



# Optimum Cost Design of Reinforced Concrete Cantilever Retaining Wall using Genetic Algorithm

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## Abstract

During last decades, economical criteria are the most important factor in civil engineering projects. Cantilever retaining walls, as reinforced concrete retaining structures, are required to resist against a combination of lateral earth pressure and hydrostatic stress. "Genetic Algorithms" (GAs) method is a general search and optimization algorithms inspired by "Darwin's Evolution Theory". In the recent years, GA is rapidly extended in many fields such as criminal suspect recognition, music composition, earthquake epicenter detection and many other fields. The application of algorithm genetic (GA) for nonlinear constraint optimum cost design of reinforced concrete cantilever (RCC) retaining wall is argued in the present research. A genetic algorithm is applied to achieve the optimized design of the RCC retaining wall. The main feature of GA is the ability to change nonlinear constrains problems to linear with no constraint problems. It is well established that genetic algorithm can be successfully applied to the optimum cost design of RCC walls. The results of optimization process of 6 RCC retaining walls show 30% to 5% reduction in total cost with respect to the same walls with initial design. The difference in percents of reduction respect to height of wall is proportion to the rate of steel to concrete and their prices. In high walls the rate of steel to concrete is more since their reduction of cost is less. During the optimization all stabilities controls respond to overturning, sliding, bearing capacity, the location of resultant, minimum and maximum steel rate in sections are satisfied.

**Keywords:** Retaining wall, Algorithm Genetic, optimization.

## 1. INTRODUCTION

A retaining wall is defined as a structure whose primary purpose is to provide lateral support for soil or rock. Retaining walls have traditionally been constructed with plain or reinforced concrete, with the purpose of sustaining the soil pressure arising from the backfill. This study is concerned with reinforced concrete cantilever (RCC) retaining walls. A schematic view of RCC is shown in Figure 1.

Earth retaining structures constitute an integral part of the infrastructure and reinforced concrete retaining walls as earth structures are frequently constructed for a variety of applications, most commonly for bridge abutments, road, transportation systems, lifelines and other constructed facilities. In order to economize the cost of the reinforced concrete retaining walls under design constraints, the designer needs to vary the dimensions of the wall several times, making design process rather tedious and monotonous. Since it is extremely difficult to obtain a design satisfying all the safety requirements, it is beneficial to cast the problem as an optimization problem. Some studies have been made in this direction by Dembicki & Chi [1], Keskar & Adidam [2], Saribas & Erbatur [3], Rhomberg & Street [4], Basudhar & Lakshman [5], Sivakumar & Munwar [6], and Yepes [7]. Although some mathematical programming based methods have been developed for optimum design problems, however, their applications are limited due to the fact that they require gradient information and usually seek to improve the solution in the neighborhood of a starting point. In recent years, structural optimization has witnessed the emergence of some novel and innovative design techniques. These stochastic search techniques make use of the ideas adopted from the nature, and do not suffer the discrepancies of mathematical programming based optimum design methods. The basic idea behind these techniques is to simulate the natural phenomena such as survival of the fittest, immune system, swarm intelligence and the cooling process of molten metal into a numerical algorithm. In this study, the genetic algorithms are used to determine the optimum design of reinforced concrete retaining walls. The objective function considered is taken as the cost of the retaining wall, and design is based on ACI 318-08.



This function is minimized subjected to design constraints. A numerical example together with sensitivity analysis is presented to illustrate the performance of the provided algorithms.

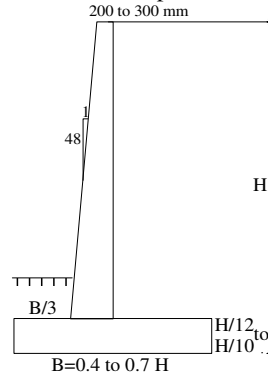


Figure 1. A schematic view of Reinforced Concrete Cantilever (RCC) retaining walls

## 2. OPTIMUM DESIGN PROCESS

Design of conventional retaining walls consists of two separate steps including stem and base optimization. In the first step we optimize the wall stem with three variables and the cost of stem will be determined. The dimensions and the reinforced concrete details of stem with optimum values will be constant for the second step. In the second step the base will be optimized with five variables so that the final design of RCC wall will be with minimum cost.

### 2.1 DESIGN VARIABLES FOR STEM OPTIMIZATION

The design variables in the first step of optimization, (i.e. the stem optimization), shown in Figure 2, are as follows:

X1:  $t_t$  (Stem thickness at the top of the wall)

X2 :  $t_b$  (Stem thickness at the bottom of the wall)

X3 :  $AS_{st}$  (Area of the main flexural steel of the stem per unit length of the wall)

As pointed out earlier after obtaining the optimum value for the X1 to X3 variables, the second step of optimization for base of the wall will be started. It is evident that the above variables will be constant parameters for the second step of optimization.

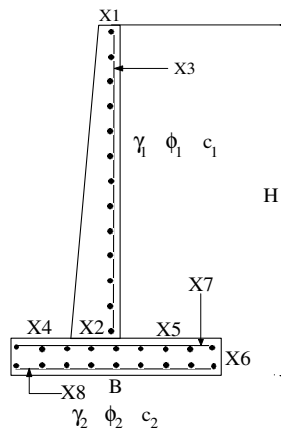


Figure 2. The design variables for stem and base of (RCC) retaining walls optimization

### 2.2 DESIGN VARIABLES FOR BASE OPTIMIZATION

The design variables in the second step of optimization, i.e. the base optimization are as follows:

X4:  $L_{toe}$  (Width of the toe)



X5 : $L_{heel}$	(Width of the heel)
X6 : $h_1$	(Thickness of the base slab)
X7 : $AS_{toe}$	(Area of the main flexural steel of the toe)
X8 : $AS_{heel}$	(Area of the main flexural steel of the heel)

### 2.3 CONSTANT DESIGN PARAMETERS

The main design parameters include soil properties, concrete and steel specifications, geometry dimensions, safety factors. These parameters are considered as the initial input of the optimization process and will be constant during this process.

### 2.4 DESIGN CONSTRAINS

Based on the Bowles' suggestions [9] and according to the ACI Code [10], the design constraints may be classified as geotechnical and structural requirements which are summarized in the following sections. These requirements represent the failure modes as a function of the design variables.

#### a: Sliding Failure Mode

The net horizontal forces must be such that the wall is prevented from sliding along its foundation. The factor of safety against sliding may be calculated as:

$$\Sigma \text{Resisting forces} / \Sigma \text{Sliding forces} \leq FS_{sliding} \quad (1)$$

The minimum acceptable limit for F.S sliding is 1.5. The most significant sliding force component usually comes from the lateral earth pressure acting on the active (backfill) side of the wall. Such force may be intensified by the presence of vertical or horizontal loads on the backfill surface.

#### b: Overturning Failure Mode

The stabilizing moments must be greater than the overturning moments to prevent rotation of the wall around its toe. The stabilizing moments result mainly from the self-weight of the structure, whereas the main source of overturning moments is the active earth pressure. The factor of safety against overturning can be calculated as:

$$\Sigma \text{Stabilizing moment} / \Sigma \text{Overturning moment} \leq FS_{overturning} \quad (2)$$

The factor of safety against overturning is usually considered greater than 1.5.

**c: Bearing Failure Mode:** The bearing capacity of the foundation must be large enough to resist the stresses acting along the base of the structure. The factor of safety against bearing capacity failure may be written as:

$$q_{max} \leq q_a \quad (3)$$

where  $q_{ult} = q_{max} = 3q_a$  is the ultimate bearing capacity of the foundation soil and  $q_{max}$  is the maximum contact pressure at the interface between the wall structure and the foundation soil. The minimum acceptable value for safety factor is 3.

#### d: Eccentricity Failure Mode

For stability, the line of action of the resultant force must lie within the middle third of the foundation base. For safety against eccentricity failure the following equation should be satisfied:

$$e - X \leq B/3 \quad (4)$$

where  $e$  is eccentricity of the resultant force.

#### e: Toe Shear Failure Mode

Toe slab of the wall has to be designed as a cantilever slab to resist moments and shear forces. The net loading acts upwards and flexural reinforcement has to be provided at the bottom of the toe slab. To prevent toe shear failure, Nominal shear stress at the junction of stem with toe slab should be less than shear strength of concrete:

$$\tau_c \leq \tau_{toe} \quad (5)$$

#### f: Heel Shear Failure Mode

Heel slab of the wall has to be designed as a cantilever slab to resist moments and shear forces. The net loading acts downwards and flexural reinforcement has to be provided at the top of the heel slab. Critical section for the shear is considered at the junction of stem with heel slab. Thus Nominal shear stress at this section should be less than shear strength of concrete:



$$\tau_c \leq \tau_{heel} \quad (6)$$

**g: Toe Moment Failure Mode**

A critical section for the moment is considered at the junction of stem with toe slab. So Maximum bending moment at a vertical section at the junction of the stem with toe slab must be less than the moment of resistance of toe slab:

$$MR_{toe} \leq M_{toe} \quad (7)$$

**h: Heel Moment Failure Mode**

A critical section for the moment is considered at the junction of stem with heel slab. So Maximum bending moment at a vertical section at the junction of the stem with heel slab must be less than the resistance moment of the heel slab:

$$MR_{heel} \leq M_{heel} \quad (8)$$

**i: Stem Shear and Moment Failure Mode**

The stem of the wall has to be designed as cantilever slab to resist moments and shear forces. Nominal shear stress of stem and bending moment must be less than shear and moment strength of concrete respectively:

$$\tau_c \leq \tau_{stem} \quad (9)$$

$$MR_{stem} \leq M_{stem} \quad (10)$$

### 3. UPPER BOND AND LOWER BOND CONSTRAINTS

For initial estimation of the retaining wall dimensions, there exist many recommendations (e. g. Bowles, 1982 and ACI, 2008). The proposed dimensions have usually minimum and maximum values (upper and lower bonds). The upper and lower bonds of all design variables for the current research are tabulated in Table 2.

**Table 1- Design variables of RCC walls (see Figure 2)**

	Lower bound	Upper bound	Symbol	Optimization variable
Stem thickness at the top	30 cm	20 cm	$t_t$	X1
Stem thickness at the bottom	H/10	H/12	$t_b$	X2
Vertical steel area of the stem	0.0035(t+ t <sub>b</sub> -0.07)	0.016(t+ t <sub>b</sub> -0.07)	AS <sub>st</sub>	X3
Width of toe	0.4H/3	0.7H/3	L <sub>toe</sub>	X4
Width of heel	0.18H	0.37H	L <sub>heel</sub>	X5
Thickness of base slab	H/12	H/10	$h_1$	X6
Horizontal steel area of the toe	0.0035(h-0.07)	0.016(h-0.07)	AS <sub>toe</sub>	X7
Horizontal steel area of the heel	0.0035(h-0.07)	0.016(h-0.07)	AS <sub>heel</sub>	X8
Width of footing	0.4H	0.7H	B	X4+t <sub>b</sub> +X5

### 4. OBJECTIVE FUNCTION

By minimizing a suitable and explicit *cost function*, one can reach to an optimum solution for a concrete cantilever retaining wall. The optimal design of a concrete cantilever retaining wall is proposed to be determined by the minimum costs of concrete and reinforcement steel. Since the cost of framework in comparison with concrete and steel is negligible it has been excluded from the project total cost. The *objective function* can then be expressed as:

$$C_W = C_V \times C_P + S_W \times S_P$$

in which:

$C_W$ : Total cost of CCR wall

$C_V$ : Volume of concrete

$C_P$ : Price of unit volume of concrete

$S_W$ : Weight of steel

$S_P$ : Price of unit weight of steel



## 5. GENETIC ALGORITHM

In all genetic algorithms three basic operators including reproduction, crossover and mutation are presented. By reproduction operators, it is decided that a string should survive or not. In addition, how many of that string, should be placed in the mating pool, to produce the next generation of strings. Decision is done based on the fitness of any string by different methods. In fact, fitness shows the ability to survive and reproduction in the next generations. In the structural optimization problems, the fitness function is a combination of objective function and constraints [10]. Because of the importance of this operator in genetic search strategies, researchers have looked for new operator of this type, so that by using it, optimization process has little fluctuation and high convergence. The important point is that the number of reproductions should be in such a way so that unordinary strings will not dominate the population. The process of optimization is outlined in Section 1. For the process of design and optimization, a computer program is provided in Mat-Lab Software.

## 6. NUMERICAL EXAMPLES

In this section, based on the proposed procedure 6 optimization examples, including different size and shape variables, are presented. These examples have been selected such that they are being employed in practical situations. Following are 6 examples with different shape and properties.

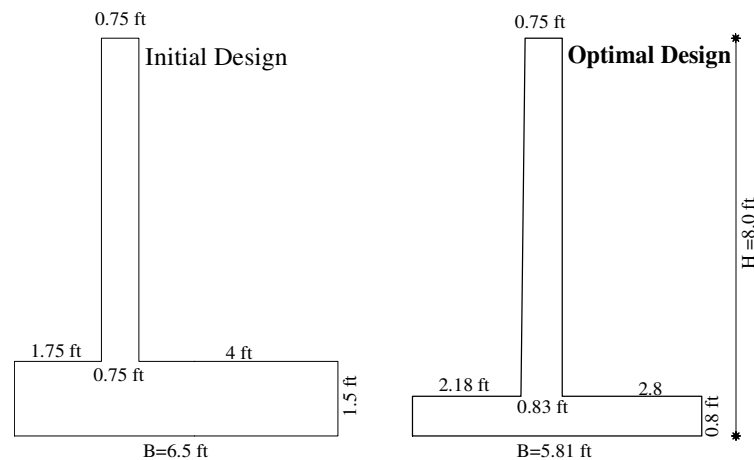
**Example 1:** The first example has been selected from reference No. 9 (Bowles, 1982) which is a common and popular reference among engineers and students. The allowable bearing capacity of foundation subsoil is 5 ksf. The safety factor for both overturning and sliding is 1.5. Other soil and concrete properties are as follows:  $f'_c=3$  ksi,  $f_y=30$  ksi,  $\gamma_c=150$  pcf,  $\phi=36^\circ$ . Also the price of steel and concrete considered as 0.45\$ per lb and 1.42\$ per cubic foot respectively. The height of the wall is 8 ft.

The outlined procedure was applied for this example to find the optimal design. The dimensions and cost of the *initial* and *optimal* design are tabulated in Table 2.

**Table 2- Comparison the cost of the *Initial* and *Optimal* design.**

Design case	Design variables							
	X1 $t_1$ (ft)	X2 $t_b$ (ft)	X3 $AS_{st}$ (in <sup>2</sup> )	X4 $L_{toe}$ (ft)	X5 $L_{heel}$ (ft)	X6 $h_1$ (ft)	X7 $AS_{toe}$ (in <sup>2</sup> )	X8 $AS_{heel}$ (in <sup>2</sup> )
<b><i>Initial</i></b>	0.75 ft	0.75 ft	0.62	1.75	4	1.5	0.59	0.59
<b><i>Optimal</i></b>	0.75 ft	0.833	0.35	2.18	2.8	0.8	0.27	0.27
Cost (\$/per unit length)	<b><i>Initial</i> Design</b>				<b><i>Optimal</i> Design</b>			
	<b>42.5</b>				<b>26.2</b>			

The proposed results clearly indicate that the cost of optimal design is about 38% less than the initial design. It is evident that the optimal design complies with all ACI requirements.



**Figure 3. A comparison between *initial* and *optimal* design**

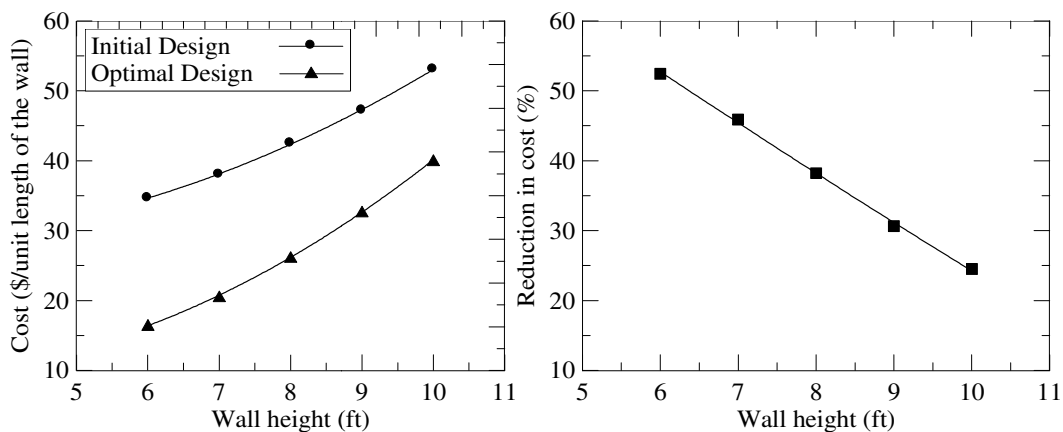


**Example 2 to 6:** The next five examples concern with RCC walls which their heights are different. For all these examples the soil and concrete properties and other requirements are the same as Example 1. Again the analysis results are summarized in Table 3.

**Table 3- The initial and Optimal design of different RCC walls**

Example	H (ft)	Design case	Design variables							
			X1(ft)	X2 (ft)	X3 (in <sup>2</sup> )	X4 (ft)	X5 (ft)	X6 (ft)	X7 (in <sup>2</sup> )	X8 (in <sup>2</sup> )
2	6	<i>Initial</i>	0.75	0.75	0.28	1.10	1.65	0.60	0.59	0.59
		<i>Optimal</i>	0.75	0.75	0.31	1.25	2.51	0.54	0.23	0.23
3	7	<i>Initial</i>	0.75	0.75	0.43	1.28	1.93	0.70	0.59	0.59
		<i>Optimal</i>	0.75	0.75	0.29	1.42	2.97	0.63	0.23	0.23
4	8	<i>Initial</i>	0.75	0.80	0.62	1.47	2.20	0.80	0.59	0.59
		<i>Optimal</i>	0.75	0.83	0.35	1.98	3.02	0.71	0.27	0.27
5	9	<i>Initial</i>	0.75	0.90	0.71	1.65	2.48	0.90	0.59	0.59
		<i>Optimal</i>	0.75	0.94	0.41	2.19	3.33	0.80	0.32	0.32
6	10	<i>Initial</i>	0.85	1.00	0.78	1.83	2.75	1.00	0.59	0.59
		<i>Optimal</i>	0.75	1.04	0.47	2.73	3.35	0.89	0.37	0.37

The cost of initial and optimal design for all wall tabulated in Table 3 are depicted in Figure 4. It is evident that there a large difference in cost between two designs. Also shown in this Figure is the rate of reduction in cost with height. It can be seen with increasing height of the wall, the rate of the total cost decreases.



**Figure 4. Optimal design of (RCC) retaining walls with different heights**

## 7. CONCLUSIONS

The Genetic algorithm, as a tool for cost minimization, was introduced in this paper. An objective function is proposed to minimize the total cost of reinforced concrete cantilever (RCC) walls. The proposed method is a self adaptive genetic based technique that uses input data as a primary design to evaluate and calculate the optimal design. The results clearly indicate that this method is a useful and powerful method for cost minimization. It is shown that for many practical situations the reduction in cost reaches up to 35%.

## 8. REFERENCES

1. Dembicki E, Chi T. (1989), "System Analysis in Calculation of Cantilever Retaining Wall", International Journal for Numerical and Analytical Method in Geo-Mechanics, **13**, pp. 599-610.
2. Keskar A. V. and Adidam S. R. (1998), "Minimum Cost Design of a Cantilever Retaining Wall", The Indian Concrete Journal, Bombay, India, **7**, pp. 401-405.
3. Saribas A, Erbatur F. (1996), "Optimization and Sensitivity of Retaining Structures", Journal of Geotechnical Engineering, **8**, pp. 649-656.



4. Rhomberg E. J. and W. M. Street M. (1981), "Optimal Design of Retaining Walls", Journal of Structural Division, ASCE, **107**, pp. 992-1002.
5. Basudhar P. K. and Lakshman B. (2006), "Optimal Cost Design of Cantilever Retaining Walls", IGC 2006, Chennai, India.
6. Sivakumar B. and Munwar B. (2008), "Optimum Design of Cantilever Retaining Walls using Target Reliability Approach", International Journal of Geo-Mechanics, **8**, pp. 240-252.
7. Yepes V., Alcalá J., Perea C. and Gonzalez-Vidoso F., (2008), "A Parametric Study of Optimum Earth-Retaining Walls by Simulated Annealing, Engineering Structures", **30**, pp. 821-830.
8. Grierson, D. E. and Pak, W. H., (1993), "Optimal Sizing, Geometrical and Topological Design Using a Genetic Algorithm", Structural Optimization, **6**, pp. 151-159.
9. Bowles, J. (1982), "Foundation Analysis and Design", McGraw-Hill, New York.
10. ACI. (2005). Patent No. 318-08. Building Code Requirements for Structural Concrete and Commentary, American Concrete Institute.