STABILITY OF ONE BAY SYMMETRICAL FRAMES WITH NONUNIFORM MEMBERS

F. Irani

Faculty of Engineering, Ferdowsi University
Mashhad, Iran

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Abstract This paper deals with simple portal or gable steel frames with varying moment of inertia. Critical load for such frames is calculated by means of a very simple and approximate method through which the variation of moment of inertia for the members is considered by a quadratic function and then the equilibrium and continuity conditions have been used. The degree of precision of this method has been checked by a computer method in a numerical example. The method is applicable only for one bay steel frames.

چکیده
در این مقاله پایداری قابهای ساده با تیر افقی یا شیبدار که دارای اعضائی با لنگر لختی متغیر می‌باشند مورد بررسی قرار گرفته است. بار بحرانی چنین قابهائی با انتخاب یک تابع درجه دو برای تغییرات لنگر لختی اعضا و با اعمال شرایط تعادل و پیوستگی قاب محاسبه شده و میزان دقت آن به کمک روش ماتریسی طی مثالی عددی معین شده است. این روش را تنها می‌توان برای قابهای فولادی یک دهانه به کار برد.

INTRODUCTION

The steel frames with varied sections are made in different shapes and dimensions and used in various industrial buildings. The problem of stability analysis of compressive members with varied cross sections and also the stability analysis of rigid frames with uniform members have been treated in the past and are documented in standard text books [1, 2].

The method which already exists to determine the critical load of rigid frames with variable moment of inertia is based upon Chu Kia Wang numerical method [3]. In that method the exact solution is achieved only if the varied member of the frames are divided into very short segments which needs to provide a large number of input data.

The method described in this paper provides an approximate solution for steel I-shaped members with high degree of precision by using the equilibrium and continuity condition at the joints of rigid frames.

All computations can be performed using a hand calculator.

This method is recommended to determine the critical load of one bay steel rigid frames with hinged supports.

METHOD OF ANALYSIS

We already know that the buckling load and buckling curvature for a frame depend on the possibility of that frame to sway, and the use of equilibrium and continuity condition at top joint of column will be a possible way to determine the critical load of the frames.

In this method, the moment of inertia of the column is determined by Equation 2, (see Figure 1) which is close enough to its real value in the case of I-shaped steel section.

In appendix I, the numerical error involved in using Equation 2 for a varied I-shaped member is presented. It should be noted that γ in Figure 1 is defined by Equation 1. For definition of all other symbols see Appendix IV.

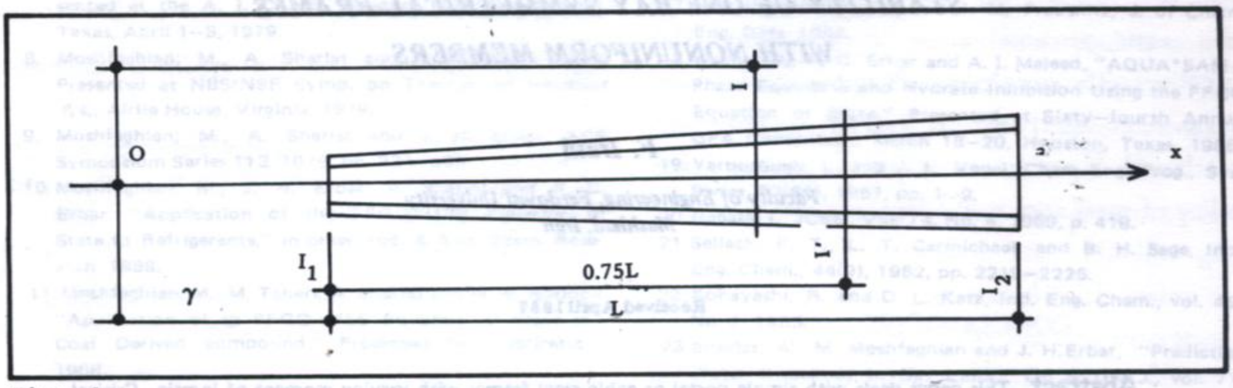


Figure 1.

$$\gamma = \frac{0.75L}{\sqrt{\frac{I'}{I_1}} - 1} \quad (1)$$

the moment of inertia of columns (Figure 1) can be defined by the following equation:

$$I = I_1 \left(\frac{x}{\gamma}\right)^2 \quad (2)$$

a: Critical Load For the Frames Under Concentrated Loads

Consider the simple frame shown in Figure 2 which is restrained against sway; with the coordinate axes shown, the equation of equilibrium of column AB will be:

$$EI_{1c}\left(\frac{x}{\gamma_c}\right)^2 \frac{d^2y}{dx^2} - m\,\frac{x-\gamma_c}{s} + \alpha P y = 0 \quad (3)$$

where

$$\alpha = \frac{s}{h} \quad (4)$$

if

$$\frac{\alpha P \gamma_c^2}{EI_{1c}} > \frac{1}{4} \quad (5)$$

then β can be expressed by the following relation:

$$\beta = +\sqrt{\frac{\alpha P \gamma_c^2}{EI_{1c}} - \frac{1}{4}} \quad (6)$$

Considering all of the boundary limits the elastic curvature of the column can be expressed as;

$$y = \frac{m}{\alpha P}\left[\sqrt{\frac{x}{\gamma_c + s}} \times \frac{\sin\left(\beta \ln \frac{x}{\gamma_c}\right)}{\sin\left(\beta \ln \frac{\gamma_c + s}{\gamma_c}\right)} - \frac{x-\gamma_c}{s}\right] \quad (7)$$

The angle of rotation at the top end "B" of the column will be equal to:

$$\theta_{BA} = -\left(\frac{dy}{dx}\right)_{x=\gamma_c+s} = \frac{m}{\alpha P(\gamma_c + s)} \times \left[1 + 2\beta \cot\left(\beta \ln \frac{\gamma_c + s}{\gamma_c}\right) - 2\,\frac{\gamma_c + s}{s}\right] \quad (8\text{-a})$$

and the angle of rotation at the B end of the beam will be expressed by:

$$\theta_{BC} = \frac{m}{EI_{e1}} \times \frac{b}{2} \quad (8\text{-b})$$

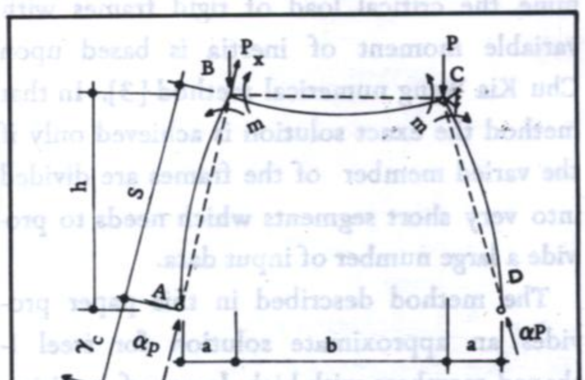


Figure 2.

where I_{e1} is "equivalent moment of inertia of beam" and its value for various types of beams is given in appendix II.

By equating (8-a) and (8-b) the buckling equation of the frames will be;

$$(\beta^2+\frac{1}{4})\left(\frac{\frac{EI_{2c}}{\gamma_c+s}}{\frac{EI_{e1}}{b}}\right) = 1+2\beta\cot\left(\beta\ln\frac{\gamma_c+s}{\gamma_c}\right) - \frac{2(\gamma_c+s)}{s} \qquad (9)$$

For the frames not prevented against sidesway (Figure 3) the equation of equilibrium of Column AB will be expressed by Relation 10. The angle of rotation for top end of column and right end of beam are Equations 11-a and 11-b;

$$EI_{1c}\left(\frac{x}{\gamma_c}\right)^2\frac{d^2y}{dx^2}+\alpha Py=0 \qquad (10)$$

$$\theta_{BA}=\left(\frac{dy}{dx}\right)_{x=\gamma_c+s}=\frac{m}{2\alpha P(\gamma_c+s)}$$

$$\left[1+2\beta\cot\left(\beta\ln\frac{\gamma_c+s}{\gamma_c}\right)\right] \qquad (11\text{-a})$$

$$\theta_{BC}=\frac{m}{EI_{e2}}\times\frac{b}{6} \qquad (11\text{-b})$$

where I_{e2} is also called "equivalente moment of inertia of beam" and its value for various types of beams is given in appendix II.

Equating the two relations (11-a) and (11-b) leads to the following buckling formula:

$$\left(\frac{\beta^2+\frac{1}{4}}{3}\right)\left(\frac{\frac{EI_{2c}}{\gamma_c+s}}{\frac{EI_{e2}}{b}}\right)=1+2\beta\cot\left(\beta\ln\frac{\gamma_c+s}{\gamma_c}\right) \qquad (12)$$

b: Critical Load of the Frames Under Distributed Load

Considering the frame in Figure 4, the differential equation of buckled column can be expressed by;

$$EI_{1c}\left(\frac{x}{\gamma_c}\right)^2\frac{d^2y}{dx^2}+Vy=\left(\frac{m}{s}-H\right)(x-\gamma_c) \qquad (13)$$

Following similar steps used for concentrated load, and using the value for β from Equation 14, we again reach Eq. 9 as the buckling formula for the frame with distributed load.

$$\beta=\sqrt{\frac{V\gamma_c^2}{EI_{1c}}-\frac{1}{4}} \qquad (14)$$

Stresses in different elements of the frame can be determined from known value of V (critical compressive load) and the reaction

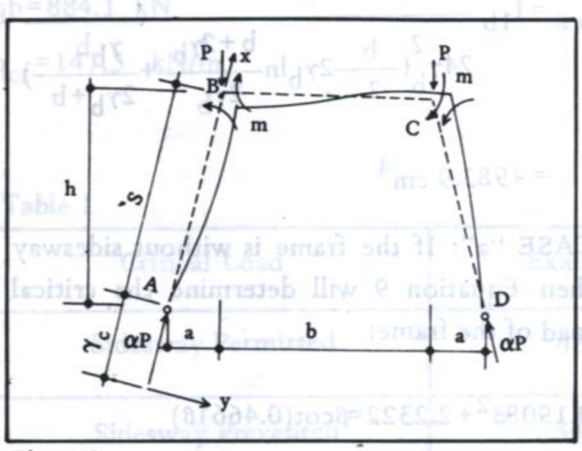


Figure 3.

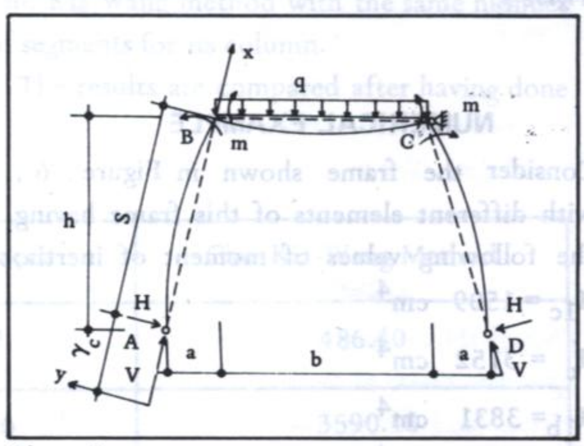


Figure 4.

"H". This can be done using classical methods of structural mechanics.

In general case H will be calculated by Equation 15;

$$H=\frac{M_{B}}{s}=\frac{qb^2}{24sN} \qquad (15)$$

Appendix III gives the value of N for several types of frames.

The differential equation of buckled column in Figure 5 which shows a frame with possible sidesway will be expressed by Equation 16;

$$EI_{1c}(\frac{x}{\gamma_c})^2\frac{d^2y}{dx^2}+Vy=-H(x-\gamma_c) \qquad (16)$$

Once again the mathematical operation will show that the Equation 12 can present the buckling formula of the frame.

The reaction H can be again determined by Equation 15.

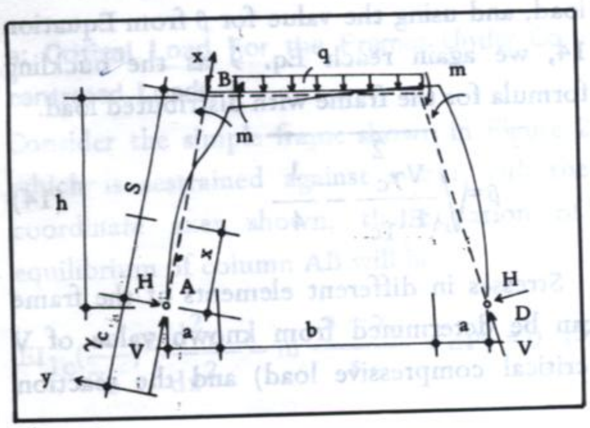


Figure 5.

NUMERICAL EXAMPLE

Consider the frame shown in Figure 6, with different elements of this frame having the following values of moment of inertia;

$I_{1c}=1509\ cm^4$

$I'_c=3152\ cm^4$

$I_{1b}=3831\ cm^4$

$I'_b=5000\ cm^4$

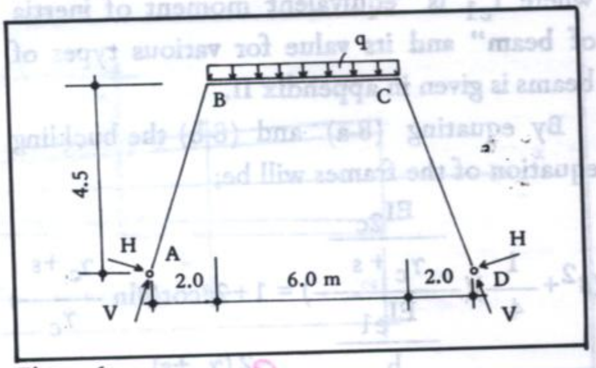


Figure 6.

The value of γ for column and beam can be calculated as below;

$$\gamma_c=\frac{0.75\ s}{\sqrt{\frac{I'_c}{I_{1c}}-1}}=829.5\ cm$$

$$\gamma_b=\frac{0.75(\frac{b}{2})}{\sqrt{\frac{I_b}{I_{1b}}-1}}=1579.7\ cm$$

I_{2c}, I_{e1} and I_{e2} for this frame will be:

$$I_{2c}=I_{1c}(\frac{\gamma_c+s}{\gamma_c})^2=3833\ cm^4$$

$$I_{e1}=I_{1b}\frac{b+2\gamma_b}{2\gamma_b}=4558.5\ cm^4$$

$$I_{e2}=I_{1b}\frac{b^3}{24\gamma_b^2(\frac{b}{2}-2\gamma_b\ln\frac{b+2\gamma_b}{2\gamma_b}+\frac{\gamma_b b}{2\gamma_b+b})}$$

$$=4983.9\ cm^4$$

CASE "a": If the frame is without sidesway then Equation 9 will determine the critical load of the frame;

$$0.1908\beta^2+2.2322=\beta\cot(0.4661\beta)$$

By trial and error method we can calculate

$\beta = 7.835$ and

$$V_{cr} = E\frac{I_{1c}}{\gamma_c^2}(\beta^2 + \frac{1}{4}) = 2838.7 \text{ kN}$$

The Equation 15 gives the reaction "H".

$$H = \frac{qb^2}{sN} = \frac{qb}{17.08}$$

$$-H\sin\alpha + V_{cr}\cos\alpha = \frac{qb}{2}$$

$qb = 4953$ kN

$q_{cr} = 825.5$ kN/m

CASE "b": If the frame is with sidesway, then the Equation 12 will determine the buckling load;

$$0.05818\beta^2 - 0.4855 = \beta\cot(0.4661\beta)$$

this equation gives $\beta = 3.279$ and

$$V_{cr} = E\frac{I_1}{\gamma_c^2}(\beta^2 + \frac{1}{4}) = 506.7 \text{ kN}$$

$$H = \frac{qb}{17.08}$$

$$-H\sin\alpha + V\cos\alpha = \frac{qb}{2}$$

$qb = 884.1$ kN

$q_{cr} = 147.3$ kN/m

DEGREE OF PRECISION OF THE METHOD

The degree of precision of this method can be determined by considering the frame in Figure 7 for which the moment of inertia of all elements is constant. The critical load of this frame is found by exact theory and by Chu Kia Wang method. In Chu Kia Wang method each column is divided into four equal segments. Both results are compared in Table 1;

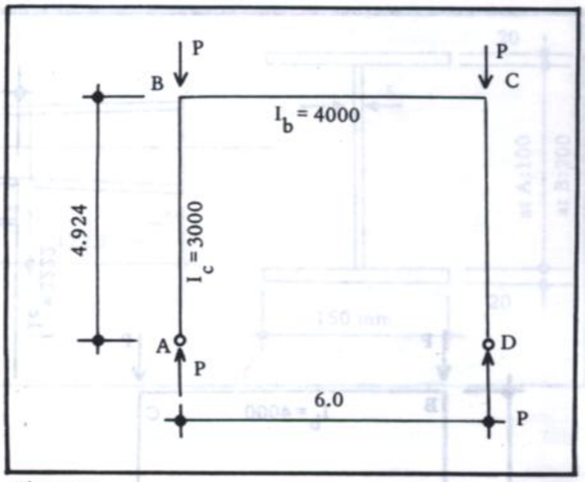


Figure 7.

Considering the above frame but with a variable moment of inertia for its column (see Figure 8). The buckling load is compared by the method of this article and by Chu Kia Wang method with the same number of segments for its column.

The results are compared after having done

Table 1

Critical Load	Exact Theory	Chu-Kia-Wang Method
Sidesway Permitted	487.70	486.40
Sidesway Prevented	3405.00	3590.10

Table 2

Critical Load	Wang Method Results		Correction Coef.		Probable Results	Article Results
Sidesway Permitted	727.64	×	$\frac{487.70}{486.40}$	=	729.60	732.00
Sidesway Prevented	4582.40	×	$\frac{3405.00}{3500.10}$	=	4346.20	4367.90

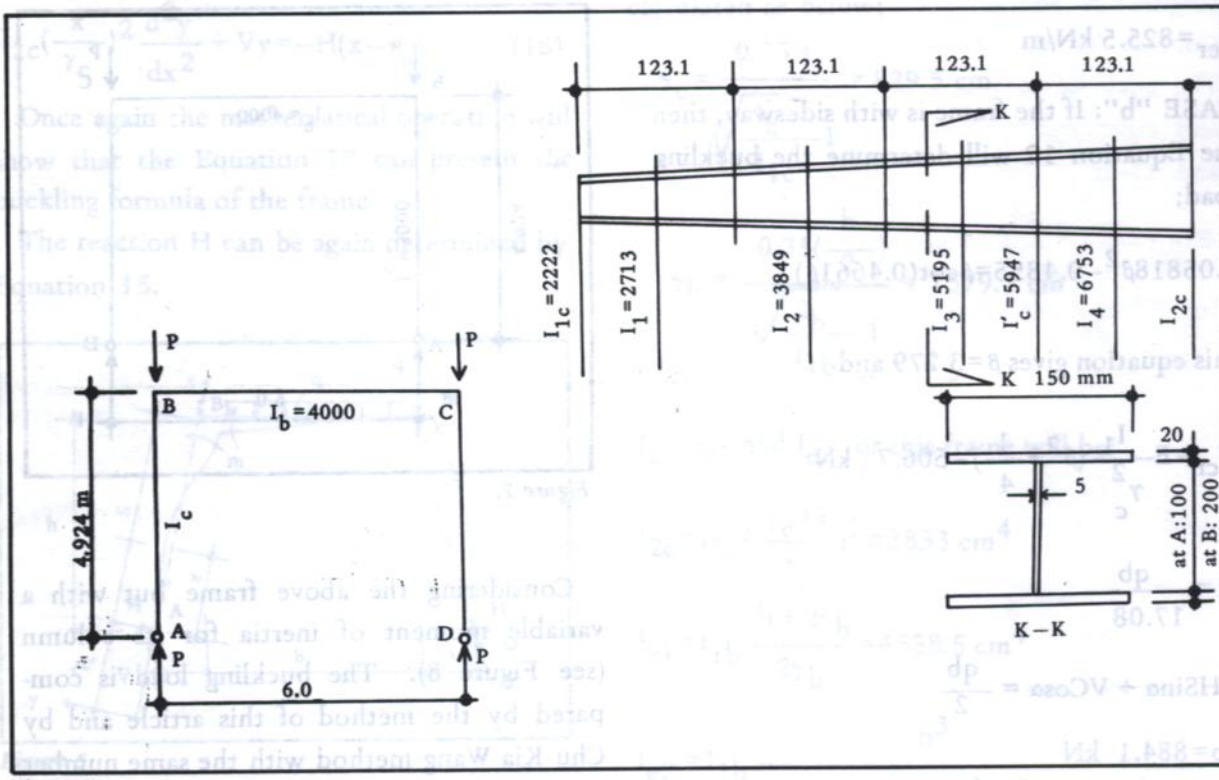


Figure 8.

the necessary corrections on Chu Kia Wang's results (see Table 2).

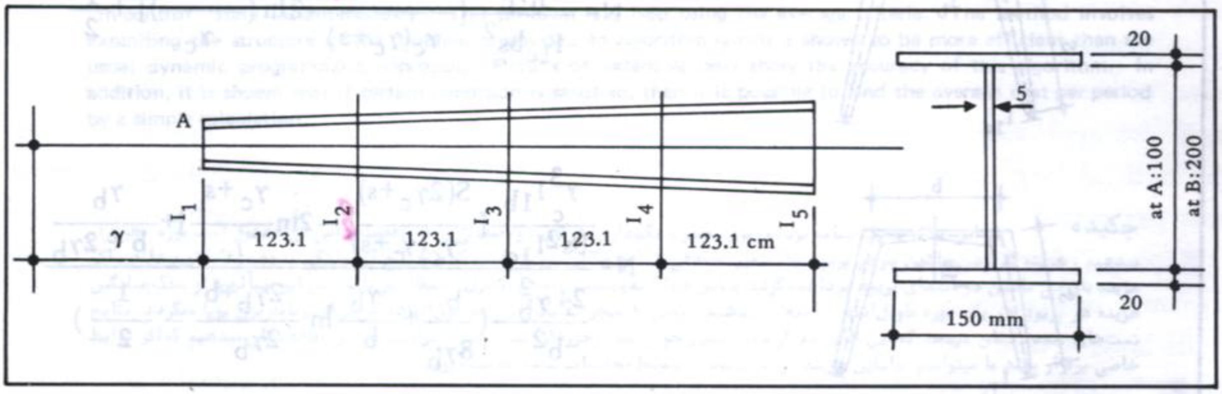
CONCLUSIONS

Though the method of this article which is based upon the classical method for determining the critical load for frames with constant moment of inertia is approximate, it is found that it is still valid for determining the critical load for frames with non-uniform moment of inertia with a good degree of precision. This method is fast and its calculations can be done only with a pocket calculator.

APPENDIX I

The erorr produced by using the Relation 2 for I shaped steel members is as follows:

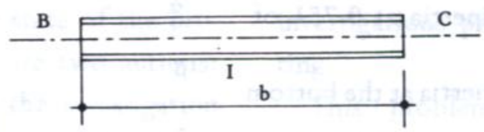
	I_1	I_2	I_3	I_4	I_5
Exact M. of Inertia	2222	3255	4496	5947	7613
M. of Inertia by Equation 2	2222	3264	4505	5947	7588
Error, %	0.0	+0.3	+0.2	0.0	−0.3



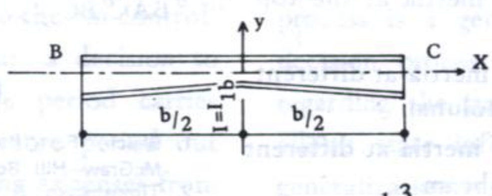
APPENDIX II

VALUES OF I_{e1} AND I_{e2} FOR DIFFERENT TYPES OF STEEL FRAMES.

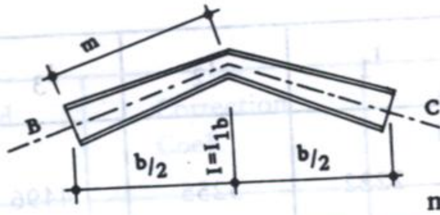
I_{e1} is the constant moment of inertia of such a beam if two flexural moments in opposit directions apply to the ends of that beam, the angle of rotation at the end of that beam equals the angle of rotation of the original beam under the same flexural moments.



$$I_{e1} = I \quad , \quad I_{e2} = I$$



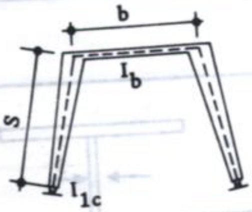
$$I_{e1} = I_{1b} \frac{b+2\gamma_b}{2\gamma_b} \;, \quad I_{e2} = I_{1b} \frac{b^3}{24\gamma_b^2\left(\frac{b}{2} - 2\gamma_b \ln\frac{b+2\gamma_b}{2\gamma_b} + \frac{\gamma_b b}{2\gamma_b + b}\right)}$$



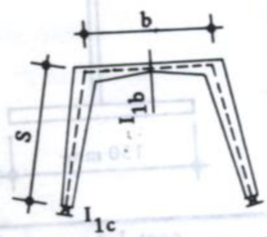
$$I_{e1} \approx I_{1b}\left(\frac{m+\gamma_b}{\gamma_b}\right), \quad I_{e2}=I_{1b}\frac{m^3}{3\gamma_b^2\left(m-2\gamma_b\ln\frac{m+\gamma_b}{\gamma_b}+\frac{\gamma_b m}{\gamma_b+m}\right)}$$

APPENDIX III

VALUE OF N



$$N=\frac{I_b\gamma_c^3}{I_{1c}bs^2}\left[\frac{s(2\gamma_c+s)}{\gamma_c(\gamma_c+s)}-2\ln\left(\frac{\gamma_c+s}{\gamma_c}\right)\right]+\frac{1}{2}$$



$$N=\frac{\frac{\gamma_c^3 I_{1b}}{bs^2 I_{1c}}\left[\frac{S(2\gamma_c+s)}{\gamma_c(\gamma_c+s)}-2\ln\frac{\gamma_c+s}{\gamma_c}\right]+\frac{\gamma_b}{b+2\gamma_b}}{\frac{24\gamma_b^2}{b^2}\left(\frac{b}{8\gamma_b}+\frac{\gamma_b}{b}\ln\frac{2\gamma_b+b}{2\gamma_b}-\frac{1}{2}\right)}$$

APPENDIX IV

NOTATIONS

E	Young's modulus
H	reaction
h	vertical projection of column
I	moment of inertia of element
I'	moment of inertia at 0.75L of element
I_1	moment of inertia at the bottom of element
I_2	moment of inertia at the top of element
$I_c, I'_c, I_{1c}, I_{2c}$	moment of inertia at different sections of column
$I_b, I'_b, I_{1b}, I_{2b}$	moment of inertia at different sections of beam
L	lenght of element
N	dimensionless parameter shown in appendix III
P	vertical load
q	uniform load
q_{cr}	critical uniform load
s	lenght of column
V	axial reaction
V_{cr}	critical axial load
α	s/h
β	$\sqrt{\alpha P\gamma_c^2 / EI_{1c}-1/4}$
γ	$0.75L/(\sqrt{I'/I_1}-1)$
θ_{BA}, θ_{BC}	angle of rotation

REFERENCES

1. Bleich, F., "Buckling Strength of Metal Structures" McGraw–Hill Book Co., Inc., New York, N. Y., 1952.
2. Timoshenko, S. P., and Gere, J. M., "Theory of Elastic Stability" McGraw–Hill Book Co., Inc., New York, N. Y., 1961.
3. Chu–Kia Wang, "Stability of Rigid Frames with Nonuniform Members" Journal of Civil Engineering, Structural Division, ASCE Feb. 1967.