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Formulation of secant and reloading soil deformation moduli using multi expression programming

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Abstract

Purpose – The purpose of this paper is to develop new constitutive models to predict the soil deformation moduli using multi expression programming (MEP). The soil deformation parameters formulated are secant (Es) and reloading (Er) moduli.

Design/methodology/approach – MEP is a new branch of classical genetic programming. The models obtained using this method are developed upon a series of plate load tests conducted on different soil types. The best models are selected after developing and controlling several models with different combinations of the influencing parameters. The validation of the models is verified using several statistical criteria. For more verification, sensitivity and parametric analyses are carried out.

Findings – The results indicate that the proposed models give precise estimations of the soil deformation moduli. The Es prediction model provides considerably better results than the model developed for Er. The Es formulation outperforms several empirical models found in the literature. The validation phases confirm the efficiency of the models for their general application to the soil moduli estimation. In general, the derived models are suitable for fine-grained soils.

Originality/value – These equations may be used by designers to check the general validity of the laboratory and field test results or to control the solutions developed by more in depth deterministic analyses.

Keywords Soils, Deformation, Modelling, Substructures, Soil deformation moduli, Multi expression programming, Plate load test, Soil physical properties, Prediction

Paper type Research paper

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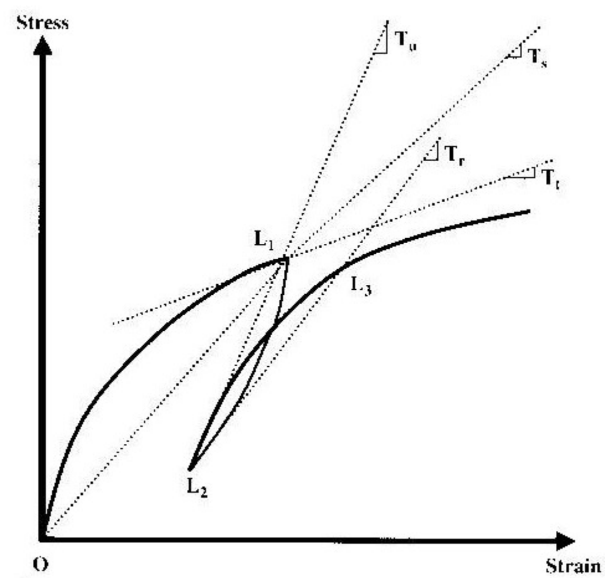
1. Introduction

It is known that both elastic and plastic deformations occur during the loading of soils. Different moduli can be obtained from the stress-strain curves because of the elasto-plastic behavior of soils (Briaud, 2001; Briaud *et al.*, 2006; Mollahasani *et al.*, 2011). Figure 1 shows a typical stress-strain curve. Referring to this figure, secant modulus (E_s) is calculated from the secant slope (T_s) corresponding to the slope from the origin (O) to L_1. Tangent modulus (E_t) is derived from the tangent slope (T_t). If the slope is drawn from L_1 to L_2, the unloading slope (T_u) is derived and the unloading modulus (E_u) is obtained from it. Reloading modulus (E_r) corresponds to the slope from L_2 to L_3 (T_r) (Briaud, 2001; Mollahasani *et al.*, 2011). The soil moduli have different applications in geotechnical engineering tasks. As an example, E_s can be used to predict the movement due to the first application of a load as in the case of a spread footing. E_u is useful in estimating the rebound of a pavement after the loading by a truck tire (resilient modulus). E_r might be employed to calculate the movement of the pavement under reloading by the same truck tire (Briaud, 2001).

Laboratory or field methods are widely used to estimate the soil deformation moduli (Murthy, 2008). One way of determining moduli is to perform triaxial compression tests on undisturbed samples. It is not practically possible to obtain undisturbed sample of cohesionless and even cohesive soils. Thus, the soil moduli values obtained from triaxial tests do not represent the actual conditions and give very approximate values (Murthy, 2008; Marsland, 1971). To overcome this limitation, field testing methods have increasingly been used to determine the soil deformation parameters. The field test results have been found to be more reliable than those of the laboratory methods (Ismael, 1985; Reznik, 1995; Mollahasani *et al.*, 2011). The field tests commonly used for this purpose include plate load test (PLT), static cone penetration test (CPT),



Source: Briaud (2001)

Figure 1.
Definitions of soil moduli

pressuremeter test (PMT), flat dilatometer test and standard penetration test (SPT) (Murthy, 2008). However, determining the soil modulus from the laboratory or field tests is not an easy task. Performing such tests is usually extensive, cumbersome and costly. Furthermore, it is not always possible to conduct the tests. Hence, different empirical and semi-empirical correlations have been derived to predict the soil modulus. The correlations developed based on *n*-value of SPT or the CPT results are mainly applicable to cohesionless soils. The PMT results can be used for cohesive soils (Murthy, 2008). Table I presents some of the E_s prediction equations in terms of N and plasticity index (PI). In this context, Platonov *et al.* (1974) and Reznik (1995, 2007) formulated oedometer deformation moduli of loessial soils in terms of soil void ratios and degrees of saturation.

PLT has been a traditional *in-situ* method for estimating the soil moduli (Mollahasani *et al.*, 2011). The effect of the scale factor and soil sample disturbance is minimized by using the PLT results (Reznik, 1993). This test is one of the best feasible choices when dealing with an unusual soil formation without prior experience (Lin *et al.*, 1998; Canadian Geotechnical Society, 1985). Although this testing method provides reliable results, very limited research has been done to develop prediction models for the soil deformation moduli using the PLT results. In this context, Reznik (1995) proposed analytical expressions describing dependence of the plate load deformation moduli of collapsible soils on soil void ratio and moisture content. Regression analysis is the approach used for developing most of the existing empirical models for the prediction of the soil moduli (Reznik, 2007). The significant limitations the traditional statistical techniques strongly affect the prediction capabilities of such models (Mollahasani *et al.*, 2011; Alavi *et al.*, 2011). The issues raised above suggest the necessity of employing more comprehensive methods to decrease errors for the soil deformation moduli estimations.

Several computer-aided data mining approaches have been developed for modeling nonlinear systems. Genetic algorithm (GA) is a powerful stochastic optimization technique based on the principles of genetics and natural selection. GA has been shown to be robust for dealing with a wide variety of engineering problems (Simpson and Priest, 1993; Yeo and Agyei, 1998; Toropov and Mahfouz, 2001; Balamurugan *et al.*, 2006; Kaveh and Shahrouzi, 2006). Genetic programming (GP) (Koza, 1992, Banzhaf *et al.*, 1998) is a specialization of GA where the solutions are computer programs rather than

Soil	E_s
Sand (normally consolidated)	$500\ (N_{cor} - 15)$, $(35{,}000\text{-}50{,}000)\ \log(N_{cor})$
Sand (saturated)	$250\ (N_{cor} + 15)$
Sand (over consolidated)	–
Gravelly sand gravel	$1{,}200\ (N_{cor} + 6)$
Clayey sand	$250\ (N_{cor} + 15)$
Silty sand	$300\ (N_{cor} + 6)$
Sands, gravels and other cohesionless soils	3.5 N
Low PI (< 12%)	2.5 N
Medium PI (12% < PI < 22%)	1.5 N
High PI (22% < PI < 32%)	1 N
Extremely high PI (PI > 32%)	0.5 N

Source: Look (2007) and Murthy (2008)

Table I. Empirical equations for predicting E_s based on SPT values and PI binary strings. GP can be considered as an alternative approach for behavioral modeling of geotechnical engineering tasks. The main advantage of GP over the conventional statistical methods and other soft computing tools is its ability to generate prediction equations without any need to assume prior form of the existing relationship (Alavi *et al.*, 2011). In contrast with ANNs and GA, application of GP in the field of civil engineering is quite new. Classical GP and its variants have been utilized to derive greatly simplified formulations for civil engineering problems (Alvarez *et al.*, 2000; Ashour *et al.*, 2003; Javadi *et al.*, 2006; Baykasoglu *et al.*, 2008, Cevik and Cabalar, 2009; Gandomi *et al.*, 2010, 2011). Recently, Mollahasani *et al.* (2011) employed a new branch of GP, called gene expression programming, to develop new prediction models for the PLT soil moduli. GP and its variants possess some obvious superiority than ANNs in dealing with geotechnical problems (Rezania and Javadi, 2007; Kayadelen *et al.*, 2009; Alavi *et al.*, 2010b). Multi expression programming (MEP) (Oltean and Dumitrescu, 2002) is a new variant of GP. The significant advantages of the MEP approach over similar techniques have been show by Oltean and Grosşan (2003a). Unlike classical GP and other soft computing tools like neural networks, applications of MEP to solve problems in civil engineering are restricted to fewer areas (Baykasoglu *et al.*, 2008, Alavi *et al.*, 2010b; Alavi and Gandomi, 2011).

This paper aims at obtaining new empirical models for determining E_s and E_r using the MEP method. Various soil properties are used as the predictor variables. The proposed models are developed based on several PLTs performed in this study. The derived models are statistically compared with some empirical equations found in the literature.

2. Genetic programming

GP is a symbolic optimization method. It uses the principle of Darwinian natural selection to evolve computer programs. GP emerged as a distinct discipline after experiments of Koza (1992) on symbolic regression. Most of the genetic operators used in GA can also be implemented in GP with minor changes. The main difference between GP and GA is the representation of the solution. The GP solutions are computer programs that are represented as tree structures and expressed in a functional programming language (such as LISP) (Koza, 1992). GA creates a string of numbers that represent the solution. The traditional optimization techniques, like GA, are generally used in parameter optimization to evolve the best values for a given set of model parameters. GP, on the other hand, gives the basic structure of the approximation model together with the values of its parameters (Javadi and Rezania, 2009; Alavi and Gandomi, 2011). GP optimizes a population of programs according to a fitness landscape determined by a program ability to perform a given computational task. The fitness of each program in the population is evaluated using a fitness function. Hence, the fitness function is the objective function that GP aims to optimize (Torres *et al.*, 2009; Alavi and Gandomi, 2011). The classical GP approach is referred to as tree-based GP. A population member in tree-based GP is a hierarchically structured tree comprising functions and terminals. The functions and terminals are selected from a set of functions and a set of terminals. The functions and terminals are chosen at random and constructed together to form a computer model in a tree-like structure with a root point with branches extending from each function and ending in a terminal (Alavi and Gandomi, 2011). Figure 2 shows a typical representation of a GP model.

MEP is a linear variant of GP. The linear variants make a clear distinction between the genotype and the phenotype of an individual. Thus, the individuals are represented

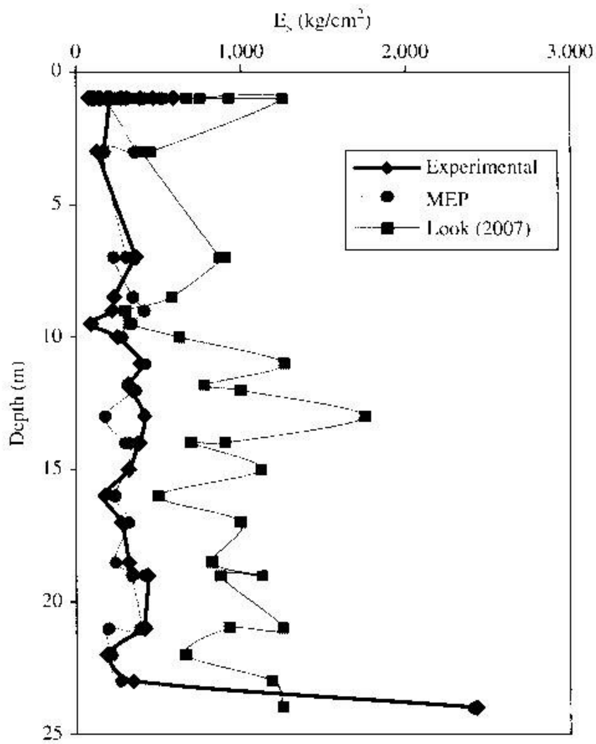


Figure 10.
Experimental versus predicted E_s values using different models for the entire database

ANNs do not give a transparent function relating the inputs to the corresponding outputs. Furthermore, determining an optimal ANN architecture (number of inputs, transfer functions, number of hidden layers and their number of nodes, etc.) is usually done through a time-consuming trial and error procedure (Shahin *et al.*, 2009; Javadi and Rezania, 2009; Alavi *et al.*, 2010a). On the other hand, MEP provides a transparent and structured representation of the system being studied. In MEP and other GP techniques, the number and combination of terms are automatically evolved during model calibration (Alavi *et al.*, 2010a).

6. Sensitivity and parametric analyses

Sensitivity and parametric analyses of the factors affecting E_s and E_r are carried out in order to test the robustness of the developed models. For the aim of sensitivity analysis, frequency values of the input variables are obtained. A frequency value equal to 100 percent for an input indicates that this variable has been appeared in 100 percent of the best 30 programs evolved by MEP. This methodology is a common approach in the GP-based analyses (Gandomi *et al.*, 2010; Alavi *et al.*, 2010b). The frequency values of the predictor variables are shown in Figure 11. According to these results, it can be found that W, D_{60} and γ_d exert dominant influence on the variations of E_s and E_r.

For further verification of the MEP-based prediction equations, a parametric analysis is performed in this study. The parametric analysis investigates the response of the predicted E_s and E_r from the MEP models to a set of hypothetical input data. The methodology is based on the change of only one predictor variable at a time while the other seismic variables are kept constant at the average values of their entire datasets. A set of synthetic data for the single varied parameter is generated by increasing the value of this in increments. These variables are presented to the prediction equations and E_s and E_r are calculated. This procedure is repeated using another variable until the model response is tested for the selected input variables. Figures 12 and 13, respectively, present the tendency of the E_s and E_r predictions to the variations of FC, D_{10}, D_{30}, D_{60}, LL, W and γ_d. As can be seen in Figure 12, E_s decreases due to increasing FC, D_{30}, LL, and W. Also, it increases with increasing D_{60} and γ_d. As shown in Figure 13, E_r increases due to increasing D_{30}, D_{60}, W, and γ_d. E_r decreases with increases in the percentages of LL. The reloading modulus does not exhibit notable sensitivity to the changes of D_{10}. The results of the parametric analysis for FC, LL, W, and γ_d are generally expected cases from a geotechnical engineering viewpoint. It is well-known that fine-grained soils with higher fine content are more compressible and have lower soil modulus. Soils with high LL have loose structures due to high clay content. The moisture content has a major influence on the soil modulus. As the moisture content increases, the water occupies more room and pushes the particles apart. Consequently, the compressibility increases and the modulus decreases. γ_d is an indicator of compressibility of a soil. If the soil particles are closely packed, the modulus tends to be high (Briaud, 2001).

7. Summary and conclusion

In the present study, the MEP approach is used to develop new design equations for predicting the soil deformation moduli (E_s and E_r). The proposed relationships are developed based on several PLT results obtained through an extensive experimental study. The best models are selected after developing and controlling several models with different combinations of the predictor variables. The developed relationships give reliable estimates of the E_s and E_r values. The E_s prediction model provides remarkably better results compared with the E_r prediction model. The validity of the MEP models was tested for a part of test results beyond the training data domain. Further, the models efficiently satisfy the conditions of different criteria considered for their external validation. The E_s prediction model produces considerably better outcomes than several empirical equations found in the literature. The developed models are mostly suitable for fine-grained soils with physical properties similar to the soil samples used in this study (i.e. ML, CL-ML, CL, SM, SW-SM, and GM).

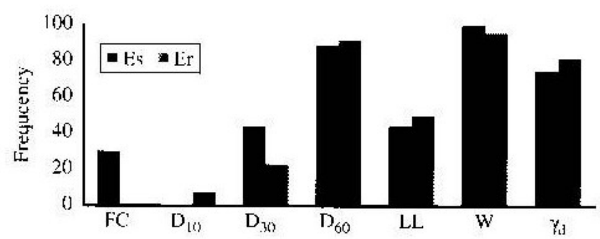


Figure 11.
Contributions of the predictor variables in the MEP models

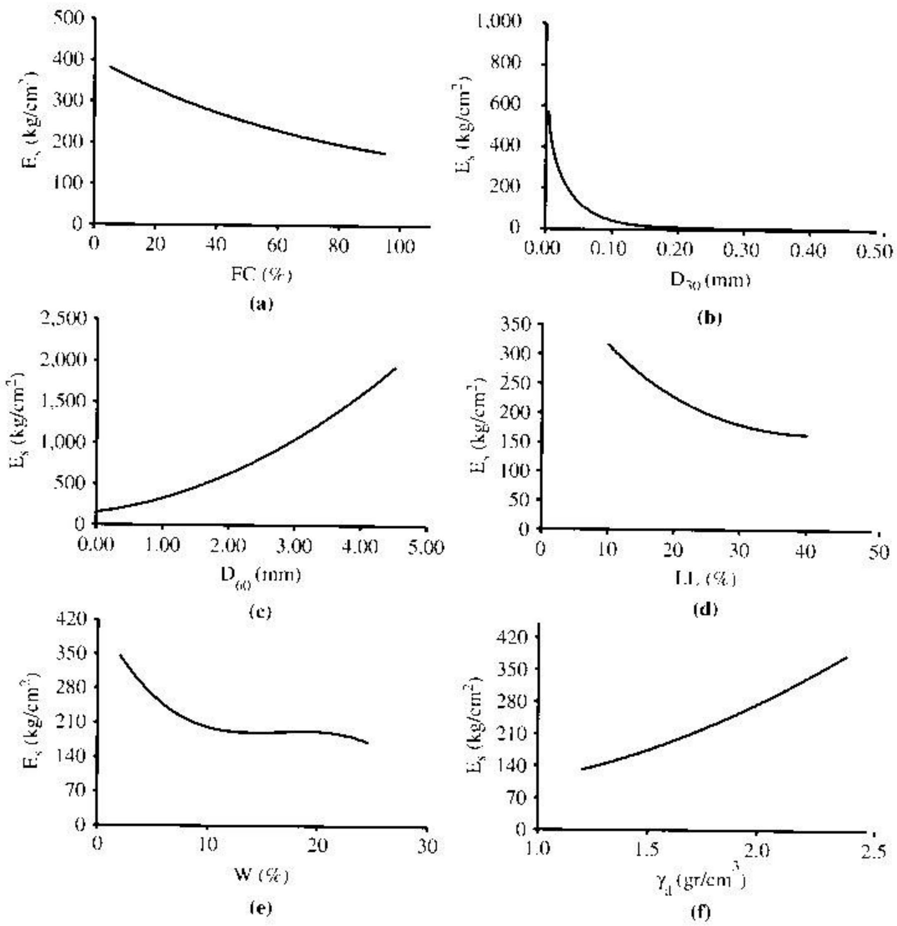


Figure 12.
Parametric analysis of E_s in the MEP model

The models reflect the results which are obtained using a round plate with a diameter of 305 mm. The proposed models simultaneously take into account the role of several important parameters (FC, D_{10}, D_{30}, D_{60}, LL, W, γ_d) representing the soil moduli behavior. Based on the results, W and γ_d can be regarded as efficient representatives of the initial state and consolidation history of the soil for determining the soil moduli. A distinctive feature of MEP-based constitutive models is that they are based on the experimental data rather than on assumptions made for developing the conventional models. The soil moduli can easily be estimated from the soil physical properties using the derived models. Unlike the existing empirical equations, there is no need to go through sophisticated and time-consuming field experiments before implementing the models. An observation from the results of the sensitivity analysis is that the soil deformation moduli are more affected by W, D_{60} and γ_d than other soil properties.

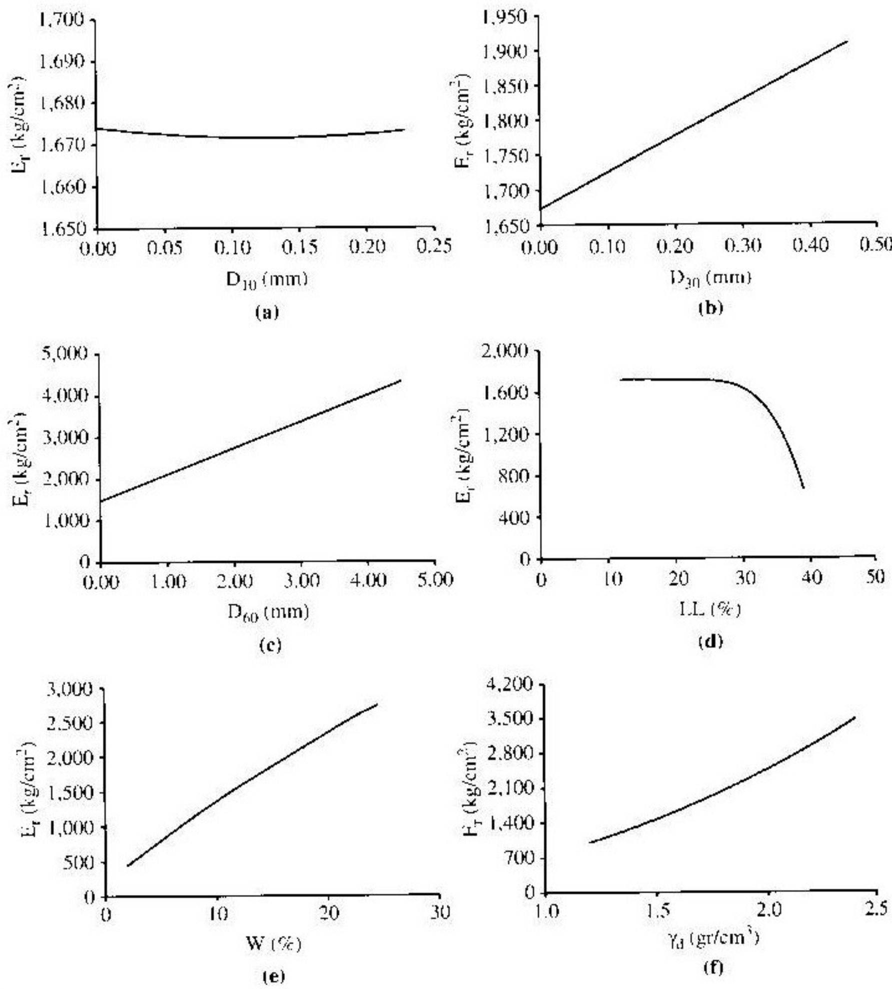


Figure 13.
Parametric analysis of E_r in the MEP model

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Appendix. The MEP algorithm

MEP uses the following steps to evolve the best expression until a termination condition is reached (Oltean and Grosşan, 2003a; Alavi *et al.*, 2010b):

(1) selecting two parents using a binary tournament procedure and recombining them with a fixed crossover probability;

(2) obtaining two offspring by the recombination of two parents; and

(3) mutating the offspring and replacing the worst individual in the current population with the best of them (if the offspring is better than the worst individual in the current population).

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