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Reliability-based Design for Damping Behavior of Inner Mass Single-unit Impact Dampers

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In this paper, the dynamic response of a vibratory system equipped with a single-unit impact damper is analyzed. The reliability behavior of this system is investigated when the stress and strength are distributed lognormally. The standard deviation of lognormal stress is realistically calculated by applying external noise and an uncertainty parameter in the simulated model. Variations of the damping inclination and the reliability of the system, with respect to the coefficient of restitution, are obtained. Finally, damping inclination, safety factor and system reliability are presented in a user-oriented diagram. Such reliable systems equipped with impact dampers can strongly suppress the undesired vibrations. Copyright © 2012 John Wiley & Sons, Ltd.

Keywords: impact damper; reliability; damping inclination; lognormal distribution

1. Introduction

An impact damper is a small loose mass within a main mass. These systems can be extensively applied to attenuate the undesirable vibration of robot arms, turbine blades, and so on.[1,2] It is shown that the impact dampers would operate more efficiently than classical dynamic vibration dampers.[3] In the past few years, the behavior of impact dampers has been investigated experimentally, analytically, and numerically.[4–7] Son *et al*.[8] proposed active momentum exchange impact dampers to suppress the first large peak value of the acceleration response because of a shock load. Bapat and Sankar[9] showed that the coefficient of restitution has a great effect on the performance of impact dampers. They demonstrated that in the case of single-unit impact dampers, optimized parameters at resonance are not necessarily optimal at other frequencies. Cheng and Xu[10] obtained a relation between the coefficient of restitution and impact damping ratio. They showed that optimal initial displacement is a monotonically increasing function of damping.

Although the impact damper has been investigated in the past, there are still several shortcomings in this area of research that need to be addressed. The effect of impact dampers on the reliability of vibratory systems is one of the important aspects in mechanics which has not been fully investigated. To the best of our knowledge, there has been no consideration towards this issue, and hence, the main thrust of this paper lies in this subject.

Reliability is defined as the probability that the system will perform its intended function successfully for a specified interval of time under stated operating and environmental conditions.[11] The structural analysis and design have traditionally been based on deterministic methods. However, in reality, stress and strength are statistically distributed and strongly tied to the reliability of the stressed component. It can be shown that the failure times of mechanical components follow the lognormal distribution if the natural logarithms of the lifetime data follow a normal distribution.[12] The lognormal distribution is formed by the multiplicative effects of random variables. Multiplicative interactions of variables are found in many natural and physical processes and are, in fact, observed in many frequently encountered failure mechanisms. This characteristic of the lognormal distribution makes it a good choice for the analysis of failure rates of many failure mechanisms.[13]

Nowadays, there is a growing interest toward simulating the reliability-based designs. The reason for this interest lies in the fact that reliability-based design can lead to low-cost and dependable results. Steenackers *et al*.[14] provided an optimization method according to reliability-based design. M. Cazuguel *et al*.[15] combined the time-variant reliability and nonlinear finite-element methods and showed the feasibility of such a combination.

In this paper, first the mathematical model of a single-unit impact damper is presented. Then, on the basis of reliability concepts, the relations for safety factor and reliability of the system under consideration are derived. Finally, the effects of varying the coefficient of restitution on the reliability of the system are investigated and discussed.

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2. Mathematical model of vibratory system

An impact damper consists of a small loose mass within a main mass that freely moves through an enclosure to suppress undesirable vibrations. Consider an impact damper with impact mass *m*, clearance *d*, oscillator with linear stiffness *K*, main mass *M* and viscous damping *C*. A free vibratory system equipped with an impact damper is shown in Figure 1.

The governing differential equation of the vibratory motion of the main mass between impacts is as follows:

$$M\ddot{x} + C\dot{x} + Kx = 0 \tag{1}$$

The above differential equation can be easily solved to formulate the displacements of the main mass between the impacts *i* and $i+1$.[16] In this analysis, *x* and *y* are the main mass and impact mass displacements, respectively. As shown in Figure 1, it is clear that impacts occur only when the following holds.

$$|x - y| = d/2 \tag{2}$$

It is clear that in an inelastic collision, some kinetic energy is transformed into heat, sound and other forms of energy. The coefficient of restitution (denoted by *e* in this paper) is proportional to the loss of energy during the collision. In a perfectly elastic collisions, $e = 1$; in a perfectly plastic collision, $e = 0$. The ratio of the differences in the velocities before and after the collision is equal to the coefficient of restitution. In other words, $e = |\dot{Z}_+ / \dot{Z}_-|$, where $\dot{Z}_-$ and $\dot{Z}_+$ are the relative velocities of the colliding masses just before and after impact, respectively, and $\dot{z} = \dot{y} - \dot{x}$.

Main mass and impact mass velocities after the *i*th impact can be determined using the concept of the coefficient of restitution, as shown in Equations 3 and 4.[9]

$$\dot{x}_{ia} = \left(\frac{M - m.e}{M + m}\right)\dot{x}_{ib} + \left(\frac{M + M.e}{M + m}\right)\dot{y}_{ib} \tag{3}$$

$$\dot{x}_{ia} = \left(\frac{M + M.e}{M + m}\right)\dot{x}_{ib} + \left(\frac{m - M.e}{M + m}\right)\dot{y}_{ib} \tag{4}$$

In the above, subscripts *ia* and *ib* represent the values of variables $\dot{y}$ and $\dot{x}$ just after and before the *i*th impact, respectively, and *e* is the coefficient of restitution.

3. Investigating free vibrations of the vibratory system

The vibratory behavior of the system is presented using a hypothesis example. Consider a vibratory system as shown in Figure 1, with the nominal values of the model parameters listed in Table I.

A computer program was written in MATLAB software to simulate the vibratory response of the discussed vibro-impact system in real time. The waveforms of free vibrations with and without the impact damper are shown in Figure 2. As illustrated, the decay of the maximum displacement in free vibrations of a viscously damped vibratory system is nearly exponential. However, in the case of a vibratory system equipped with an impact damper, the decay is initially linear, and after a considerable decrease in displacement amplitude, it tends to be exponential.[9]

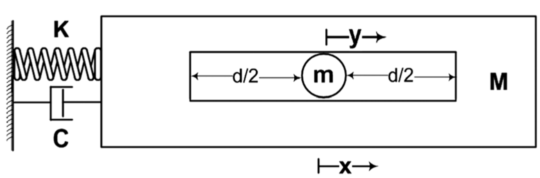

Figure 1. Model of an impact damper.

Table I. Parameters for the vibratory system with an impact damper

$M = 0.1$ kg	$m = 0.02$ kg
$K = 400$ N/m	$C = 0.1$ N. s/m
$d = 1$ cm	$x_0 = y_0 = 0$
$\dot{x}_0 = 2$ m/s	$\dot{y}_0 = 0$

M: main mass; *m*: impact mass; *C*: viscous damping; *K*: oscillator with linear stiffness; *d*: clearance.

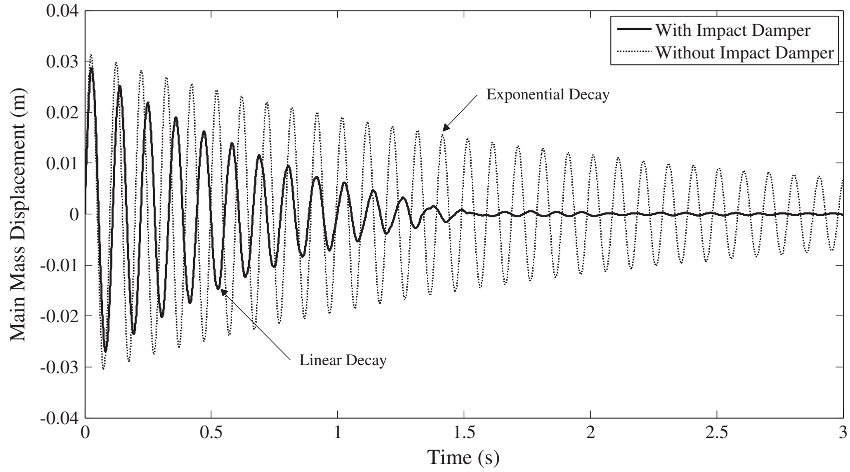

Figure 2. Waveforms of free vibrations without an impact damper and with an impact damper.

The initially linear decrease in the maximum displacement of the vibratory system with impact dampers is usually termed 'damping inclination', defined as follows:

$$DI = \frac{X_1 - X_2}{t_2 - t_1} \tag{5}$$

where t_1 and t_2 are the time of occurrences of the maximum positive displacements X_1 and X_2, respectively.

4. Reliability theory

4.1. Safety factor and reliability

The lognormal distribution will be relevant where a good fit is important for the central part of the distribution. In fact, stresses are quite likely to have a lognormal distribution because the multiplication of variables that are normally distributed produces a result that approaches lognormal.[12,13] If both stress and strength follow the lognormal distribution, the reliability relation is given by:[12]

$$R = 1 - \varphi\left(-\frac{\mu_{\ln\delta} - \mu_{\ln s}}{\sqrt{\sigma^2_{\ln\delta} + \sigma^2_{\ln s}}}\right) \tag{6}$$

where φ is the area under standard normal distribution, $\mu_{\ln\delta}$and $\mu_{\ln s}$ are the mean of lognormal strength and stress, respectively. Similarly, $\sigma_{\ln\delta}$ and $\sigma_{\ln s}$ are the standard deviation of lognormal strength and stress. Generally, the safety factor is taken to be$n = \mu_\delta/\mu_s$. However, it should be noted that, in reality, both stress and strength are random variables. Therefore, the safety factor for the lognormal distribution of strength and stress can be formulated as follows:[13]

$$\mu_n = \exp\left[C_n\left(\frac{C_n}{2} + \frac{\mu_{\ln\delta} - \mu_{\ln s}}{\sqrt{\sigma^2_{\ln\delta} + \sigma^2_{\ln s}}}\right)\right] \tag{7}$$

In the previous formula, the variable C_n is calculated by

$$C_n = \sqrt{\frac{(\sigma_{\ln\delta}/\mu_{\ln\delta})^2 + (\sigma_{\ln s}/\mu_{\ln s})^2}{1 + (\sigma_{\ln s}/\mu_{\ln s})^2}} \tag{8}$$

4.2. Standard deviation of stress

In practice, the gap size and the impact mass are the two most important decision variables in designing the impact dampers. In this work, for the vibratory system equipped with an impact damper, the standard deviations of the lognormal stresses are obtained by randomly varying the gap size and impact mass. In addition, the output results are subjected to external noises, which are created numerically using the MATLAB software. To do this, 'white noises' with a constant noise power (CNP) and different starting seeds are added to the output displacement response. It is noted that, on the basis of the specific application of the vibratory system, a proper value for the CNP can be selected. In this paper, the CNP is assumed to be 5×10^{-6} mm. The computational results of this process are shown in Table II.

On the basis of the data presented in Table II, the mean value of the maximum displacements (x_{max}) of the main mass with an impact damper is equal to $\mu_{x\,max} = 2.667$ cm. Furthermore, the standard deviation of the maximum displacement may be calculated as follows:[13]

$$\sigma_x = \sqrt{\frac{\sum x_i^2 - \left(\sum x_i\right)^2/N}{N-1}} = 0.063 \text{ cm} \tag{9}$$

where N is the number of data and x_i is the ith maximum displacement. The previous mean value and standard deviation will be used to calculate the mean value and standard deviation of the beam-bending stress.

The reason for selecting the bending of a cantilever beam as the main design consideration lies in the fact that this phenomenon can be used to investigate the failure behavior of many important machine parts and mechanisms. Cutting tools, robot arms, and turbine blades are among several industrial parts involving bending stress. Figure 3 is a schematic representation of a vibratory system in which the cantilever beam acts as a spring in lateral direction. This model is commonly used in many researches in the area of dynamics/vibration analysis.[17,18]

The maximum displacement can be easily related to the maximum bending stress by using the strength of material equations.[19] Consider a cantilever circular cross section beam equipped with an impact damper at its tip (as shown in Figure 3). The maximum value of the bending stress at the clamped side of the beam is equal to

$$S_{max} = \frac{3Er}{L^2} x_{max} \tag{10}$$

Table II. Random variations of impact damper parameters. ($e = 0.9$)

	Noise initial seed	Δm (gr)	Δd (mm)	x_{max} (cm)
Ideal working	—	0	0	2.594
Work with noise and/or Δm and/or Δd	—	0	+1	2.619
	—	0	−1	2.550
	[2 3 3 4 1]	0	0	2.768
	[3 2 3 2 4]	0	0	2.687
	[2 2 4 1 3]	0	0	2.645
	[4 1 3 1 3]	0	0	2.709
	[3 2 3 2 4]	0	0	2.687
	[4 1 3 1 3]	+1	+1	2.653
	[2 2 4 1 3]	+1	−1	2.716
	[2 2 4 1 3]	−1	−1	2.742
	[2 3 3 4 1]	−1	+1	2.637

x_{max}: maximum displacements; m: impact mass; d: clearance.

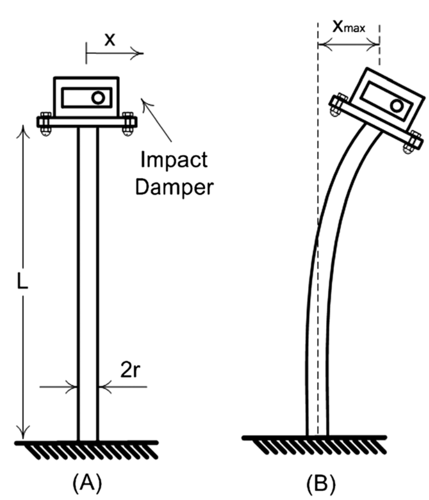

Figure 3. Schematic representation of the vibratory model.

In the previous equation, r, L and E are the radius, length, and Young modulus of the beam, respectively (Figure 3). The above relation will be used in reliability calculations. In this study, the nominal values of the model parameters are assumed to be: $E = 200$ GPa, $r = 3.4$ mm, and $L = 0.5$ m.

5. Result and discussion

In section 2, a theoretical model used to predict the vibratory behavior of a system equipped with an impact damper was presented. It is noted that, in case of contact with high values of the coefficient of restitution (near to 1), energy loss in each collision is small. Figure 4 illustrates the variation of the damping inclination with the coefficient of restitution. As shown, the value of the damping inclination decreases as the coefficient of restitution increases. This is due to a very low energy loss at each impact when the coefficient of restitution is near unity, even though many impacts occur. As a result, the total energy loss remains low in this case. The jump in the value of the damping inclination, shown in Figure 4, is a consequence of the sudden increase in the first maximum displacement of the vibratory mass. As a result, in the range of the coefficient of restitution where the jump occurs (between $e = 0.73$ and 0.77), the impact damper cannot effectively suppress the first peak of the main mass displacement.

The damping inclination relates to the effects of impacts on the displacement amplitude of the vibratory systems with impact dampers. In other words, the damping inclination shows the capability of impact dampers for fast attenuating undesired vibrations. This ability is clearly shown in Figure 5. As shown in this figure, impact damper with a higher amount of damping inclination (DI) suppresses the vibration amplitude more strongly. Furthermore, the first peak of vibrations is magnified in this figure. As demonstrated, for the system with greater (better) damping inclination, the first peak of vibrations is suppressed more strongly.

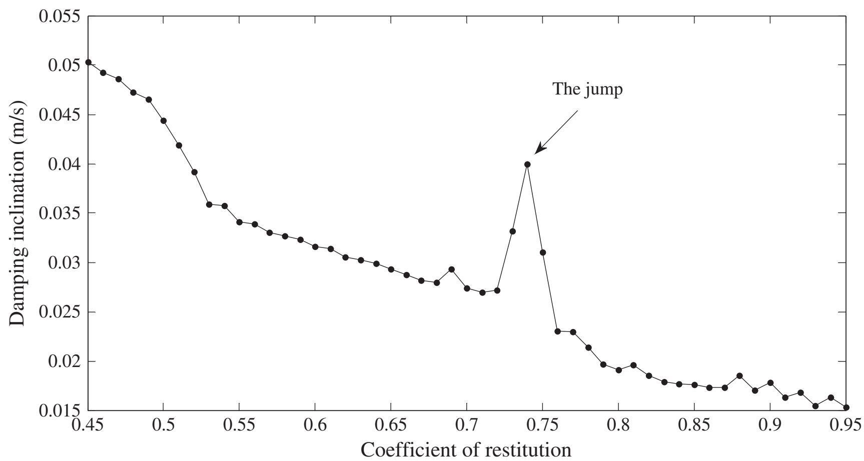

Figure 4. Variations of the damping inclination with the coefficient of restitution.

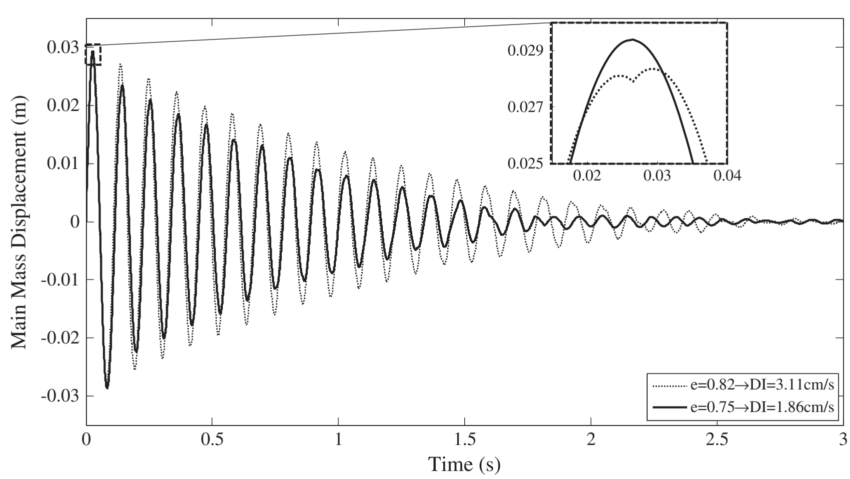

Figure 5. Effects of varying the damping inclination on free vibrations and the first peak vibrations.

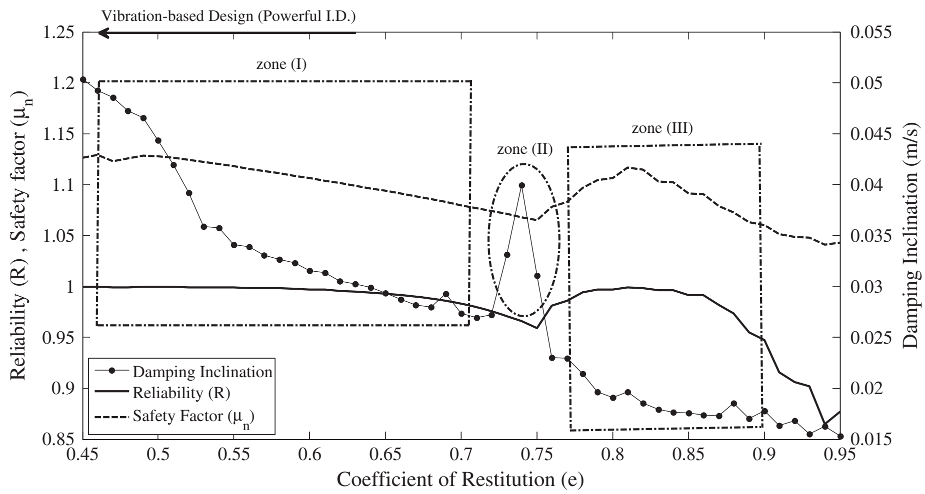

Figure 6. The reliability-based design and vibratory-based design.

Table III. Classifications of the vibratory and reliability-based zones

Zone number	Coefficient of restitution	Reliability	DI (m/s)	Perfectibility (%)
I	0.45–0.70	0.9965	0.0361	100
II	0.70–0.77	0.9737	0.0298	90.1
III	0.77–0.90	0.9861	0.0187	67.6

DI: damping inclination.

The reliability (R) and safety factor (μ_n) of the discussed system, obtained by Equations 6 and 7 versus the coefficient of restitution are illustrated in Figure 6. As shown in Zone III of this figure, in spite of the damping inclination (shown in Figure 4), the reliability increases as the coefficient of restitution increases. The reason for this behavior is that, in zone III, the impact damper is capable of suppressing only the first peak of vibrations (x_{max}).

The designing an impact damper may be divided into two parts. One is designing the impact damper to effectively suppress the vibratory response. For convenience, this type of design may be referred to as 'vibratory-based' design. Note that the damping inclination (shown by dots in Figure 6) relates to the vibratory-based design. The other part, so called 'reliability-based' design, leads to designing an impact damper which would result in a safe and reliable system. For the best design practice, the reliability-based and the vibratory-based approach should be combined. In this way, the best design may be selected on the basis of the relative importance of these criteria. Therefore, Figure 6 may serve as a user-oriented chart that shows the various zones of the reliability-based and vibratory-based design combinations.

Three zones of the reliability-based and the vibratory-based designs are illustrated in Figure 6. Zone I provides a relatively practical range of coefficient of restitution, which can reasonably satisfy both vibratory-based and reliability-based design requirements. Zone III represents reliable and safe systems, but the impact damper cannot strongly work in this range of coefficient of restitutions. Furthermore, it can be concluded that the coefficient of restitution, located in Zone II, are not generally reliable in designing the impact dampers (because of the jump shown in Figure 5). The above results can conveniently be summarized in Table III. In this table, the design 'adequacy' is defined as follows:

$$\text{Perfectibility} = \left(\frac{R}{R_{\max}} \times WF_1 + \frac{DI}{DI_{\max}} \times WF_2\right) \times 100 \tag{11}$$

where WFs are the weight factors (taken to be $WF_1 = WF_2 = 0.5$ in this work)

6. Conclusion

A mathematical model has been used to predict the vibratory behavior of a system equipped with an impact damper. The effects of the coefficient of restitution on the reliability of the system with impact damper have been studied. The safety factor and reliability of the proposed system are investigated using the lognormal distribution.

It is shown that increasing the coefficient of restitution, between 0.45 and 0.95, results in a relatively uniform decrease of the damping inclination. Unlike the damping inclination, the reliability does not change monotonically with the coefficient of restitution. In other words, increasing the coefficient of restitution may lead to the increase of safety factor and reliability.

Therefore, to design a vibratory system with an impact damper, two independent criteria should be simultaneously considered: reliability of the system and the system capability to strongly suppress the undesired vibrations. In this work, these two factors are combined into a single objective called 'adequacy' of the design. Furthermore, it is shown that both the vibratory-based and the reliability-based designs can be illustrated in a user-oriented chart.

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