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Some criteria for detecting capable Lie algebras

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ABSTRACT

In virtue of a recent bound obtained in [P. Niroomand, F.G. Russo, A note on the Schur multiplier of a nilpotent Lie algebra, Comm. Algebra 39 (2011) 1293–1297], we classify all capable nilpotent Lie algebras of finite dimension possessing a derived subalgebra of dimension one. Indirectly, we find also a criterion for detecting noncapable Lie algebras. The final part contains a construction, which shows that there exist capable Lie algebras of arbitrary big corank (in the sense of Berkovich–Zhou).

1. Introduction

The theory of p-groups (p prime) is playing a fundamental role in several areas of research in the last ten years. There are some classical results due to Lazard (see [9,19]), which allow us to associate a p-group with a given Lie algebra. This means that we may do analogies between groups and Lie algebras mutatis mutandis, but the analogies are not perfect most of the times and careful distinctions shall be done. It seems that Barnes [3–7] has originally formulated a formation theory of Lie algebras on the prototype of the well-known formation theory of groups in the sense of Gaschütz. In particular, he and Stitzinger later (see [8,17]) have shown that it is possible to apply certain specific methods of investigation of the p-groups to the classification of nilpotent Lie algebras. We have followed the

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same line of research in [20–23] and have found criteria for recognizing Lie algebras by the size of their low dimensional homology.

In the present paper we focus on the notion of *capable Lie algebra*, which originates from the concept of capable group, given by Baer [2]. A group G is *capable* if there exists some group H such that $G \simeq H/Z(H)$. Baer classified all capable groups among the direct sums of cyclic groups and hence determined all capable finitely generated abelian groups. Beyl and others [10] described capable groups among extra special p-groups. These authors introduced the *epicenter* $Z^*(G)$ of G which is the uniquely determined central subgroup of G that is minimal subject to being the image of G of some central extensions of G, that is,

$$Z^*(G) = \bigcap_{(E,\phi) \text{ is a central extension of } G} \phi\big(Z(E)\big).$$

The importance of $Z^*(G)$ is due to the fact that G is capable if and only if $Z^*(G) = 1$ (see [10]). Another notion having relation to capability is the exterior square of groups which was introduced in [12]. Using this concept Ellis [15], introduced the *exterior center* $Z^\wedge(G)$ to be the set of all elements g of G for which $g \wedge h = 1$ for all $h \in G$. He proved $Z^\wedge(G) = Z^*(G)$ which is an interesting result.

Recently, [1] introduced the analogous notion of epicenter $Z^*(L)$ for a Lie algebra L and it was proved that a Lie algebra L is capable if and only if $Z^*(L) = 0$. Here we continue to prove the other analogies which one expects to have. First of all, we show that $Z^\wedge(L) = Z^*(L)$ for a finite dimensional Lie algebra L, finding a more general context for the result of Ellis in [15]. This will help us to find a necessary and sufficient condition for an abelian finite dimensional Lie algebra to be capable and so we generalize the aforementioned results of Baer in [2]. Successively, we classify capable Heisenberg Lie algebras which have a similar role of extra special p-groups, so we generalize the aforementioned results of Beyl and others in [10]. Moreover, we combine the results which we obtain with some other results in [20] and are able to prove a necessary and sufficient condition of capability for nilpotent finite dimensional Lie algebras with small derived subalgebra L^2, namely $\dim L^2 = 1$.

2. Preliminaries

Throughout this paper all Lie algebras are finite dimensional. To convenience of the reader, we recall that a Lie algebra L is *Heisenberg* provided that $L^2 = Z(L)$ and $\dim L^2 = 1$. Such algebras are odd dimensional with basis $v_1, \dots, v_{2m}, v$ and the only nonzero multiplication between basis elements is given by $[v_{2i-1}, v_{2i}] = -[v_{2i}, v_{2i-1}] = v$ for $i = 1, \dots, m$. $A(n)$ and $H(m)$ will denote the *abelian Lie algebra* of dimension n and the *Heisenberg Lie algebra* of dimension $2m + 1$, respectively. The *Schur multiplier* $M(L)$ of a Lie algebra L is, as usual, the second homology Lie algebra $H_2(L, \mathbb{Z})$ with integral coefficients.

We recall two lemmas.

Lemma 2.1. *(See [1], Theorem 4.4.) Let L be a Lie algebra and N a central ideal of L. Then $N \subseteq Z^*(L)$ if and only if the natural map $M(L) \longrightarrow M(L/N)$ is a monomorphism.*

Lemma 2.2. *(See [14], Proposition 13, and [1], Proposition 4.1(iii).) Let L be a Lie algebra and N be a central ideal of L. Then the following sequences are exact.*

(i) $L \wedge N \longrightarrow L \wedge L \longrightarrow L/N \wedge L/N \longrightarrow 0.$
(ii) $M(L) \longrightarrow M(L/N) \longrightarrow N \cap L^2 \longrightarrow 0.$

Here is an easy consequence.

Corollary 2.3. *$N \subseteq Z^\wedge(L)$ if and only if the natural map $L \wedge L \longrightarrow L/N \wedge L/N$ is a monomorphism.*

Another useful result is below.

Lemma 2.4. *(See [13].) Let L be a Lie algebra. Then*

$$0 \longrightarrow M(L) \longrightarrow L \wedge L \longrightarrow L^2 \longrightarrow 0$$

is a central extension.

The following is another easy consequence. It can be deduced also in different ways, using homological methods of abstract algebra in [24].

Corollary 2.5. *We have $M(A(n)) \cong A(n) \wedge A(n)$.*

Schur multipliers of abelian and Heisenberg algebras are well known.

Lemma 2.6. *(See for instance [20].) We have*

(i) $\dim M(A(n)) = \frac{1}{2}n(n-1)$.
(ii) $\dim M(H(1)) = 2$.
(iii) $\dim M(H(m)) = 2m^2 - m - 1$ *for all* $m \geqslant 2$.

Now there are some rules of homological nature on the Schur multipliers of direct sums of Lie algebras, which we recall below.

Theorem 2.7. *(See [8,17,14,20,21].) Two Lie algebras H and K satisfy the condition*

$$M(H \oplus K) = M(H) \oplus M(K) \oplus \big(H/H^2 \otimes K/K^2\big).$$

Moreover,

$$(H \oplus K) \wedge (H \oplus K) \cong (H \wedge H) \oplus (K \wedge K) \oplus \big(H/H^2 \otimes K/K^2\big),$$
$$Z^\wedge(H \oplus K) \subseteq Z^\wedge(H) \oplus Z^\wedge(K).$$

3. Main results

We are going to classify all nilpotent Lie algebras with derived subalgebra of dimension at most 1. We start from an observation in [1], where it is proved that a Lie algebra L is capable if and only if $Z^*(L) = 0$, and show $Z^*(L) = Z^\wedge(L)$ so that L is capable if and only if $Z^\wedge(L) = 0$. These are two fundamental ingredients of the main proofs of the present section.

Lemma 3.1. *For any (finite dimensional) Lie algebra L, we have $Z^*(L) = Z^\wedge(L)$.*

Proof. Considering Lemmas 2.1 and 2.2(i), we can see that $M(L) \longrightarrow M(L/Z^\wedge(L))$ is a monomorphism, so $Z^\wedge(L) \subseteq Z^*(L)$. On the other hand, by Lemmas 2.1 and 2.2(ii), we have $\dim M(L/Z^*(L)) = \dim M(L) + \dim L^2 \cap Z^*(L)$. But Lemma 2.4 shows that

$$\dim L \wedge L = \dim M(L) + \dim L^2 \quad \text{and}$$
$$\dim L/Z^*(L) \wedge L/Z^*(L) = \dim M\big(L/Z^*(L)\big) + \dim\big(L/Z^*(L)\big)^2.$$

Using the isomorphism theorems of Lie algebras,

$$\big(L/Z^*(L)\big)^2 \cong \big(L^2 + Z^*(L)\big)/Z^*(L) \cong L^2/\big(Z^*(L) \cap L^2\big)$$

and we have

$$\dim L \wedge L = \dim L/Z^*(L) \wedge L/Z^*(L),$$

hence $Z^*(L) \subseteq Z^\wedge(L)$ due to Corollary 2.3. □

The following information on Heisenberg Lie algebras will be useful.

Lemma 3.2. *Let $m \geqslant 1$. Then*

(i) $H(1) \wedge H(1) \cong A(3)$.
(ii) $H(m) \wedge H(m) \cong A(2m^2 - m)$ *for all* $m \geqslant 2$.

Proof. Since $\dim H(m)^2 = 1$, Lemma 2.4 implies that $H(m) \wedge H(m)$ is abelian. Invoking Lemmas 2.4 and 2.6, we have $\dim H(1) \wedge H(1) = 3$ and $\dim H(m) \wedge H(m) = 2m^2 - m$ for all $m \geqslant 2$. The result follows. □

The following result describes all capable abelian Lie algebras and is a natural extension to the context of Lie algebras of a famous theorem of Baer [2] in case of abelian groups. In few words, we prove that abelian Lie algebras cannot be one dimensional if we are looking for capable Lie algebras. We offer a proof based on the computation on the Schur multipliers of abelian Lie algebras, then, indirectly, we involve homological methods. This technique differs very much from the corresponding argument in [2], where Baer uses some well-known theorems of structure of abelian groups, proved with elementary methods.

Theorem 3.3. *$A(n)$ is capable if and only if $n \geqslant 2$.*

Proof. Since $M(A(1)) = 0$, Lemma 2.1 implies $A(1)$ is not capable. Now, let $n \geqslant 2$ and I be a k-dimensional ideal of $A(n)$. Then, we have

$$\dim M\big(A(n)/I\big) = \frac{1}{2}(n-k)(n-k-1) \quad \text{and} \quad \dim M\big(A(n)\big) = \frac{1}{2}n(n-1),$$

so Lemma 2.1 implies $I \subseteq Z^\wedge(A(n))$ if and only if $k = 0$. Hence, we should have $Z^\wedge(A(n)) = 0$ and the result holds. □

Ignoring abelian Lie algebras, Heisenberg Lie algebras are probably the simplest Lie algebras to work with. The following theorem classifies all capable Heisenberg Lie algebras and gives the corresponding formulation of the results of Beyl and others [10] in case of extra special p-groups. The reader can note that we are going to show the contrary of Theorem 3.3 in a certain sense: While capable abelian Lie algebras cannot be small, capable Heisenberg Lie algebras must be small.

Theorem 3.4. *$H(m)$ is capable if and only if $m = 1$.*

Proof. First suppose that $m = 1$. From Lemma 3.2, we have $\dim H(1) \wedge H(1) = 3$. On the other hand, for any nonzero ideal of $H(1)$ such as $I = H(1)^2$, Corollary 2.5 and Lemma 2.6(i) imply that $H(1)/I \wedge H(1)/I$ is one dimensional, that is,

$$\dim H(1)/I \wedge H(1)/I = 1.$$

Hence $Z^\wedge(H(1))$ contains no nonzero ideal and must be trivial.

Now assume that $m \geqslant 2$. Similarly to the case of $m = 1$, we may use Lemma 3.2 in order to compute the dimension of $H(m) \wedge H(m)$ and we find that

$$\dim H(m) \wedge H(m) = 2m^2 - m,$$

then the dimension of the exterior square $H(m)/H(m)^2 \wedge H(m)/H(m)^2$ of the abelianization $H(m)/H(m)^2$ is

$$\begin{aligned}&\dim H(m)/H(m)^2 \wedge H(m)/H(m)^2\\ &\quad = \dim M\big(H(m)/H(m)^2\big) = \frac{2m(2m-1)}{2} = 2m^2 - m\end{aligned}$$

which implies $H(m)^2 = Z^\wedge(H(m))$. □

The direct sum of an abelian Lie algebra and a Heisenberg Lie algebra has the derived subalgebra of dimension one and it is interesting to decide whether it is capable or not. The following theorem gives a necessary and sufficient condition for capability of such Lie algebras. We will see that it is a generalization of Theorems 2.1 and 2.4, but some careful considerations shall be done.

Theorem 3.5. *For any value of $k \geqslant 1$, the nilpotent Lie algebra $H(m) \oplus A(k)$ is capable if and only if $m = 1$.*

Proof. We consider three cases as follows

(i) $m = k = 1$;
(ii) $m = 1$ and $k \geqslant 2$;
(iii) $m \geqslant 2$.

In case (i), $L \cong H(1) \oplus A(1)$ and Theorems 2.7, 3.3 and 3.4 imply that

$$Z^\wedge(L) \subseteq Z^\wedge\big(H(1)\big) \oplus Z^\wedge\big(A(1)\big) = A(1).$$

But

$$\begin{aligned}\dim M(L) &= \dim M\big(H(1)\big) + \dim M\big(A(1)\big) + \dim H(1)/H(1)^2 \otimes A(1)\\ &= 2 + 0 + 2 = 4.\end{aligned}$$

On the other hand, $\dim M(L/A(1)) = \dim M(H(1)) = 2$, and so $A(1) \nsubseteq Z^\wedge(L)$ which implies that $Z^\wedge(L) = 0$.

In case (ii), again Theorems 2.7, 3.3 and 3.4 allow us to conclude that

$$Z^\wedge(L) \subseteq Z^\wedge\big(H(1)\big) \oplus Z^\wedge\big(A(k)\big) = 0,$$

as required.

Finally in case (iii), we claim that

$$L \wedge L \cong L/H(m)^2 \wedge L/H(m)^2,$$

and hence L is not capable.

Since $L/H(m)^2 \cong H(m)/H(m)^2 \oplus A(k)$, we have

$$L/H(m)^2 \wedge L/H(m)^2 \cong \big(H(m)/H(m)^2 \wedge H(m)/H(m)^2\big) \oplus \big(A(k) \wedge A(k)\big) \oplus \big(H(m)/H(m)^2 \otimes A(k)\big).$$

Thus

$$\dim L/H(m)^2 \wedge L/H(m)^2 = \frac{2m(2m-1)}{2} + \frac{k(k-1)}{2} + 2km.$$

On the other hand,

$$\dim L \wedge L = \dim\big(H(m) \oplus A(k)\big) \wedge \big(H(m) \oplus A(k)\big).$$

Now using Theorem 2.7

$$\begin{aligned}\dim L \wedge L &= \dim H(m) \wedge H(m) + \dim A(K) \wedge A(K) + \dim H(m)/H(m)^2 \otimes A(k) \\ &= \frac{2m(2m-1)}{2} + \frac{k(k-1)}{2} + 2mk.\end{aligned}$$

Hence $\dim L/H(m)^2 \wedge L/H(m)^2 = \dim L \wedge L$, and the result holds. □

The first and the third author gave recently an upper bound for the Schur multipliers of nilpotent Lie algebras in [20] and this allows us to describe the scenario for arbitrary nilpotent Lie algebras possessing one dimensional derived subalgebras.

Theorem 3.6. *Let L be a nilpotent Lie algebra of dimension n such that $\dim L^2 = 1$. Then $L \cong H(m) \oplus A(n-2m-1)$ for some $m \geqslant 1$ and L is capable if and only if $m = 1$, that is, $L \cong H(1) \oplus A(n-3)$.*

Proof. We recall from [22, Lemma 3.3] that a nilpotent Lie algebra L of dimension n such that $\dim L^2 = 1$ must be of the form $L \cong H(m) \oplus A(n-2m-1)$ for some $m \geqslant 1$. Now the result follows from Theorem 3.5. □

The following theorem improves [9, Proposition 21.18] with a different type of argument. This is another successful application of the linear methods of the study of p-group to the context of nilpotent Lie algebras. Roughly speaking, we find a criterion for detecting noncapable Lie algebras, only looking at the size of the section $L/Z(L)$ of L when L^2 is one dimensional.

Theorem 3.7. *A nilpotent Lie algebra L with $\dim L^2 = 1$ and $\dim L/Z(L) > 2$ is not capable.*

Proof. By using Theorem 3.5, we have $L \cong H(m) \oplus A(n-2m-1)$ and so

$$\begin{aligned}\frac{L}{Z(L)} &\cong \frac{H(m) \oplus A(n-2m-1)}{Z(H(m)) \oplus A(n-2m-1)} \\ &\cong \frac{H(m)}{Z(H(m))} \oplus \frac{A(n-2m-1)}{A(n-2m-1)} \\ &\cong \frac{H(m)}{Z(H(m))} \cong A(2m).\end{aligned}$$

Since $\dim L/Z(L) > 2$, we shall have $m > 1$ and so Theorem 3.5 shows L is not capable. □

4. Applications to the corank in the sense of Berkovich–Zhou

In [16] Ellis and Wiegold gave a positive answer to a problem of Jones (see for instance [23]), who asked whether a p-group G of order p^n (p prime and $n \geqslant 1$), achieving the bound $|M(G)| = p^{\frac{1}{2}n(n-1)-t(G)}$ for an integer $t(G) \geqslant 0$, was subject to a classification in terms of structure theorems (notice that some bounds of [16] are sharpened in [23]). The question of Jones was supported by several evidences and later $t(G)$ was called *corank* of G. Up to $t(G) \leqslant 5$, some authors gave interesting classifications and [23] contains details about this problem. In the corresponding classifications we have always a list of finitely many groups, which are characterized by a prescribed value of corank. Now the natural question is whether we could grow as much as we want with $t(G)$, keeping a finite list of groups in the corresponding classification. Fortunately, the answer is positive. In fact [16, Corollary 4] shows that for each prime p and integer $t(G) \geqslant 0$, there exists at least one p-group of given corank $t(G)$. Furthermore, for $t(G) \geqslant 1$, there are only finitely many p-groups of given corank $t(G)$.

Now it is interesting to note that there are corresponding classifications up to $t(L) \leqslant 6$, for nilpotent Lie algebras (see [8,17]). More precisely, the *corank* $t(L)$ of a nilpotent Lie algebra L is defined by

$$\dim M(L) = \frac{1}{2}n(n-1) - t(L).$$

Some classical results are known from more than twenty years and we summarize them below.

Theorem 4.1. *(See [17].) A nilpotent Lie algebra L of $\dim L = n$ is characterized for $t(L) = 0, 1, 2, 3, 4, 5, 6$. More precisely,*

(i) *$t(L) = 0$ if and only if L is abelian.*
(ii) *$t(L) = 1$ if and only if $L = H(1)$.*
(iii) *$t(L) = 2$ if and only if $L = H(1) \oplus A(1)$.*
(iv) *$t(L) = 3$ if and only if $L = H(1) \oplus A(2)$.*
(v) *$t(L) = 4$ if and only if either $L = H(1) \oplus A(3)$ or $L = L(3,4,1,4)$ or $L = L(4,5,2,4)$.*
(vi) *$t(L) = 5$ if and only if either $L = H(1) \oplus A(4)$ or $L = H(2)$.*
(vii) *$t(L) = 6$ if and only if L is one of the following algebras: $H(1) \oplus A(5)$, $H(2) \oplus A(1)$, $L(4,5,1,6)$, $L(3,4,1,4) \oplus A(1)$, $L(4,5,2,4) \oplus A(1)$.*

The two successive cases $t(L) = 7, 8$ are described in [18] (and we haven't noted these cases in Theorem 4.1, because they are quite technical to formulate) and the classification for $t(L) \geqslant 9$ is an open problem, to the best of our knowledge. On the other hand, Bosko [11] classified the structure of a filiform nilpotent Lie algebra L up to $t(L) = 16$ and her methods show interesting applications of the theory of the finite p-groups of maximal class to the theory of the nilpotent Lie algebras of finite dimension. At the same time, we don't know what happens for a nonfiliform nilpotent Lie algebra L when $9 \leqslant t(L) \leqslant 16$ and, honestly, these Lie algebras seem to be quite hard to describe, in terms of generators and relations.

Unfortunately, the case of Lie algebras involves the notion of dimension and this is different under several aspects from the notion of exponent (and of order) in p-groups. Therefore we cannot expect to have always perfect analogies. However we have the following result in our direction of research.

Theorem 4.2. *There exists at least one capable nilpotent Lie algebra (of finite dimension) with arbitrary corank.*

We are going to provide an explicit family of nilpotent Lie algebras for which the growth of the corank is linear and will check when these are capable in view of the result of the previous section. Firstly, we apply Theorem 2.7 in order to obtain a rule for the corank of the sums of two nilpotent Lie algebras.

Lemma 4.3. *A Lie algebra H of* $\dim H = h$ *and a Lie algebra K of* $\dim K = k$ *satisfy the condition*

$$t(H \oplus K) = t(H) + t(K) + hk - \dim H/H^2 \otimes K/K^2.$$

Proof. From Theorem 2.7,

$$\dim M(H \oplus K) = \frac{1}{2}(h+k)(h+k-1) - t(H \oplus K)$$

must be equal to

$$\begin{aligned}
&\dim M(H) + \dim M(K) + \dim H/H^2 \otimes K/K^2 \\
&\quad = \frac{1}{2}h(h-1) - t(H) + \frac{1}{2}k(k-1) - t(K) + \dim H/H^2 \otimes K/K^2,
\end{aligned}$$

then

$$\begin{aligned}
&t(H) + t(K) + \frac{1}{2}\big(h^2 + 2hk + k^2 - h - k\big) \\
&\quad = t(H \oplus K) + \frac{1}{2}\big(h^2 + k^2 - h - k\big) + \dim H/H^2 \otimes K/K^2
\end{aligned}$$

and so

$$t(H) + t(K) + hk = t(H \oplus K) + \dim H/H^2 \otimes K/K^2. \qquad \square$$

We note a precise formula in case of Heisenberg algebras.

Lemma 4.4. $t(H(m)) = 2m + 1$ *for all* $m \geqslant 2$.

Proof. We note that for $m \geqslant 2$,

$$\dim M\big(H(m)\big) = \frac{1}{2}(2m+1)(2m) - t\big(H(m)\big) = m(2m+1) - t\big(H(m)\big),$$

but we have also

$$m(2m+1) - t\big(H(m)\big) = 2m^2 - m - 1$$

then

$$2m^2 + m = 2m^2 - m - 1 + t\big(H(m)\big),$$

from which the result follows. $\square$

We have all is necessary to prove the main result of the present section.

Proof of Theorem 4.2. From Theorem 2.7 and the definition of corank, we have easily that $t(A(k)) = 0$ for all $k \geqslant 1$. Now, for all $m \geqslant 2$ Lemmas 4.3 and 4.4 imply

$$t\big(H(m)\oplus A(k)\big)=(2m+1)+0+(2m+1)\cdot k-\dim H(m)/H(m)^2\otimes A(k)$$
$$=2m+1+2mk+k-2mk=2m+k+1.$$

We conclude that $H(m)\oplus A(k)$ is a nilpotent Lie algebra of corank $\geqslant 6$ of the form $2m+k+1$, which is arbitrary and depending only on m and k. Now Theorem 3.5 implies that $H(1)\oplus A(k)$ is capable and such Lie algebra has arbitrary corank of the form $k+3$.

On the other hand, the list of Theorem 4.1 contains explicit constructions for values of corank between 0 and 5 and, having in mind Theorem 3.5, we note that there is always at least one capable Lie algebra for $t(L)=0,1,2,3,4,5,6$ in Theorem 4.1. The result follows. □

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