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Progressive Collapse Analysis of Steel Frames: Simplified Procedure and Explicit Expression for Dynamic Increase Factor

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Abstract

Progressive collapse refers to a phenomenon in which local damage in a primary structural element leads to total or partial structural system failure. When investigating the progressive collapse of structures, nonlinear dynamic procedures lead to more accurate results than static procedures. However, nonlinear dynamic procedures are very complicated and the evaluation or validation of the results can become very time consuming. Therefore, it is better to use simpler methods. For static analyses, the gravity force applied to the removed column bay should be multiplied by a constant factor of two. However, using a constant dynamic increase factor (DIF) is only appropriate for elastic systems. According to the optimal design of structures, the assumption of elastic behavior after column removal is conservative. Thus, it is necessary to establish an expression for DIF that considers inelastic responses. In this paper, a simplified analysis procedure for the progressive collapse analysis of steel structures is presented using the load displacement and capacity curve of a fixed end steel beam. The results of the proposed method are in good agreement with nonlinear dynamic analysis results. Also, the capacity curve, obtained by dividing the accumulated area under the nonlinear static load displacement curve by the corresponding displacement of the column removed point, is used to predict the progressive collapse resistance of the column removed structure. Finally, an explicit expression for the DIF is established for elastic-perfectly plastic and elastic plastic with catenary action behavior.

Keywords: progressive collapse, steel frames, simplified analysis, dynamic increase factor, capacity curve, catenary action

1. Introduction

Progressive collapse refers to the phenomenon in which the local damage of a primary structural element leads to total or partial structural system failure, without any proportionality between the initial and final damage. Although the probability of structural collapse is low, if it occurs, it can cause significant losses. In the past few decades, many examples of the total or partial collapse of structures due to fire, explosions or impacts have occurred.

The progressive collapse phenomena was first brought to engineers' attention due to the collapse of a 22 story building in Ronan Point, London (UK), as a result of a gas explosion in 1968 (McGuire, 1974; Leyendecker and Ellingwood, 1977). Furthermore, research in this area has been accelerated due to two significant terrorist attacks in the United States that resulted in the structural collapse of the buildings: the Alfred P. Murrah Federal Building bombing in 1995 in Oklahoma City (USA) (Corley *et al.*, 1998) and the destruction of the World Trade Center (WTC), in New York (USA) in 2001 (Bazant and Zhou, 2002; Bazant and Verdure, 2007; Bazant *et al.*, 2008; Seffen, 2008).

In the United States, the Department of Defense (DOD, 2005) and General Service Administration (GSA, 2003) issued design guidelines in an attempt to mitigate the potential for progressive collapse in structures. Both guidelines use the alternate load path (ALP) method for progressive collapse mitigation. The ALP method is an event independent procedure that does not consider the reason for the removal of the column; instead, a single column is typically assumed to be suddenly missing and an analysis is conducted to determine whether or not the structure can bridge support across the missing column. Progressive collapse is naturally dynamic and causes large displacements in the structure. The DOD and GSA allow three analysis methods: linear static (LS), nonlinear static (NLS), and nonlinear dynamic (NLD) methods. In the linear static analysis, the load combination according to Eq. (1) or (2) is applied to all beams of the frame:

$$\gamma\times(DL+0.25LL)\ \text{(GSA)} \tag{1}$$

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$\gamma\times(1.2DL+0.5LL)+0.2WL$ (DOD) (2)

where

DL: dead load

LL: live load

WL: wind load

γ: load factor that considers the dynamic effects in static analysis.

γ is 2 for the beams in bays adjacent to the removed column and to all floors above the removed column; it is equal to 1 for all other beams.

The GSA proposed that the demand to capacity ratio (DCR), the ratio of the member force and member strength, be used as the criterion to determine the failure of the main structural members using linear analysis procedures:

$$DCR=\frac{Q_{UD}}{Q_{CE}} \quad (3)$$

where

Q_{UD}: acting force determined in the member

Q_{CE}: expected ultimate unfactored capacity of that member.

The acceptance criterion is 3 for all members.

In the DOD regulations, the DCR is not evaluated, instead the design strength of member (the nominal strength multiplied the strength reduction factor) is multiplied by the over strength factor (1.1) and is then compared with the member force to determine the failure of members.

In the nonlinear static analysis, the applied load is similar to the linear static analysis; however, in the nonlinear dynamic analysis, γ is 1 for all beams. The failure criteria in nonlinear static and dynamic analyses are based on the beam chord rotation ($\theta\leq12^{o}$) and ductility of the column removed point ($m\leq20$). Beam chord rotation is the ratio of the vertical displacement of the removed column point to the length of the beam. Ductility is the ratio of the vertical displacement of the removed column point to the final elastic displacement of the same point. Therefore, in nonlinear analyses, evaluating the vertical displacement of the removed column point is very important.

Among the three analysis procedures designated by the DOD and GSA as suitable for structural analysis, the nonlinear dynamic method obtains the most accurate results (Marjanishvili and Agnew, 2006; Fu, 2009; Ruth *et al.*, 2006; Powel, 2005; Tsai and Lin, 2008). As the nonlinear dynamic analysis of 3D frames is very time consuming, most researches are performed on 2D frames (Sucuoglu *et al.*, 1994, Kim and Kim, 2009; Khandelwal *et al.*, 2009; Kim and Dawoon, 2009). Performing simpler and approximate analyses not only allows the analyst to disregard nonlinear dynamic analyses but provides reliable significant results (Grierson *et al.*, 2005; Izzuddin *et al.*, 2008a; Izzuddin *et al.*, 2008b; Lee *et al.*, 2009).

This paper presents the formulation of a simple model to evaluate the displacement of the removed column point. In this regard, the work done on the structure due to column removal is equal to the energy dissipated by the structure. Although in general this method may be applied to structures with any type of materials, the formulation presented here is only valid for structures with mild steel. To demonstrate the capability and validity of the proposed method, three 2D steel frames with different number of bays and stories were chosen. Nonlinear dynamic analysis of these structures has been performed by other researchers. It is shown that the results of the proposed method is in good agreement with nonlinear dynamic analysis results.

Finally, an explicit expression for the DIF is established for elastic-perfectly plastic and elastic plastic with catenary action behavior.

2. Previous Works

In Marjanishvili and Agnew (2006), an explanation of four methods used to perform progressive collapse analysis (LS, NLS, LD, and NLD) in SAP2000 is presented. Fu (2009) performing nonlinear dynamic analyses of a 20 story 3D structure, found that the columns that are adjacent to the removed column should be designed with an axial force twice that of the static axial force obtained when applying the DL+0.25 LL load combination. Furthermore, Fu (2009) found that column removal in the top stories leads to higher vertical deformations because fewer stories participate in the absorption of the released energy. Mohamed (2009) analyzed 3D concrete structures and investigated the shear stresses that resulted from the torsion in the beam connected to the corner column being removed. The shear stresses in these scenarios lead to brittle failure of the beam, but the 2D analysis models could not trace them.

Ruth *et al.* (2006) analyzed 2D and 3D steel frames: they illustrated that using load factor of 2 may be conservative, whereas using a load factor of approximately 1.5 captures better dynamic effects when static analyses are performed. However, the authors mentioned that if the behavior of the materials was not elastic-perfectly plastic, if the materials harden after yielding, and if the ductility of structure is high, using load factor of 2 may be more appropriate. As a result, their research suggested that a load factor of 2 should be used for important structures and 1.5 for other structures.

Powel (2005) compared LS, NLS, and ND analyses and found that if a load factor of 2 is used in static analyses, it can display very conservative results. Tsai and Lin (2008) evaluated the progressive collapse resistance of RC buildings and demonstrated that nonlinear static analyses provide more conservative estimations for the collapse resistance than nonlinear dynamic analyses. They also found that the load factor decreases with increases in the displacement of the removed column point. Sucuoglu *et al.* (1994) found that the frames that contained a removed column sustained most of the load that was created due to the column removal. Therefore, to evaluate the vertical displacement, the plastic hinge distribution, and rotation in 3D frames, it is sufficient to analyze the 2D frames that contain the removed column.

Kim and Kim (2009) studied the progressive collapse of the steel moment resisting frames. It was observed that the nonlinear static analyses provide smaller structural responses than nonlinear dynamic analyses. However, the linear procedure provides a more conservative decision for the potential of the structure. Khandelwal *et al.* (2009) analyzed braced frames designed for seismic activity and found that the eccentrically braced frame was less vulnerable to progressive collapse than the concentrically braced frame.

Kim and Dawoon (2009) investigated the effect of the catenary action on the progressive collapse potential of steel moment frame structures. According to the nonlinear static push-down analysis results, the contribution of the catenary action to the progressive collapse demonstrated that the potential of the structures increases as the number of stories and bays increases. Grierson *et al.* (2005) presented a method for conducting a linear static progressive collapse analysis. They modeled the reduced stiffness during the progressive collapse using an equivalent spring method.

Izzuddin *et al.* (2008a,b) presented a simplified method for nonlinear static analysis of steel structures. In their research, four stages of simplification were applied to the method. Lee *et al.* (2009) also developed a simplified tri-linear model for the relationship of the vertical resistance and chord rotation of the double span beam. This model depends explicitly on the beam length (l) and beam section depth (d). A response was obtained for three values of l/d: 10, 15, and 20. Lee *et al.* (2009) state that for other values of l/d, linear interpolation should be used.

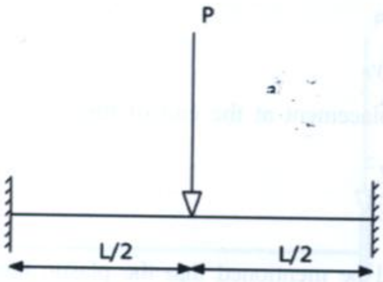

Figure 1. Fix ended beam under a concentrated load.

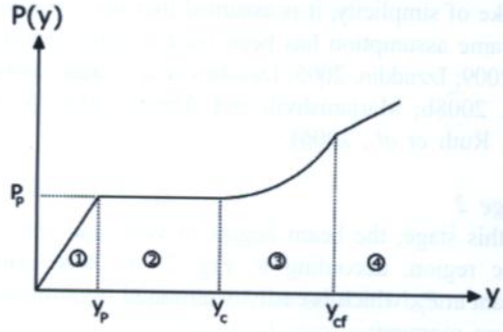

Figure 2. Load displacement response of a fixed end beam under a concentrated load (Valid for a mild steel).

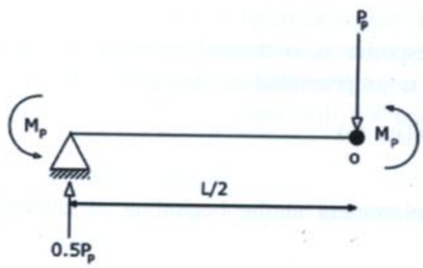

Figure 3. Formation of plastic hinges due to load increase.

3. Initial Formulation

3.1. Middle column removal

Consider a fix-ended mild steel beam with a concentrated load in the mid-span, as presented in Fig. 1. The load deflection curve of this beam has four stages, as shown in Fig. 2.

Stage 1

The beam has an elastic response and the load deflection curve in this stage is linear. The following equations represent the situation:

$$P(y)=Ky \tag{4}$$

$$K=\frac{192EI}{L^3} \tag{5}$$

where

K: bending stiffness of the beam

E: modulus of elasticity of the beam

I: moment of inertia of the beam

L: length of the beam

y: mid-span displacement of the beam

P: concentrated load applied at the middle of the beam

By increasing P, the bending moment of the beam increases until the beam reaches its maximum bending resistance at load P_p and plastic hinges form. According to Fig. 3:

$$\sum M_o=0\rightarrow\frac{P_P}{2}\times\frac{L}{2}=2M_P\rightarrow P_P=\frac{8M_P}{L} \tag{6}$$

where

M_p: the plastic moment of the beam

P_P: concentrated load applied at the middle of the beam which causes the beam to enter plastic region.

L: length of the beam

obtained:

$$P(y_P)=Ky_P \tag{7}$$

The displacement at the end of this stage is:

$$y_P=\frac{M_PL^2}{24EI} \tag{8}$$

It should be mentioned that the plastic hinges at the ends and mid-span of the fix-ended beam are not formed simultaneously and stage 1 is bilinear, but in this paper, for sake of simplicity, it is assumed that stage 1 is linear. The same assumption has been used in other researches (Fu, 2009; Izzuddin, 2005; Izzuddin *et al.,* 2008a; Izzuddin *et al.,* 2008b; Marjanishvili and Agnew, 2006; McKay, 2008; Ruth *et al.*, 2006).

Stage 2

In this stage, the beam begins to yield and enters the plastic region. according to Fig. 2, the load remains constant at P_p, which is easily determined from the plastic bending moment:

$$P(y)=\frac{8M_P}{L} \tag{9}$$

This response is continued until the displacement is equal to y_c as presented in (Izzuddin, 2005):

$$y_c=\max(y_P, r_P) \tag{10}$$

where

y_c: displacement at the beginning of catenary action region

r_P: plastic interaction radius

For typical I sections r_P is approximately half the cross section depth ($r_P=\frac{d}{2}$).

When displacement reaches y_c, the beam enters Stage 3.

Stage 3

In this stage, the catenary action is activated and the load deflection response is (Izzuddin, 2005):

$$P(y)=\frac{8}{L}\left[M_p+K_e\frac{2(y-y_c)(y+y_c-2r_p)(y-y_p)}{L}\right] \tag{11}$$

where K_e (effective axial stiffness) is determined from $\frac{1}{K_e}=\frac{1}{K_{hr}}+\frac{L}{EA}+\frac{1}{K_{hl}}$, EA is the elastic axial rigidity of the beam, and K_{hr} and K_{hl} are the stiffness of right and left frames of the removed column bays, respectively. As an example, Fig. 4 is considered to calculate K_{hr} and K_{hl}. To determine the stiffness of the left bay, the stiffness of the frame in Fig. 5 should be considered.

If the number of stories is high, the frame in Fig. 5 can be simplified into frame in Fig. 6.

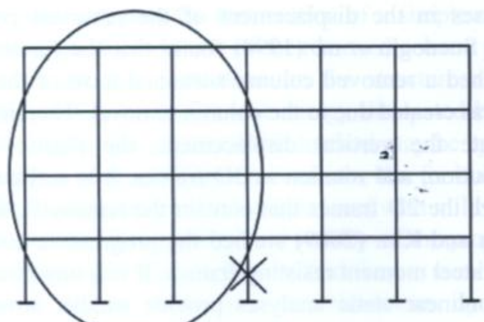

Figure 4. Frame on the left side of the removed column.

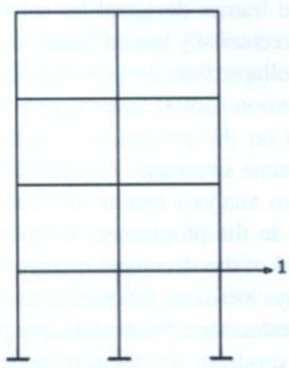

Figure 5. Stiffness calculated for the left frame.

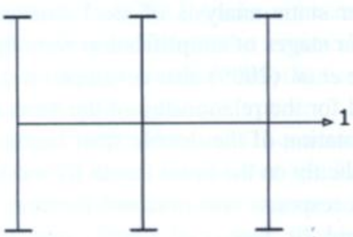

Figure 6. Simplified model for stiffness calculation.

Figure 7. Simplified model of a fix-ended beam.

The restraint condition for each column is according to Fig. 7.

The stiffness of the column in Fig. 7 is $\frac{12EI}{L^3}$. For general cases in which there are n columns in each side, stiffness is calculated using:

$$K_{hr,l}=\frac{24nEI}{L^3} \tag{12}$$

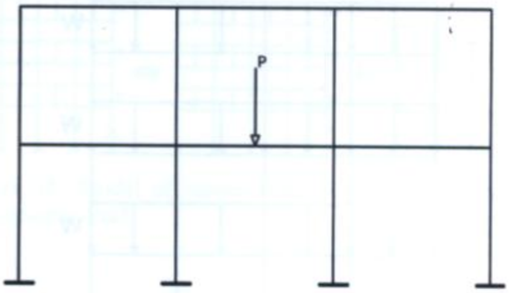

Figure 8. Frame under concentrated load.

This response is valuable until y_{cf} reaches the value in Eq. (13) (Izzuddin, 2005):

$$y_{cf}=y_c+\sqrt{(y_c-r_p)^2+\frac{F_pL}{2K_e}} \tag{13}$$

where

F_p: plastic tension force

Using the linear interaction between the moment and axial force, $F_P=\dfrac{M_P}{r_P}$ is obtained.

Stage 4

Beyond y_{cf}, the beam reaches its full plastic axial capacity with a zero bending moment. Therefore, the load deflection response in the final catenary stage may be expressed as (Izzuddin, 2005):

$$P(y)=\frac{8F_Py}{L} \tag{14}$$

Using the response of a single beam, the response of a beam in a frame must be formulated prior to the analysis. Consider the frame in Fig. 8.

If the deflections of the adjacent beams and columns are neglected, according to Fig. 1, the response of the beam may be calculated using equations above.

It should be noted that simplifying the frame, as in Fig. 1, presents the beam behavior as stiffer than the actual situation. As the beam in a frame is not completely restrained at the ends, the effects of rotations of the joints should be considered in the calculations. Assume that the inflection point of the columns is in the middle of their height (only the first and last story columns do not have this limitation): the load deflection response of the beam is corrected as follows (Dussenberry and Hamburger, 2006):

$$P(y)=\frac{192EI}{L^3(1+3R_b)}y \tag{15}$$

$$y_P=\frac{M_PL^2(1+3R_b)}{24EI} \tag{16}$$

Figure 9. Fixed end beam under a concentrated load at one end.

where R_b is:

$$R_b=\frac{I}{I+4I_{ab}+6I_c\frac{L}{H}} \tag{17}$$

where

I_{ab}: moment of inertia of the adjacent beam

I_c: moment of inertia of adjacent columns

H: length of adjacent columns

As in many practical cases, the connections do not fail until the end of catenary action behavior region (Lee *et al.*, 2009), in this study the maximum expected displacement of the column removed point is assumed to be y_{cf} and it is assumed that connection failure will not occur.

3.2. Corner column removal

Consider a beam with the load and restrain conditions as presented in Fig. 9.

The difference between the load deflection response of this beam and that in the beam in Fig. 2 is that, this beam has moment resistance only and does not have catenary action. By increasing P, plastic hinges are formed at both ends of the beam. According to Fig. 10, the following is obtained:

$$\sum M_o=0\rightarrow P_P\times L=2M_P\rightarrow P_P=\frac{2M_P}{L} \tag{18}$$

where

M_p: plastic moment of the beam

According to the beam stiffness in the elastic stage ($K=\dfrac{12EI}{L^3}$), the final displacement in this stage is:

$$y_P=\frac{M_PL^2}{6EI} \tag{19}$$

At the end of the elastic stage, the beam enters a plastic stage and results in the following:

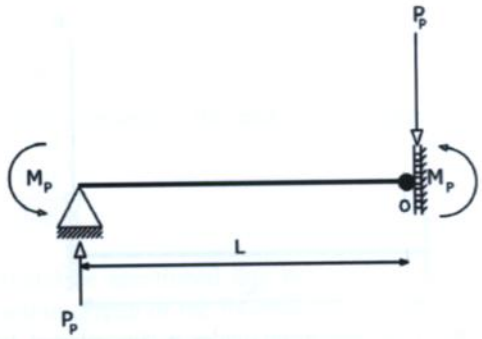

Figure 10. Plastic hinge formation due to an increasing load.

$$P(y)=\frac{2M_P}{L} \tag{20}$$

Displacement continues until y_c reaches the value in Eq. (10) and because the beam does not enter the catenary action, y_c is its final value.

According to the restraint rotations, the load deflection response in the elastic stage may be corrected as follows:

$$P(y)=\frac{24EI}{L^3(2+3R_b)}y \tag{21}$$

$$y_P=\frac{M_PL^2(1+3R_b)}{12EI} \tag{22}$$

where R_b is:

$$R_b=\frac{I}{I+4I_{ab}+12I_c\frac{L}{H}} \tag{23}$$

4. Progressive Collapse Formulation

Consider the frame in Fig. 11 under distributed load W. The gravity axial force in column C is R; therefore, the behavior of this frame is similar to the frame depicted in Fig. 12. The removal of column C is almost equivalent to replacing it by load R, as in Fig. 13.

After removal of the column according to Fig. 14, vertical displacements occur in the middle of removed column bay. If it is assumed that these displacements are equal in all stories, the behavior of the beam in the removed column bay can be considered equivalent to that of the beam in Fig. 8. Thus, the equations developed in the previous section may be used.

In a similar manner, corner column removal may be modeled according to Fig. 15.

If the displacement of column removed point is y, the external work done by load R after column removal scenario is:

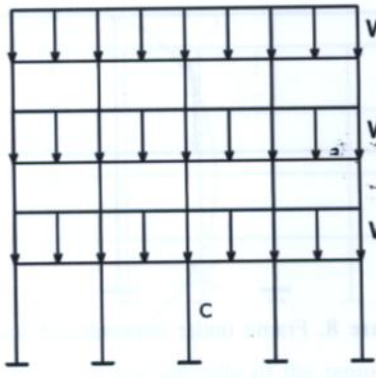

Figure 11. A frame under a distributed load.

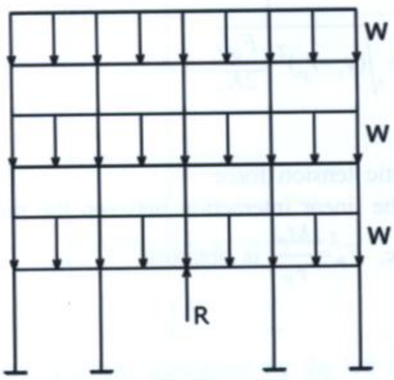

Figure 12. Axial force in the column due to the applied load.

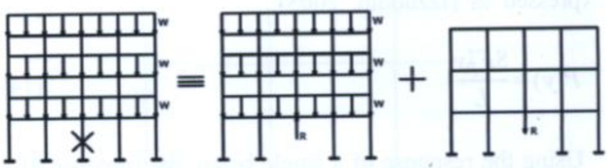

Figure 13. Model of the removal of the middle column by applying a transverse load.

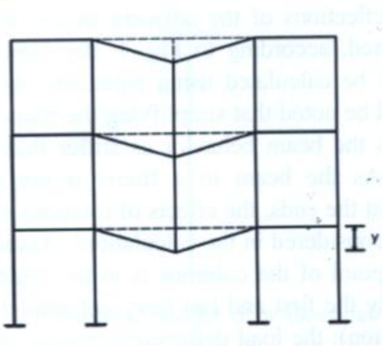

Figure 14. Vertical displacement due to column removal.

$$W_E=R\cdot y \tag{24}$$

The internal work of the beams for each story is equal to the area under the load deflection response (Fig. 2) according to displacement y. Thus, the following can be calculated:

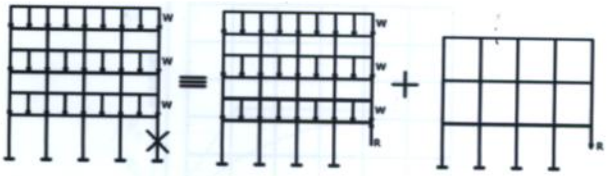

Figure 15. Model of corner column removal by applying a transverse load.

$$w_i = \int_0^y P dy \tag{25}$$

Because the structural system of Fig. 14 is a parallel system, the total internal work is the sum of the internal work of each beam:

$$W_I = \sum_N \int_0^y P dy \tag{26}$$

where N is the number of stories above the removed column point.

Then, equating Eqs. (24) and (26) leads to:

$$y = \frac{1}{R}\sum_N \int_0^y P dy \tag{27}$$

It can be seen that instead of performing a dynamic nonlinear analysis using Eq. (27), the displacement of the removed column point is easily calculated.

5. Structural Safety Control

By equating Eqs. (24) and (26), the following is obtained:

$$R = \frac{1}{y}\sum_N \int_0^y P dy \tag{28}$$

where $R(y)$ is the capacity curve.

According to Eq. (28), the capacity curve can be constructed by dividing the accumulated stored energy by its corresponding displacement. It can be shown that this curve is identical to the nonlinear dynamic response curve of structures (Tsai and Lin, 2008; Izzuddin *et al.*, 2008a,b). The maximum point of this curve represents the ultimate resistance of the structure against progressive collapse. Therefore, if this curve intersects with the applied load line, it indicates that the structure is safe against progressive collapse; if not, the structure risks severe damage (Abruzzo *et al.*, 2006).

6. Proposed Method Verification

6.1. Example 1

The steel frame with f_y=325 (MPa) presented in Fig. 16 is under a distributed load of 70 kN/m; the beams dimensions are: L=6.1 m, A=7.742×10^{-3} m^2, I=8.325×10^{-4} m^4, F_P=2001690 N, and M_P=734.4 kN/m. The column dimensions are as follows: L=3.66 m and I=4.995×10^{-4} cm^4. Under an applied distributed load, the axial forces in the right column and middle column are R=427 kN and R=854 kN, respectively.

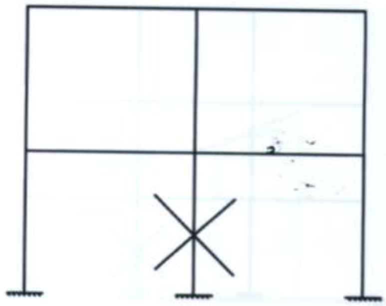

Figure 16. Example 1 frame.

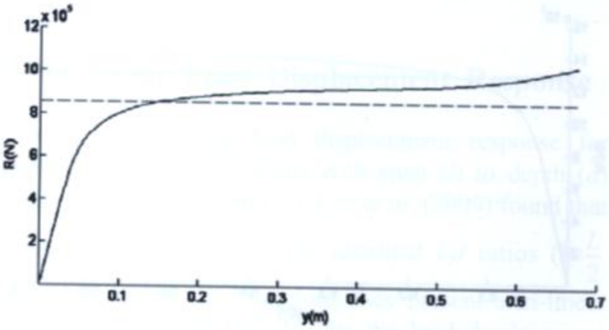

Figure 17. Capacity curve of the structure in Example 1.

Using the proposed formulation, the y values for the removal of the right and middle columns are 0.13 (m) and 0.15 (m), respectively. The responses obtained from the nonlinear dynamic analysis for the right and middle column removals were 0.14 (m) (Kaewkulchai and Williamson, 2004) and 0.17 (m) (Kaewkulchai, 2003), respectively.

In Fig. 17, the capacity curve of this structure for middle column removal and the line of axial force of the removed column are plotted. As shown, these curves intersect each other, so this structure is safe against progressive collapse.

6.2. Example 2

Consider the steel frame presented in Fig. 18. The loading, beam, and column identifications are similar to Example 1. Under an applied distributed load, the axial force in the removed column is R=1281 kN. Using the proposed formulation, y is 0.143 (m). The response obtained from the nonlinear dynamic analysis is 0.16 (m) (Kaewkulchai, 2003).

The capacity curve of this structure for middle column removal and the line of axial force of the removed column are plotted in Fig. 19. As shown, these curves

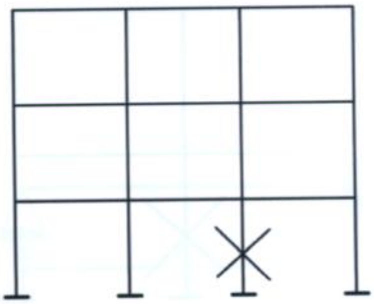

Figure 18. Example 2 frame.

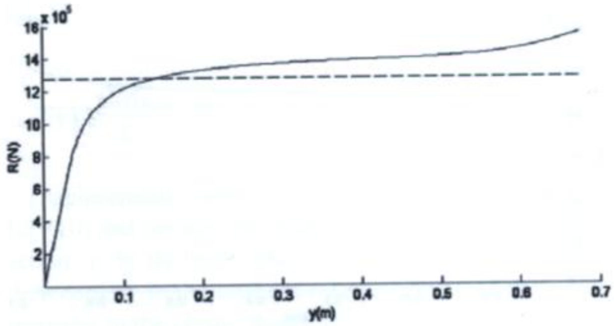

Figure 19. Capacity curve of Example 2.

intersect each other, so the structure is safe against progressive collapse.

6.3. Example 3

Consider the nine story steel frame with f_y=345 (Mpa) depicted in Fig. 20. Distributed loads of 32.37 kN/m and 12.67 kN/m are applied to the floor and roof beams, respectively. The beam lengths and column lengths are 8.25 and 4.3 m, respectively. All beams are W21×57. Under the applied load, the axial force in the removed column is R=2240.947 kN. Using the proposed formulation, y is 0.14 m. The response obtained from the nonlinear dynamic analysis was 0.132 m (Lee *et al.*, 2009). A more detailed calculation of this example is presented in Appendix 1.

The capacity curve of the structure in Fig. 20 for middle column removal and the line of axial force of the removed column are plotted in Fig. 21. As shown, these curves intersect each other, so the structure is considered safe against progressive collapse.

7. Dynamic Increase Factor

The load factor, γ, in Eqs. (1) and (2), which considers the dynamic effects in static analyses, is called the dynamic magnification factor (Biggs, 1964; Clough and Penzin, 1993) or the displacement response factor (Chopra, 2007) in structural dynamic books. Chopra (2007) expresses the displacement response factor for a single degree of freedom system with an elastic behavior under a step force according to Fig. 22 as follows:

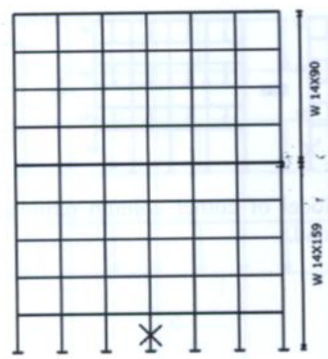

Figure 20. Example 3 frame.

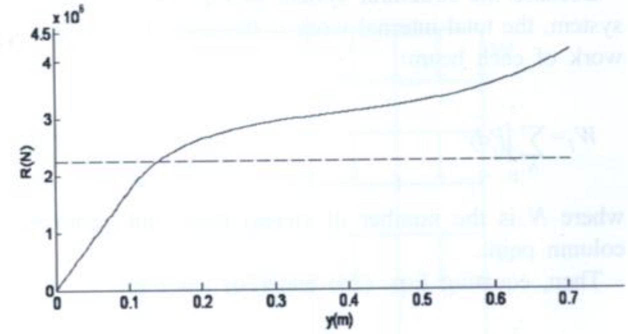

Figure 21. Capacity curve of Example 3.

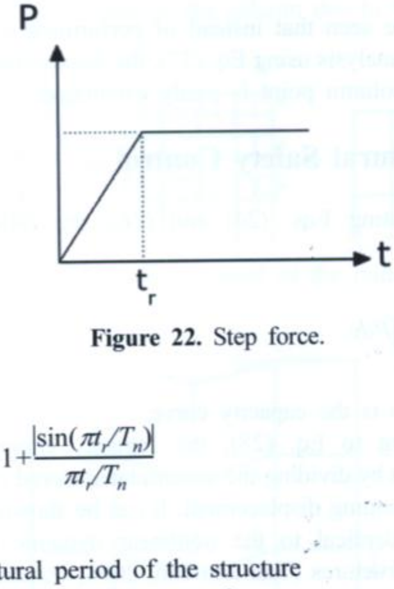

Figure 22. Step force.

$$DIF = 1 + \frac{|\sin(\pi t_r/T_n)|}{\pi t_r/T_n} \tag{29}$$

where

T_n: natural period of the structure

t_r: rising time of the step force

The load R in Fig. 13 may be assumed as a step force with very finite t_r. In this case, the value of t_r/T_n tends towards zero and the value of the DIF tends towards 2. The constant value used in Eqs. (1) and (2) is obtained by assuming elastic behavior. However, if the material has plastic behavior, the value of the DIF is less than 2 (Ruth *et al.*, 2006). According to the optimal design of structures, the assumption of elastic behavior after column removal scenario is conservative.

After analyzing 2D and 3D frames, McKay (2008)

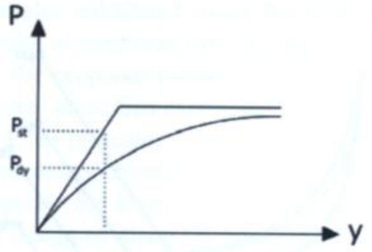

Figure 23. DIF according to load.

suggested the following equations:

$$DIF=1.04+\frac{0.45}{m-0.52}\ \text{(concrete structures)} \tag{30}$$

$$DIF=1.08+\frac{0.76}{m-0.17}\ \text{(steel structures)} \tag{31}$$

where m is the displacement ductility of the structure.

Stevens *et al.* (2008) expresses the following equation:

$$DIF=1.44(m-1)^{-0.12} \tag{32}$$

The DIF for a SDOF system is defined as:

$$DIF=\Delta_{dy}/\Delta_{st} \tag{33}$$

where Δ_{dy} and Δ_{st} are the responses obtained from dynamic and static analysis, respectively, under the same load. The DIF can be defined, according to Fig. 23, as:

$$DIF=P_{st}/P_{dy} \tag{34}$$

where P_{st} and P_{dy} are static and dynamic forces for a same displacement, respectively.

For any value of displacement in a load displacement curve, the value of the static to dynamic force may result in the DIF. As a result, with the load displacement curve under nonlinear static and nonlinear dynamic analyses and using Eq. (34), the DIF is obtained according to the post yield behavior of the structure. Thus, if the load displacement response of the structure in Fig. 14 is presented, the dynamic load displacement response is obtained using Eq. (28) (Tsai and Lin, 2008); according to Eq. (34), the DIF is the ratio of these two responses.

In the previous section, using the load displacement of a fixed end steel beam, the behavior of the frame after the removal of the column is simulated. As the load displacement response of the beam in the catenary action region is nonlinear, and because many parameters contribute to its response, it is not possible to obtain an explicit expression for DIF. Therefore, a simpler tri-linear load displacement curve is used. Finally, using Eq. (34), an explicit expression for DIF is expressed. To compare these results with previous researches (Eqs. (30)-(32)), an expression is derived for the elastic-perfectly plastic behavior.

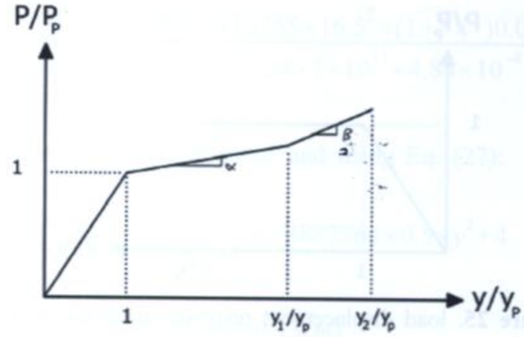

Figure 24. Tri-linear load displacement response of a fix-ended beam.

8. Tri-linear Load Displacement Response

While plotting the load displacement response for twelve compact steel beams with span (l) to depth (d) ratios equal to 10, 15 and 20, Lee *et al.* (2009) found that these curves are similar for identical l/d ratios ($l=\frac{L}{2}$ according to Fig. 1). Therefore, they present a tri-linear curve, according to Fig. 24, for the load displacement response, as the areas below the presented curves and real curves are similar. It should be noted that the limiting value for displacement is at the end of the catenary action region.

The parameters are defined as follows (Lee *et al.*, 2009):

for $l/d=10$: (35)

$\alpha=0.033$, $\beta=0.085$, $y_1/l=0.110$, $y_2/l=0.172$,

for $l/d=15$: (36)

$\alpha=0.071$, $\beta=0.179$, $y_1/l=0.073$, $y_2/l=0.132$,

for $l/d=20$: (37)

$\alpha=0.135$, $\beta=0.335$, $y_1/l=0.060$, $y_2/l=0.116$,

For other values, $10\le l/d\le 15$, or $15\le l/d\le 20$ linear interpolation is required. Therefore, the expressions below are derived:

$$\alpha=\begin{cases}(1/131.58)(l/d-5.66) & (10\le l/d<15)\\ (1/78.125)(l/d-9.453) & (15\le l/d\le 20)\end{cases} \tag{38}$$

$$\beta=\begin{cases}(1/53.19)(l/d-5.479) & (10\le l/d<15)\\ (1/32.05)(l/d-9.263) & (15\le l/d\le 20)\end{cases} \tag{39}$$

$$y_1/l=\begin{cases}(-1/135135)(l/d-24865) & (10\le l/d<15)\\ (-1/384615)(l/d-43077) & (15\le l/d\le 20)\end{cases} \tag{40}$$

$$y_2/l=\begin{cases}(-1/125)(l/d-31.5) & (10\le l/d<15)\\ (-1/312.5)(l/d-56.25) & (15\le l/d\le 20)\end{cases} \tag{41}$$

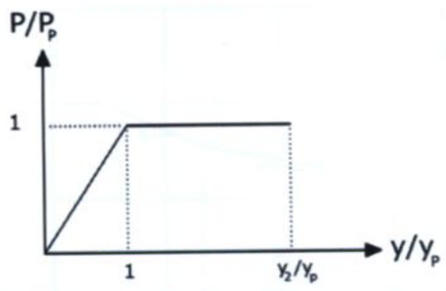

Figure 25. load displacement response as elastic perfectly plastic.

where

y_1: displacement at the beginning of catenary action behavior in the tri-linear load-displacement curve

y_2: displacement at the end of catenary action behavior in the tri-linear load-displacement curve

α: slope of plastic region in the tri-linear load-displacement curve

β: slope of catenary action region in the tri-linear load-displacement curve

According to the above expressions, in addition to the linear relations between the load and displacement, the response only depends on l/d which simplifies the derivation of an explicit expression for the DIF.

9. DIF Formulation

An explicit expression for the DIF is derived using the tri-linear load displacement curve that was introduced in the previous section. According to the calculations (detailed calculations are presented in Appendixes 2 and 3), the DIF for elastoplastic behavior with catenary action is:

$$DIF=\frac{m\{\beta(m-a)+\alpha(a-1)+1\}}{-0.5+0.5\alpha(a-1)^2+0.5\beta(m-a)^2+m+(a-1)(m-a)\alpha} \tag{42}$$

where $m=y_2/y_p$ is the displacement ductility and $a=y_1/y_p$ is:

$$a=m/\lambda \tag{43}$$

for $10\leq l/d\leq 15$: (44)

$$\lambda=1.0811+\frac{0.0531}{0.184-7.4\times10^{-3}\times l/d}$$

for $15\leq l/d\leq 20$: (45)

$$\lambda=1.2308+\frac{0.0421}{0.112-2.6\times10^{-3}\times l/d}$$

where $\lambda=y_2/y_1$ is the ratio of the displacement at the end of the catenary action to the displacement at the beginning of the catenary action.

To compare the results of the proposed method with

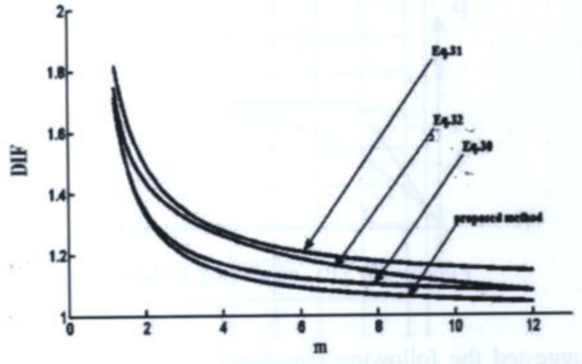

Figure 26. DIF vs. *m* for an elastic-perfectly plastic response.

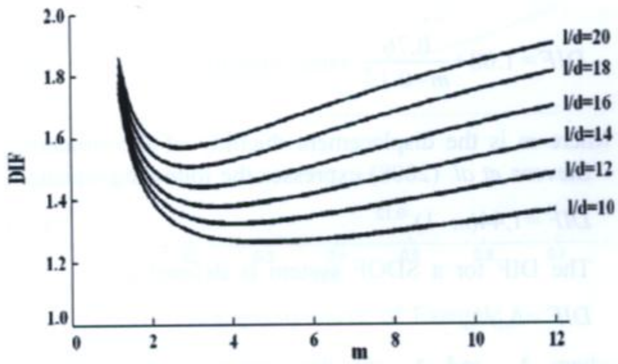

Figure 27. DIF vs. *m* for elastoplastic with a catenary action response.

previous researches, the DIF for elastic-perfectly plastic behavior (Fig. 25) is derived as:

$$DIF=\frac{2m}{2m-1} \tag{46}$$

It should be noted that for $m=1$, which represents the elastic behavior, the DIF value is equal to 2. Figure 26 plots the DIF against m for the values of m between 1.2 and 12 for Eqs. (30), (31), (32), and (46).

Figure 27 plots the DIF against m for the values of m between 1.2 and 12 for Eq. (42) for six values of l/d. As can be seen, an increasing l/d causes the DIF to increase. Also, until a ductility value around 4, the increasing ductility causes the DIF to decrease; however, after 4, the DIF value increases, which is comparable with the results presented in Ruth *et al.* (2006).

10. Conclusion

In this paper, the formulation of a simplified method for the progressive collapse analysis of steel moment resisting frames is proposed using the load deflection response of a fix- ended beam. This formulation is obtained by equating the external work of an applied load with the internal work of the beams in the removed column bay. According to the examples calculated using the proposed method and the results of nonlinear dynamic analyses from other researches, the proposed method is sufficiently accurate.

Furthermore, an explicit expression for dynamic increase factor (DIF) was presented. Using a constant DIF equal to two for considering dynamic effects in static analysis is only appropriate for elastic systems while according to the optimal design of structures, assuming elastic behavior after column removal is conservative. The DIF is defined as the ratio of the static force to dynamic force for the same displacement. As a result, having a load displacement curve under nonlinear static and nonlinear dynamic analyses, the DIF is obtained according to the post yield behavior of the structure. For nonlinear static analyses, a tri-linear load displacement curve of a fix-ended steel beam is used. The capacity curve is used for the nonlinear dynamic responses of the structure. The DIF is plotted against the ductility curve for elastic-perfectly plastic and elastoplastic with catenary action behaviors. If the catenary action is not considered, the DIF may increase as the ductility increases. However when the ductility reaches 12, the DIF approaches 1. If the catenary action is considered, for a ductility value of up to 4, an increasing ductility causes the DIF to decrease; however, after that point, the DIF increases. For a ductility value equal to 12, the value of the DIF is approximately 2. As a result, if the catenary action is considered and the ductility of structure is high, using load factor of 2 is not conservative.

Appendix 1:

Detailed calculation of example 3 is presented here.

(for first 5 story)

$$R_b=\frac{4.87\times10^{-4}}{4.87\times10^{-4}+4\times4.87\times10^{-4}+6\times7.908\times10^{-4}\times\frac{16.5}{4.3}}=0.0236$$

(for last 4 story)

$$R_b=\frac{4.87\times10^{-4}}{4.87\times10^{-4}+4\times4.87\times10^{-4}+6\times4.158\times10^{-4}\times\frac{16.5}{4.3}}=0.0406$$

(for first 5 story)

$$P_1(y)=\frac{192\times2\times10^{11}\times4.87\times10^{-4}}{16.5^3(1+3\times0.0236)}y=3887764y$$

(for last 4 story)

$$P_1(y)=\frac{192\times2\times10^{11}\times4.87\times10^{-4}}{16.5^3(1+3\times0.0406)}y=3711016y$$

$$M_P=Z.F_y=2.114\times10^{-3}\times384.7\times10^{6}=813255.8$$

$$y_P=\frac{M_PL^2(1+3R_b)}{24EI}=\frac{813255\times16.5^2\times(1+3\times\ )0.0406}{24\times2\times10^{11}\times4.84\times10^{-4}}$$
$$=0.107(m)$$

Assuming elastic response and using Eq. (27):

$$y=\frac{1}{R}\sum_N\int_0^y Pdy=\frac{1}{2240947}(5\times3887764\times0.5\times y^2+4$$
$$3711016\times0.5\times y^2)\Rightarrow y=0.13(m)$$

As $y>y_P$, the above assumption is not true and the response may be plastic.

(for all stories)

$$P(y)=\frac{8M_P}{L}=\frac{8\times813255}{16.5}=394305.94$$

$$y_c=\max(y_P,\ r_P)=\max(0.107,\ \frac{0.5359}{2})=0.268$$

Assuming plastic response and using Eq. (27):

$$y=\frac{1}{2240947}(5\times3887764\times0.5\times0.107^2+4\times3711016$$
$$\times0.5\times0.107^2+9\times394305.94\times(y-0.107))\Rightarrow y=0.14(m)$$

As $y<y_c$, the response may be plastic and the answer is y=0.14(m).

Appendix 2:

(a) Dynamic Increase Factor expression by considering catenary action

$$P=\beta(y-a)+1+\alpha(a-1)$$

$$R=\frac{1}{y}\left\{\int_0^1 ydy+\int_1^a[\alpha(y-1)+1]dy+\int_a^y[\beta(y-a)+1+a(a-1)]dy\right\}$$

$$=\frac{1}{y}\{-0.5+0.5\alpha(a-1)^2+0.5\beta(y-a)^2+y+(a-1)(y-a)\alpha\}$$

$$DIF=\frac{P}{R}\bigg|_{y=m}$$

$$DIF=\frac{m\{\beta(m-a)+\alpha(a-1)+1\}}{-0.5+0.5\alpha(a-1)^2+0.5\beta(m-a)^2+m+(a-1)(m-a)\alpha}$$

(b) Dynamic Increase Factor expression for elastic-perfectly plastic behavior

$$R=\frac{1}{y}\left\{\int_0^1 ydy+\int_1^a dy\right\}=\frac{1}{y}\{y-0.5\}$$

$$DIF=\frac{P}{R}\bigg|_{y=m}=\frac{2m}{2m-1}$$

Appendix 3:

Demonstration of Eqs. (43) and (44):

For $10 \le l/d \le 15$, according to Eqs. (40) and (41):

$$l/d = \begin{cases} 24.865 - 135.135 \times y_1/l \\ 31.5 - 125 \times y_2/l \end{cases}$$

$$y_1/l = 0.925 \times y_2/l - 0.0491$$

$$1 = 0.925\frac{y_2}{y_1} - \frac{0.0491}{y_1/l}$$

According to Eq. (40):

$$y_1/l = 0.184 \times 7.4 \times 10^{-3} \times l/d$$

Therefore we have:

$$\lambda = y_2/y_1 = 1.0811 + \frac{0.0531}{0.184 - 7.4 \times 10^{-3} \times l/d}$$

As $\lambda = y_2/y_1$ and $m = y_2/y_p$:

$a = m/\lambda$

For $15 \le l/d \le 20$, Eq. (45) may be proved in a similar way.

Notation

- γ: Load factor.
- DL: Dead load.
- LL: Live load.
- WL: Wind load.
- DCR: The ratio of member force and member strength.
- Q_{UD}: Acting force determined in the member.
- Q_{CE}: Expected ultimate unfactored capacity of that member.
- l: Span length.
- d: Beam section depth.
- K: bending stiffness of the beam.
- E: Modulus of elasticity of the beam.
- I: Moment of inertia of the beam.
- L: Length of beam.
- y: Mid-span displacement of the beam.
- P: Concentrated load applied at the middle of the fix-ended beam.
- P_P: concentrated load applied at the middle of the beam which causes the beam to enter plastic behaviour.
- M_P: Plastic moment of the beam.
- y_p: Displacement at the beginning of plastic behaviour.
- r_p: Plastic interaction radius.
- K_e: Effective axial stiffness.
- y_c: Displacement at the beginning of catenary action behaviour.
- EA: Elastic axial rigidity of the beam.
- K_{hr}: stiffness of right frame of the removed column bay.
- K_{hl}: stiffness of left frame of the removed column bay.
- n: Number of columns in each side of the column removed bay.
- y_{cf}: Displacement at the end of catenary action behaviour.
- F_p: Plastic tension force.
- R_b: Factor to consider the rotations of beams and columns which are adjacent to removed column.
- I_{ab}: Moment of inertia of beams adjacent to removed column.
- I_c: Moment of inertia of columns adjacent to removed column.
- H: Length of columns adjacent to removed columns.
- R: Axial force in the removed column.
- W_E: the external work done after column removal scenario.
- w_i: The internal work done by each beam.
- W_I: Total internal work done after column removal scenario.
- N: Number of stories above the column removed point.
- f_y: Yield tension of steel.
- A: Area of beam section.
- t_r: Rising time of the step force.
- T_n: Natural period of the structure.
- m: Displacement ductility.
- $_{dy}$: Response obtained from dynamic analysis.
- $_{st}$: Response obtained from static analysis.
- P_{st}: Static force that cause a known displacement.
- P_{dy}: Dynamic force that cause a known displacement.
- y_1: Displacement at the beginning of catenary action behavior in the tri-linear load-displacement curve.
- y_2: Displacement at the end of catenary action behavior in the tri-linear load-displacement curve.
- α: The slope of plastic region in the tri-linear load-displacement curve.
- β: The slope of catenary action region in the tri-linear load-displacement curve.
- γ: The ratio of the displacement at the end of the catenary action to the displacement at the beginning of the catenary action.
- a: The ratio of the displacement at the end of the plastic behavior to the displacement at the beginning of the plastic behavior.
- Z: Section modulus.

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