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Providing a New Empirical Failure Criterion for Intact Rock and Comparing it With Three Criteria Bieniawski, Ramamurthy and Hook-Brown

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***Abstract* - In recent decades several empirical criteria have been presented, in order to simulate the triaxial behavior of rock samples. These factors have mostly been associated with some limitations, because of the natural complexity in rock sample's behavior in deformation and physical form. Some of these parameters are appropriate for a specific type of rock or special condition in laboratory. By comparison with other parameters, although Bieniawski, Ramamurthy and Hoek criteria show minimal limitations, but according to this paper, they are not accurate enough in correlation with the results of the tests. In this paper a new empirical criteria is introduced and compared to the three mentioned criteria and as a result, the advantages of this newly introduced parameter in correlation with the test results are interpreted.**

***Keywords* - Intact rock, Empirical criteria, Hook criteria, Bieniawski criteria, Ramamurthy criteria**

I. Introduction

After extensive studies on coal rocks that Hobbs did the first empirical criteria for failure to deliver in 1964, although this measure was only applicable to coal- rock, but it was as a starting point for researchers in this area. Over time and with more advances in rock mechanics, these measures became vital so that within less than 20 years 15 criteria were presented by researchers of rock mechanics. In this study, the three most commonly discussed and compared the behavior of rock samples has been presented with a new frame. The first criterion is the criterion of Bieniawski that was presented in 1974 as logarithmic. This standard was revised in 1983 by Yadhbir and colleagues that made the measure more complete. The second criterion is Hook and Brown criterion that was provided exponentially. Because this criterion could be used both in soil and rock samples, was used more. However, the third criterion that was provided by Ramamurthy 5 years later showed that could be better than the one provided by Hook and Brown ([1][2][3][4][5]).

The applicability of Bieniawski, Ramamurthy and Hoek-Brown empirical strength criteria has been assessed for various types of intact rocks by using a vast number of published triaxial test data from various places.

Analysis of individual data sets revealed that the traditional forms of the criteria do not have a perfect agreement with the data. A strong negative correlation has been observed between B in Bieniawski's criterion and m in Hoek and Brown's criterion with uniaxial compressive strength of materials [6]. To estimate the strength of rock and rock mass a failure criterion is required. The theoretical triaxial strength criteria based on the actual mechanism of fracture do not fit the experimental results properly and to overcome this problem, many empirical criteria have been formulated for rocks.

Stone is one of the most important materials. As it is widely mined around the world, and receiving attention on the current mining, developing an appropriate strength criterion for use in stone seams to be of considerable value.

One criterion proposed for stone is that suggested by Sheorey (See [7][8]).

II. Strength Criteria For Intact Rock

Under a given situation, geological, lithological, physical, environmental and most of the mechanical aspects remain constant and the influence of confining pressure is predominant. The effect of confining pressure on the strength of intact rock has been investigated extensively starting with von Karman (1911) who conducted pioneering experiments on Carrara marble in copper jackets and observed a nonlinear variation of strength with confining pressure. All the subsequent investigations conducted to study the influence of confining pressure confirm this nonlinear response. An in behavior from brittle to ductile nature at high confining pressures ([9][10][11] [12]).

The general form of a strength criterion is:

$$\sigma_1 = f(\sigma_2 . \sigma_3)$$

Where σ_1, σ_2 and σ_3 are the principal stresses at failure.

Because the available data indicate that the intermediate principal stress, σ_2, has very little influence on strength than the minor principal stress, σ_3, all of the criteria used in practice are reduced to the form:

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$$\sigma_1 = f(\sigma_3) \quad (1)$$

Or in its normalized form;

$$\frac{\sigma_1}{\sigma_c} = f\left(\frac{\sigma_3}{\sigma_c}\right) \quad (2)$$

Hoek-Brown's criterion for intact rocks

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + S\sigma_c^2)^{0.5} \quad (3)$$

m is a constant value for each rock type. Published strength data examined by Hoek and Brown (1980) suggested that *m* increased with rock quality. Representative values for different rock groups are given below.

- Group a $m \approx 7$ for carbonate rocks with well-developed crystal cleavage (dolomite, limestone, marble);
- Group b $m \approx 10$ for lithified argillaceous rocks (mudstone, siltstone, shale, slate);
- Group c $m \approx 15$ for arenaceous rocks with strong crystals and poorly developed crystal cleavage (sandstone, quartzite);
- Group d $m \approx 17$ for fine-grained polymineralic igneous crystalline rocks (andesite, dolerite, diabase, rhyolite);
- Group e $m \approx 25$ for coarse-grained polymineralic igneous and metamorphic rocks (amphibolite, gabbro, gneiss, granite, norite, quartz-diorite).

Bieniawski's criterion for intact rocks

$$\sigma_1 = \sigma_c + B\sigma_c\left(\frac{\sigma_3}{\sigma_c}\right)^{\alpha} \quad (4)$$

α is slope of the plot between $\left(\frac{\sigma_1}{\sigma_3} - 1\right)$ versus $\left(\frac{\sigma_3}{\sigma_c}\right)$ on log-log plot and B is a constant value for each rock type. From a study of a range of South African rocks, Bieniawski had the distinction of linking up the constants of the failure criterion with the lithology of some rocks. He suggested that B = 3.0 for siltstone and sandstone, 4.0 for sandstone, 4.5 for quartzite and 5.0 for norite.

Ramamurthy's criterion for intact rocks

$$\sigma_1 = A\sigma_3\left(\frac{\sigma_c}{\sigma_3}\right)^{\alpha} + \sigma_3 \quad (5)$$

Where B is rock material constant; function of rock type and quality and α is slope of plot between $\left(\frac{\sigma_1-\sigma_3}{\sigma_3}\right)$ and $\left(\frac{\sigma_c}{\sigma_3}\right)$ on log-log plot.

III. Data Selected For Analysis

For a discussion and comparison of the forms and the new form presented in this study, triaxial data of 80 samples were collected from different sources. These data are homogeneous and on specimens of almost the same size. Thus, they are to a reasonable extent free from the effects of specimen size. Following the theory of Mogi (1966) [13], the transmission range from brittle to ductile mode is shown with σ_1=4.4σ_3, For each set of experimental data used in this study to analyze all the data returned by the condition of σ_1<4.4σ_3 have been exhibited. For best results, only those test results that are at least 5 pairs of σ_1, σ_3 data, has been accepted. We have discussed tension largely as a parameter defining the stress fracture criteria from two variables of the stress and strain. Maximum stress that can be tolerated by a rock is defined as its resistance. Thus, in the new criterion defined, failure is as the failure strength and a failure criterion is not associated with the strain. At last, for the 4 criteria for each type of rock, two parameters were calculated to determine the degree of compliance of each criterion in each rock type.

1. The regression parameter (r^2) defined by the following equation :

$$r^2 = 1 - \frac{\sum\left[\sigma_{1j} - F(\sigma_{3j})\right]^2}{\sum \sigma_{1j}^2 - \frac{\left(\sum \sigma_{1j}\right)^2}{n}} \quad (6)$$

For a criteria of failure we have $\sigma_1 = f(\sigma_3)$, (σ_{3j}, σ_{1j}), that is for the Jth pair and n pairs of the information provided.

2. Comparison of tested and estimated tensile and compressive stress values.

IV. Analysing The Applicability Of The Criteria For Intact Rock

Analysis of individual data sets revealed that none of the existing criteria shows perfect agreement with experimental values of stone strength. Although unique values of the constants in third criteria have been determined with good coefficients of determination for overall data, a wide variation has been noticed in the values of the constants when individual data sets have been analyzed. The analysis was carried out for different rock types, namely, limestone, granite, granodiorite, shale, sandstone, claystone and liparite.

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Although the data were from various sources with differences in techniques, size and shape of specimens, the results indicate that the parameters cannot be regarded as constant values. For each particular rock type there found to be a correlation between B in the Bieniawski criterion and m in the Hoek-Brown criterion with σ_c.

A. Igneous rocks

Ability to convergent input data, necessary but insufficient condition for achieving any measure of the true values obtained from the tests are considered. The first parameters of each of these four criteria are for definition of variables in both horizontal and vertical axis, so that we can measure all the difference in the choice of the two main variables considered. As the parameters of these two axes make more converging numbers, they are more likely to be closer to the values obtained from the test results.

In the first comparison of the data available in the first 80 groups, five groups of the 4 criteria are chosen for comparing regression. The result on igneous rocks has been shown in Fig. 1-A, to emphasize that defined values cannot be assigned to constants in Eq. 3, 4 and 5.

The second comparison between the values obtained from the four criterions for igneous rocks of the experiment is to measure the actual values. The numerical value of the gap is much less than the normal standard and is closer to the actual values.

B. Clastic rocks

Anticipating maximum and minimum values of the stresses in clastic rocks and providing a useful measure because of grain, cement matrix that are the three main components, is very difficult.

Some researchers that conduct studies on these rocks have tried several times to provide a more comprehensive measure for this group of rocks but due to the complexity of the acquisition, they were not successful until now. In this study, regression analyses of clastic rocks of 13 groups of data have been used. The result on clastic rocks has been shown in Fig. 1-B.

C. Chemical rocks

Factors contributing to this type of inter-basin sedimentary rocks that are formed under very different physical and chemical environments are very varied. The variation in quantities of α and B, causes severe disorganization of rock material coefficients. These disorganization make difficulty in providing a useful measure of the chemical rocks in comparison with groups of igneous rocks and clastic rocks, Therefore, for more accurate results and a more complete comparison of 19 groups of four benchmark data is used for regression analysis. The result on chemical rocks has been shown in Fig. 1-C.

D. Argillite rocks

In Argillite rocks tensions are so high that even the distribution of the minimum and maximum stress values obtained from triaxial tests are also very high, so we propose a criterion for this group of rocks much harder than the previous 3 groups. For better evaluation of each criterion in the rocks of the information 19 groups has been used. In Figure 1-D the results of the regression each parameter is shown.

Also the values of α and the relationships between A, B and m with σ_c for these three cases are as follows:

Table 1-
values of α and the relationships between A, B and m with σ_c

Rock type	α	A	B	m
granite	0.70	$-7.05\ln(\sigma_c) + 45.9$		
	0.68		$-15.04\ln(\sigma_c) + 95.2$	
				$-9.951\ln(\sigma_c) + 82.1$
quartzite	0.71	$-2.12\ln(\sigma_c) + 12.46$		
	0.71		$-4.59\ln(\sigma_c) + 24.3$	
				$-7.4\ln(\sigma_c) + 52.4$
limestone	0.78	$-0.94\ln(\sigma_c) + 6.5767$		
	0.76		$-1.07\ln(\sigma_c) + 7.7$	
				$-5.40\ln(\sigma_c) + 33.30$

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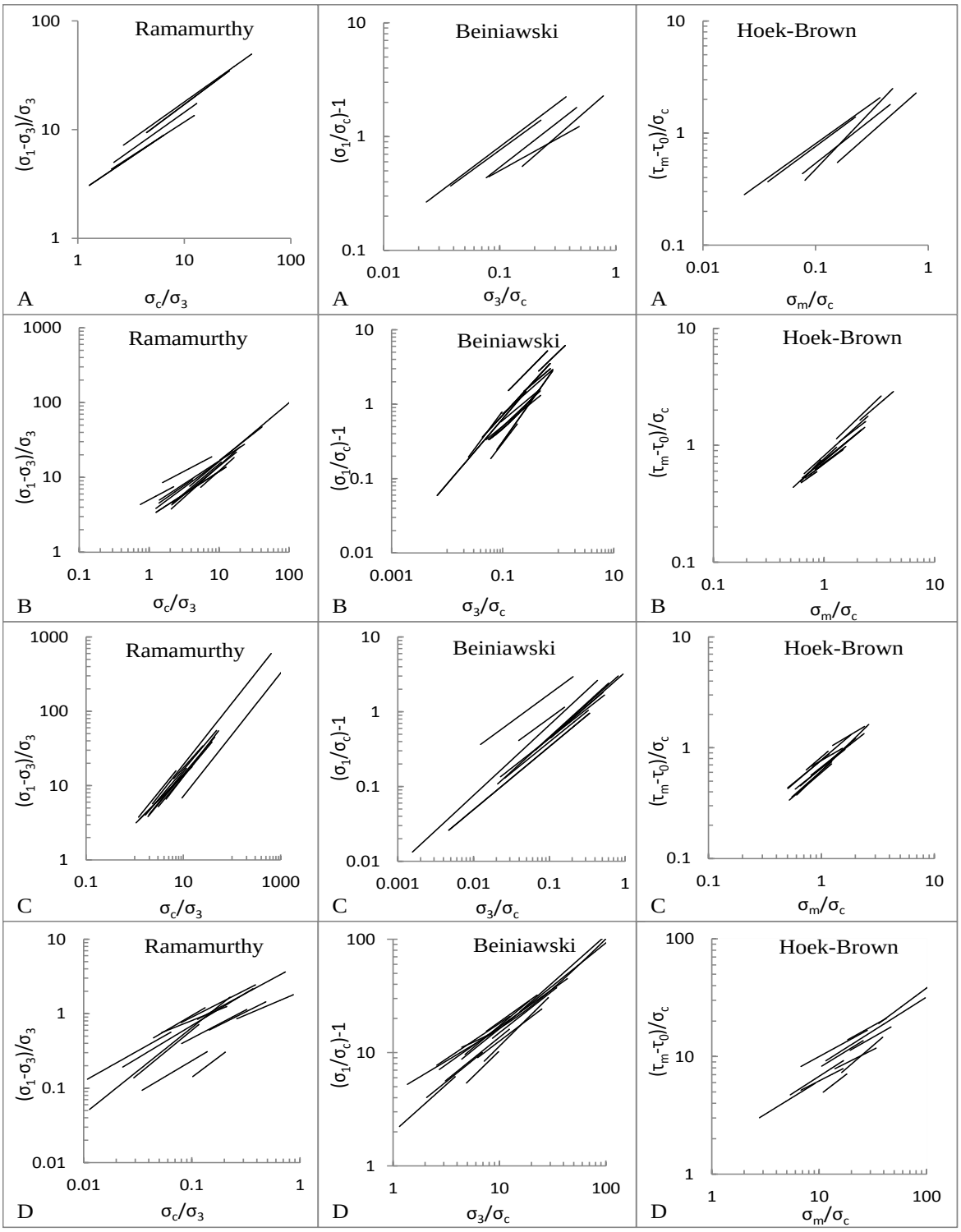

Figure 1- Plot of Ramamurthy, Bieniawski and Hoek-Brown for types of rocks

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V. Proposed Strenth Criterion

In 1773 Coulomb proposed for masonry and soil a shear resistance expression of the form

$$S = ca + \frac{1}{n}N \qquad (7)$$

Where *c* is the (non-directional) cohesion per unit area; *a* is the area of the shear plane; *N* is the normal force on the shear plane; 1/*n* is the coefficient of internal friction.

In modern terms n = cot φ and equation 7 is usually written in the form

$$\tau_f = c + \sigma_n tan\varphi \qquad (8)$$

Where τ_f is the shear strength per unit area; *c* is the unit cohesion; σ_n is the normal stress on the shear plane; φ is the angle of shearing resistance.

For all practical purposes at least, the validity of equation 8 is now universally accepted, but parameters *c* and φ may take many different values for the same soil, depending on stress path, stress level and drainage conditions.

Equation 8 can also be applied in rock mechanics for shear along joints and discontinuities and in some cases to the intact rock itself.

The combination of the Mohr stress circle with the Mohr–Coulomb failure criterion not only gives a valuable understanding of stress conditions at failure, but also provides a very powerful tool in geotechnical analyses.

In order to develop a simple mathematical expression with would enable prediction of strength sufficiently accurate for intact rocks covering the entire brittle and ductile regions, an attempt has been made through Mohr-Columb failure criterion.

In axial compression, exhibiting strength characteristics *c*, φ, the following useful expressions can be deduced:

$$\frac{\sigma_1}{\sigma_3} = \frac{2c\ cos\emptyset}{\sigma_3(1-sin\emptyset)} + \frac{1+sin\emptyset}{1-sin\emptyset} \qquad (9)$$

Where c is cohesion intercept and φ is friction angle.

When $\sigma_3 = 0$, the term $\frac{2c\ cos\emptyset}{(1-sin\emptyset)}$ is equal to σ_c.

To take care of the variation in c and φ with increase of confining pressure σ_3 and also to account for the non-linear behavior, Eq. 9, is modified as:

$$(\frac{\sigma_1}{\sigma_3}) = \beta(\frac{3\sigma_c - 0.5\sigma_3}{\sigma_3})^{\alpha} \qquad (10)$$

Where β is rock material constant; function of rock type and quality and α is slope of plot between $(\frac{\sigma_1}{\sigma_3})$ and $(\frac{3\sigma_c - 0.5\sigma_3}{\sigma_3})$ on log-log plot.

The above expression is applicable for all values of $\sigma_3 > 0$.

In plotting the results from conventional laboratory triaxial tests, in which the test specimens are compressed axially, it is usual to plot only the semicircle and envelope above the $\tau_{=0}$ axis. A method of deducing the best fit envelope from experimental data using the least squares method has been described by Balmer (1952).

In Figure 2, the results of the regression of this criterion are shown. Finally in Figure 3, which compares the results, obtained from the four criterions with actual values obtained from triaxial tests on samples, it is better visible.

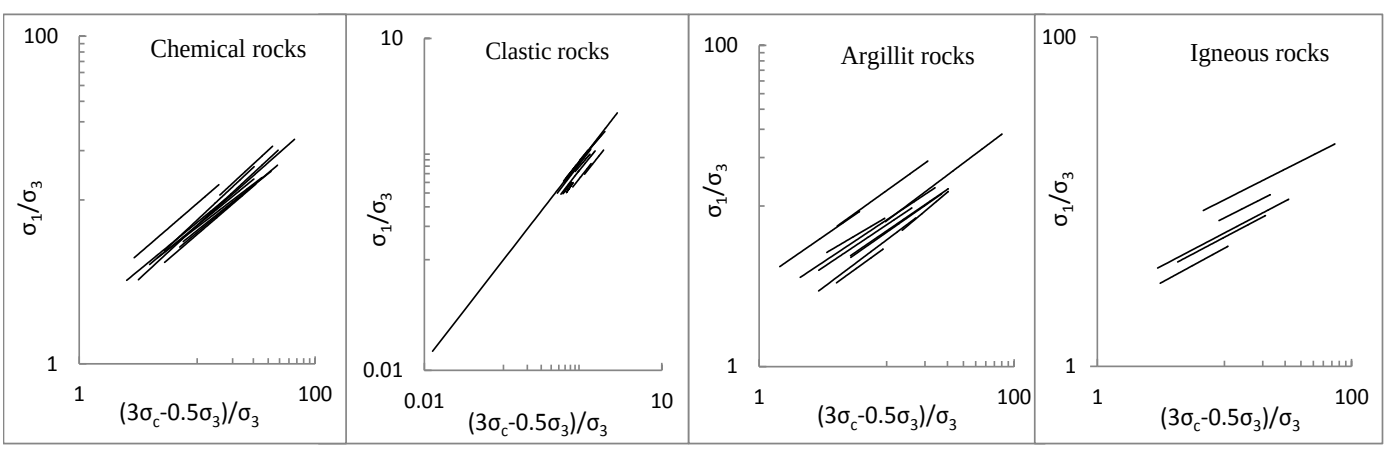

Figure 2- Plot of proposed criterion for types of rocks

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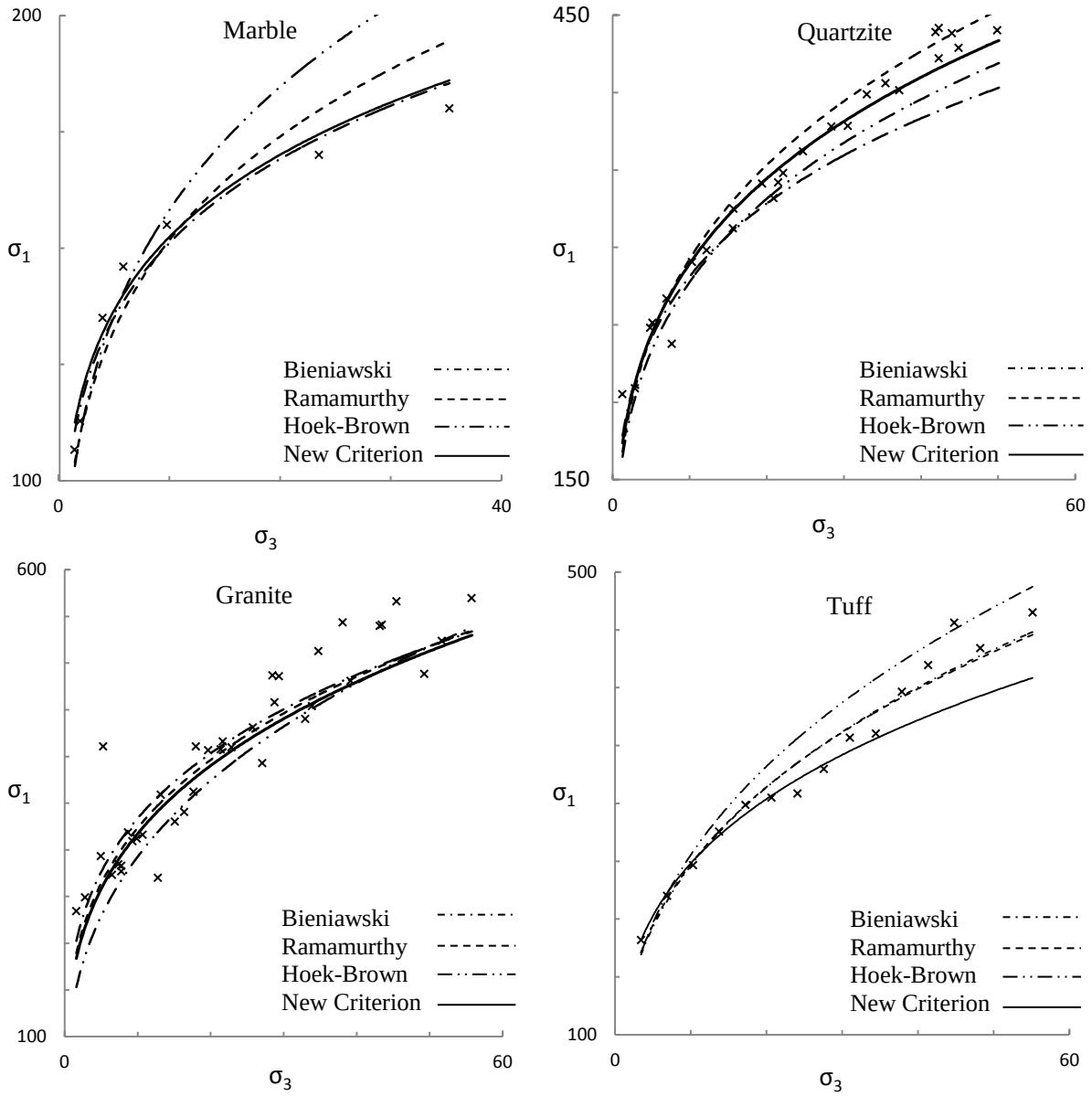

Figure 3- Cmparison between predicted and measured strength of marble, quartzite, granite and tuff

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VI. Summary And Conclusions

Although eventual modifications to the selected criteria for intact rocks requires more investigations in which more proper data groups must be analyzed within any rock type, the first conclusion coming out from the assessments done in this paper implies that treating criteria parameters with constant values would result in considerable inaccuracy, even for intact materials.

A strength criterion must be capable to deal with different conditions of a certain type of rock having different properties. Such a criterion mayor may not provide the best estimation for a large number of mixed data from various collieries and seams around the world.

In practice, a design engineer is faced with a certain type of rock with its particular properties. The characteristics of any rock type may change from place to place or even from one part to another part of the same seam or block.

A criterion must be flexible enough to fit the various conditions of rock properties.

The modified Hoek-Brown criterion gives good level of accuracy for rocks.

An estimate of the triaxial strength can be made by means of the Bieniawski criterion with a variable B dependent upon σ_c and α certain constant σ for each particular material. The only parameter required for this criterion is the unconfined compressive strength which can be determined simply.

The main results obtained in this study can be summarized as follows:

In igneous rocks: the relationship between these two groups of rocks, Bieniawski and new criterion is defined in the first place in this study and the next places are Ramamurthy criteria and Hook.

In Clastic rocks: the relationship between these groups of rocks, two Ramamurthy and new criterion is defined in this study in the first place and Hook and Bieniawski criteria are the following positions.

In chemical rocks: in these two groups of rocks Bieniawski relationship between this new definition and criteria are being investigated in the first place and Ramamurthy and Hook are the next places.

In Argillite rocks: the relationship between these two groups of rocks, Bieniawski and Ramamurthy are defined in the first place in this study and the next places are new Criterion is defined in this study and Hook-Brown.

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