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HOMOGENIZATION FOR CELLULAR BODIES WITH PERIODIC MICROSTRUCTURES

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1. Introduction

The homogenization theory is used to determine the average effective mechanical properties of composite materials with a large number of heterogeneities. This rigorous mathematical theory was innovated in the 1970's and since then much research has been undertaken in this area of applied mathematics.

The cellular material is formed by repetition of very small microstructures comprised of solids and voids, called 'microscopic cells'. Assuming a very small period for these microscopic cells allows us to compute the equivalent material properties by using the limiting concept where the size of the cells is reduced to zero. This does not mean that from a macroscopic point of view the dimensions of voids could not be varied, but the variation should be smooth enough. Considering one such microstructure for cellular material in a general elasticity problem, the resulting displacements and stresses will rapidly and drastically vary within a small neighbourhood of a point. In this case using finite element methods would be almost impossible, because to capture the real behaviour of such a material we need to use an incredibly large amount of finite elements [1,2]. (See Figure 1.)

The homogenization equation is solved using the finite element method. In addition an alternative engineering approach is presented. Our ultimate aim is to use the method for the topological layout optimization problems.

2. Solution of the homogenization equation

According to the mathematical theory of homogenization, in the case of cellular body the elements of the elasticity tensor can be found by

$$E^H_{ijkl}(\mathbf{x}) = \frac{1}{|Y|} \int_{\mathbf{Y}} \Big[E_{ijkl}(\mathbf{x},\mathbf{y}) - E_{ijpq}(\mathbf{x},\mathbf{y}) \, \frac{\partial \chi^{kl}_p}{\partial y_q} \Big] dY, \qquad (1)$$

where the microscopic displacement field $\boldsymbol{\chi}$ is the $\mathbf{Y}$ - periodic solution of the following equation:

$$\int_{\mathbf{Y}} E_{ijpq}(\mathbf{x},\mathbf{y}) \, \frac{\partial \chi^{kl}_p}{\partial y_q} \, \frac{\partial v_i(\mathbf{y})}{\partial y_j} dY = \int_{\mathbf{Y}} E_{ijpq}(\mathbf{x},\mathbf{y}) \, \frac{\partial v_i(\mathbf{y})}{\partial y_j} dY, \qquad \text{for all } \mathbf{v} \qquad (2)$$

Equations (1) and (2) with different values of k and l provide enough equations for finding the elements of the homogenized elasticity matrix. For 2-D problems it would be sufficient to solve the equations in three cases with $k = l = 1$ and $k = l = 2$ and $k = 1, l = 2$.