinity of this crack tip can be written as

$$\begin{aligned}\sigma_x &= \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\\ \sigma_y &= \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\\ \tau_{xy} &= \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\end{aligned} \tag{23}$$

Where σ_x, σ_y *and* τ_{xy} are stress components, K_I is the mode I stress intensity factor and (r,θ) is the polar coordinates. Along the crack face ($\theta = 0$), the normal stress will be

$$\sigma_y\big|_{\theta=0} = \frac{K_I}{\sqrt{2\pi r}} \tag{24}$$

Theoretically, the opening mode value of K can be obtained from the σ_y stress ahead of the crack by following equation

$$K_I^* = \sigma_y\big|_{\theta=0} .\sqrt{2\pi r} \tag{25}$$

In the stress extrapolation method, K_I is found by extrapolating the K_I^* along the crack face [2]. With the values of K_I^* along the crack face, linear regression is employed to determine a "best" straight line. The stress intensity factor, K, is the y-axis intercept of a best-fit line through the data.

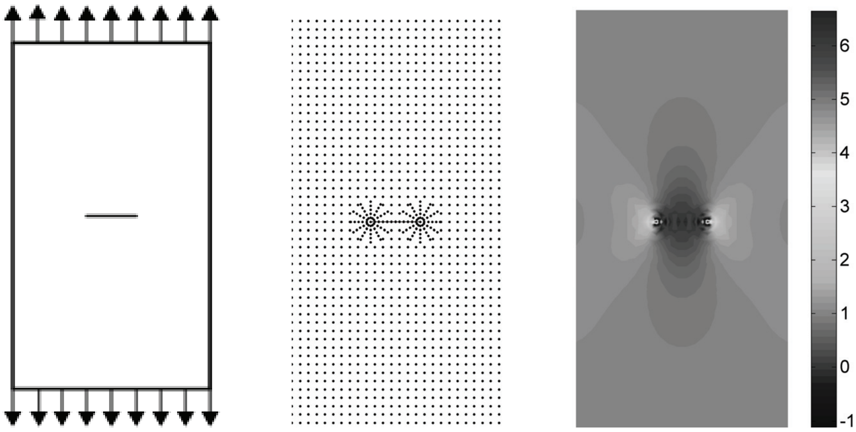
$$K_I = \lim_{r\to 0} K_I^* \tag{26}$$

4. NUMERICAL EXAMINATION AND CONCLUSIONS

The validity of the EFGM and displacement extrapolation technique is tested by an example; a rectangular rock specimen with $200\times 100\,mm^2$ dimensions containing a central horizontal crack by length of 24 mm was considered and loaded by 1 MPa tension stress (Fig. 3-a). Young module and Poison ratio of the sample are 72.4 GPa and 0.3 respectively. To model this problem by EFGM and calculate the SIF by stress extrapolation, the required programs were developed in MATLAB based on equations (1) to (26).

4. 1. Determination of SIF by EFGM model and Stress extrapolation method

To determine SIF by EFGM, 1426 nodes were distributed in problem domain (Fig. 3-b). As explained, to indicate stress intensity factor, the normal stress (σ_y) must be obtained. By the element free Galerkin method the model is solved and normal stress is calculated (see fig. 3-c).



(a) (b) (c)

Figure 3. a) Geometry of the Center-Cracked Tension specimen, b) Distribution of nodes in EFGM model, c) Normal Stress (σ_y).

As mentioned before, to obtain mode I stress intensity factor, element free Galerkin results ($\sigma_y.\sqrt{2\pi r}$) are plotted as a function of the distance from crack tip and best-fit line is found by linear regression. The intersection of this line with y-axis indicates the stress intensity factor. Figure 4 illustrates the stress extrapolation method; the stress intensity factor is obtained equal to $5.88\ \text{MPa}.\sqrt{\text{mm}}$.

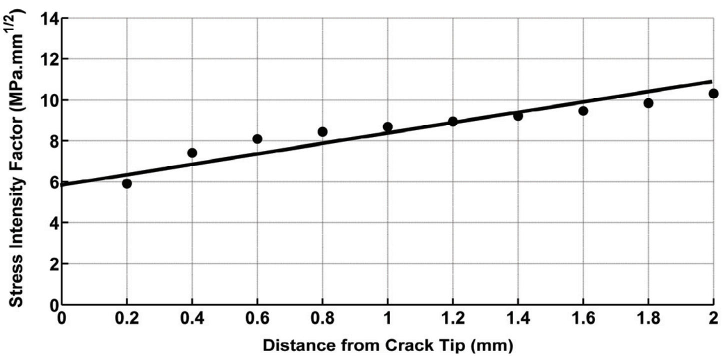


Figure 4. Illustration of the stress extrapolation method; central crack, $K_I = 5.88$.

4. 1. Determination of SIF by Analytical and Finite Element methods

By the use of analytical method available for this problem, the mode I stress intensity factor (K_I) is calculated as follows:

$$K_I = \sigma\sqrt{\pi a}\left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{1/2}$$
$$K_I = (1)\sqrt{12\pi}\left(\frac{100}{12\pi}\tan\frac{12\pi}{100}\right)^{1/2} = 6.29 \quad MPa.\sqrt{mm} \qquad (27)$$

This problem was solved using finite element method together with displacement and stress extrapolation techniques by Javidrad [10]. He obtained the SIF equal to 6.6 $MPa.\sqrt{mm}$ by displacement extrapolation method and 6 $MPa.\sqrt{mm}$ by stress extrapolation method.

4. 3. Comparison of EFGM Result with Analytical and Finite Element Methods

The result of element free Galerkin method is compared with the results of analytical and finite element methods. Table 1 shows this comparison.

Table 1. Comparison of EFGM Result with Analytical and Finite Element Methods

Method	Stress Extrapolation by EFGM	Analytical Method	Displacement Extrapolation by FEM	Stress Extrapolation by FEM
K_I $MPa.\sqrt{mm}$	5.88	6.29	6.6	6

According to Tab.1 the result of stress extrapolation technique with element free Galerkin method with %7 difference in comparison with analytical method is accurate and has excellent agreement with stress extrapolation by finite element method.

5. CONCLUSION

Stress intensity factor is the one of most important parameters in rock fracture mechanics. Determination of this parameter is necessary to study the fracture propagation and also its direction. The element free Galerkin method based on the linear elastic fracture mechanics was used to determine tension-mode stress intensity factor in jointed rock medium using stress extrapolation technique. By the use of an example the validity of the EFGM and stress extrapolation method was evaluated and the results were compared with analytical finite element methods. This comparison indicates that these methods, with %7 difference in comparison with analytical method, are high accurate and also have excellent agreement with the stress extrapolation method based upon finite element method. Consequently the EFGM together with the stress extrapolation technique can be used as significant tools in rock fracture mechanics.

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