AOQ and ATI for Double Sampling Plan with Using Fuzzy Binomial Distribution

Ezzatallah Baloui Jamkhaneh*
Department of Statistics, Islamic Azad University, Ghaemshahr Branch,
Ghaemshahr, Iran
e_baloui2008@yahoo.com

Bahram Sadeghpour Gildeh
Department of Statistics, Faculty of Basic Science, University of Mazandaran,
Bablosar, Iran
sadeghpour@umz.ac.ir

Abstract— in this paper we introduce the average outgoing quality (AOQ) and average total inspect (ATI) for double sampling plan when that proportion nonconforming items is fuzzy number. We have shown that AOQ and ATI curves of the plan are like a band having high and low bounds and if the process quality is very good or very bad, then AOQ bands will be lower values. With the decreasing the quality of process the bandwidth of FATI will narrower, and with the increasing the size of lot if others parameter be fixed, this width be wider.

Keywords-- Statistical quality control, acceptance single sampling, average outgoing quality, average total inspection, fuzzy number.

I. INTRODUCTION

Acceptance sampling plan is one of the most important components of statistical quality control. In this field, acceptance double sampling plan is one of the sampling methods for acceptance or rejection a lot with classical attribute quality characteristic. In different acceptance sampling plans the fraction of defective items, is considered as a precise value, but sometimes we are not able to obtain exact numerical value, and there also exist some uncertainty in the value of obtained from experiment, personal judgment or estimation. However the quality characteristic in a lot is not often exact and certain. Fuzzy set theory is a mathematical model of vague data or uncertain value that frequently generated by experiment, estimation or means of the natural language. This theory is a powerful and well-known tool to formulate and analyzing the parameters cannot be estimated accurately. In dealing with the above problem we tried to restore the uncertainty existing in the problem by defining the imprecise parameter as a fuzzy number, and achieve a result with a higher certainty. With this definition, the number of defective items in the sample has a fuzzy binomial probability distribution [4]. Classical acceptance sampling plans have been studied by many researchers. They are thoroughly elaborated by Schilling (1982). Single sampling by attributes with relaxed requirements was discussed by Ohta and Ichihashi (1988) Kanagawa and Ohta (1990), Tamaki, Kanagawa and Ohta (1991), and Grzegorzewski (2001b). Grzegrozewski (2002) also considered sampling plan by variables with fuzzy requirements. Sampling plan by attributes for vague data were considered by Hrniewicz (1992). Dodge Romig (1956) provided the rectifying single sampling plan and double sampling plans for attributes with the protection of lots tolerance percent defective (LTPD) or average outgoing quality limit (AOQL). Determination of rectifying plans for single sampling by attributes was discussed by Guenther (1984). Bebbington and Govindaraju (1998) have developed sampling schemes similar to the rectifying inspections. Kleijnen et al. (1992) have studied the application of the rectifying inspections in financial auditing. Suresh and Ramkumar (1996) justified the use of maximum allowable average outgoing quality for developing sampling plan.

The paper is organized as follows. Sampling plan with fuzzy parameter terminology introduce in section II. In section III the fuzzy average outgoing quality, was considered broadly, and its values in special case was computed. In section IV, we deal with fuzzy average total inspection. The results are summarized in the conclusion section

II. SAMPLING PLAN WITH FUZZY PARAMETER TERMINLOGY

In m independent Bernoulli experiment let us assume that p , probability of a "success" in each experiment is not known precisely and needs to be estimated, or obtained from expert opinion.

So that p value is uncertain and we substitute $\widetilde{p}$ for p and $\widetilde{q}$ for q so that there is a $p \in p[1]$ and a $q \in q[1]$ with $p+q=1$.

Now let $\widetilde{P}(r)$ be the fuzzy probability of r successes in m independent trials of the experiment. Under our restricted fuzzy algebra we obtain

$$\widetilde{P}(r)[\alpha] = \{C_m^r\, p^r\, q^{m-r} \mid s\} \qquad (1)$$

For $0 \leq \alpha \leq 1$, where now S is the statement, "$p \in \widetilde{p}[\alpha],\, q \in \widetilde{q}[\alpha],\, p+q=1$".

If $\widetilde{P}(r)[\alpha] = [P_{r_1}(\alpha), P_{r_2}(\alpha)]$ then $P_{r_1}(\alpha) = \min\{C_m^r\, p^r\, q^{m-r} \mid s\}$ and $P_{r_2}(\alpha) = \max\{C_m^r\, p^r\, q^{m-r} \mid s\}$ and if $\widetilde{P}[a,b]$ be the fuzzy probability of x successes so that $a \leq x \leq b$, then

$$\widetilde{P}([a,b])[\alpha] = \{\sum_{x=a}^{b} C_m^x\, p^x\, q^{m-x} \mid s\} \qquad (2)$$

* Corresponding author

978-0-7695-4014-6/10 $26.00 © 2010 IEEE
DOI 10.1109/ICICCI.2010.9

if $\widetilde{P}([a,b])[\alpha]=[P_1([a,b])[\alpha],P_2([a,b])[\alpha]]$ then:

$$P_1([a,b])[\alpha]=\min\left\{\sum_{x=a}^{b} C_m^x\ p^x\ q^{m-x} \mid s\right\} \text{ and}$$

$$P_2([a,b])[\alpha]=\max\left\{\sum_{x=a}^{b} C_m^x\ p^x\ q^{m-x} \mid s\right\}$$

Where, S is the same with past case. Suppose that we want to inspect a lot with the size of N, which the proportion of damaged items or the probability of the defectiveness of each of its number is not known precisely and is uncertain value. So we represent this parameter with a fuzzy number $\widetilde{p}$ which is:

$$\widetilde{p}=(a_1,a_2,a_3), p\in\widetilde{p}[1], q\in\widetilde{q}[1], p+q=1$$

A double sampling plan with a fuzzy parameter is defined by the first sample size n_1, acceptance number of first stage c_1, second sample size n_2 and acceptance number of second stage c_2. Then the operating procedure of the double sampling plan is given in the following steps:

Step1: First draw a random sample of size n_1, and observe the number of nonconforming items d_1.

Step2: If $d_1\le c_1$, the first stage acceptance number, accept the lot. If $d_1>c_2$, second stage acceptance number, reject the lot. If $c_1<d_1\le c_2$, go to Step 3.

Step3: Take a second random sample of size n_2 and observe the number of nonconforming items d_2. Cumulate d_1 and d_2, if $d_1+d_2\le c_2$, the second stage acceptance number, accept the lot. If $d_1+d_2>c_2$, reject the lot. If the size of lot was very great, the random variables d_1 and d_2 have fuzzy binomial probability distribution with parameters $(n_1,\widetilde{p})$ and $(n_2,\widetilde{p})$, in which $\widetilde{p}$ indicates the fuzzy proportion of the defective items. Fuzzy probability distributions have been studied by J.J. Buckley [4] and [5]. According to this case if we show the fuzzy probability of the lot's acceptance in combined samples with $\widetilde{p}_a$ and also the fuzzy probability of the lot's acceptance in first and second samples, respectively $\widetilde{p}_a^I, \widetilde{p}_a^{II}$ then

$$\widetilde{p}_a=\widetilde{p}_a^I+\widetilde{p}_a^{II} \tag{3}$$

Where $\widetilde{p}_a^I$ indicates the fuzzy probability of observation of $d_1\le c_1$ defective items in first random sample. Thus

$$\widetilde{p}[\alpha]=[p_1[\alpha],p_2[\alpha]]=$$
$$[a_1+(a_2-a_1)\alpha\ ,a_3-(a_3-a_2)\alpha]$$
$$\widetilde{p}_a^i=\widetilde{P}(A_i)=[P_{A_I}^L[\alpha],P_{A_i}^U[\alpha]], i=I,II$$
$$P_{A_I}^L[\alpha]=\min\{P(A_i)|s\} \tag{4}$$
$$P_{A_I}^L[\alpha]=\max\{P(A_i)|s\}, i=I,II$$

Where the event A_i is the event of acceptance of a lot in term sample i^{th} .

$$\widetilde{P}(d_1\le c_1)[\alpha]=\{\sum_{d_1=0}^{c_1} C_n^{d_1}\ p^{d_1}\ q^{n-d_1} \mid s\} \tag{5}$$
$$=[P^L[\alpha],P^U[\alpha]]$$
$$P^L[\alpha]=\min\{\sum_{d_1=0}^{c_1} C_n^{d_1}\ p^{d_1}\ q^{n-d_1} \mid s\}$$
$$P^U[\alpha]=\max\{\sum_{d_1=0}^{c_1} C_n^{d_1}\ p^{d_1}\ q^{n-d_1} \mid s\}$$

and $\widetilde{p}_a^{II}$ according to the independence of two random variables and their distribution will calculate with as follows this formula:

$$\widetilde{p}_a^{II}=\widetilde{P}(d_1+d_2\le c_2\ ,c_1<d_1<c_2) \tag{6}$$

III. FUZZY AVERAGE OUTGOING QUALITY

In programs of acceptance sampling we can do rectifying inspection in order to improving the level of lots quality. In a one rectifying inspection way, if the lot was accepted, the defective items in the sample will substitute with safe items, and if the lot was rejected, we do inspecting one hundred percent, and if we face with defective items, we substitute with safe items. The average outgoing quality is the level of lots quality after rectifying inspection process. It is the average outgoing rate of nonconforming. Assume the size of lot is N and the probability of defective items is $\widetilde{p}$, then with selecting a sample with the size n_1, with the probability $\widetilde{p}_a^I$ the lot will be accepted and the probability $1-\widetilde{p}_a^I$, we will do the secondary stage of sampling. If in this stage the lot is accepted, then n_1 items was investigated and became without deficient items, finally $N-n_1$ rested items that will accept without investigating, has $\widetilde{p}(N-n_1)$ defective items averagely. If we have to select the secondary sample, we will select the random sample in a size of n_2, then we will investigate it. If the lot was rejected in this stage, we will investigate one hundred percent, and we will substitute the defective items with the safe items. Then the number of defective items is equal to zero. If the lot is accepted, according to the inspection of n_1+n_2 unites and became without defective items, finally the $N-n_1-n_2$ rested units that is accepted without inspection has $\widetilde{p}(N-n_1-n_2)$ defective items averagely. Thus in the outgoing process, the number of defective items with the probability of $\widetilde{p}_a^I$ equal $\widetilde{p}(N-n_1)$ and with the probability of $\widetilde{p}_a^{II}$ equal $\widetilde{p}(N-n_1-n_2)$ and with the probability of $\widetilde{p}_a^{III}=1-\widetilde{p}_a^I-\widetilde{p}_a^{II}$ equal zero. There are $p_a{}^i\in\widetilde{p}_a{}^i, i=I,II,III$ such that $p_a^{\ I} + p_a^{\ II} + p_a^{\ III} = 1$. Then $\alpha -$ cut of FAOQ with the using of definition of fuzzy mean is as fallowing:

$$FAOQ[\alpha] = \left\{ \frac{[p_a^I(N-n_1) + p_a^{II}(N-n_1-n_2)]p}{N} \middle| s \right\}$$

$$= \left[FAOQ^L[\alpha], FAOQ^U[\alpha] \right] \qquad (7)$$

That

$$FAOQ^L[\alpha] = \min \left\{ \frac{[p_a^I(N-n_1) + p_a^{II}(N-n_1-n_2)]p}{N} \middle| s \right\}$$

$$FAOQ^U[\alpha] = \max \left\{ \frac{[p_a^I(N-n_1) + p_a^{II}(N-n_1-n_2)]p}{N} \middle| s \right\}$$

For $0 \le \alpha \le 1$, where S stands for the statement "$p_a^{\ i} \in \tilde{p}_a^{\ i}, i = I, II, III, p_a^I + p_a^{II} + p_a^{III} = 1$"

Example1: Suppose that $\tilde{p} = (0.01, 0.02, 0.03)$, $\tilde{q} = (0.97, 0.98, 0.97)$ and $c_1 = 0$, $c_2 = 1$, $N = 200, n_1 = n_2 = 10$, then AOQ of this lot is as follows:

$$\tilde{p}[\alpha] = [0.1 + 0.1\alpha, 0.3 - 0.1\alpha],$$

$$\tilde{p}_a^I[\alpha] = \left\{ (1-p)^{10} \middle| p \in \tilde{p}[\alpha] \right\},$$

$$\tilde{p}_a^{II} = \left\{ 10p(1-p)^{19} \middle| p \in \tilde{p}[\alpha] \right\}.$$

Then fuzzy average outgoing quality is

$$FAOQ = \left\{ 0.95p(1-p)^{10} + 9p^2(1-p)^{19} \middle| s \right\}$$

$$= [FAOQ^L[\alpha], FAOQ^U[\alpha]]$$

Then with studying function

$$f(p) = 0.95p(1-p)^{10} + 9p^2(1-p)^{19}$$

We have

$$FAOQ^L[\alpha] = 0.95(0.01 + 0.01\alpha)(0.99 - 0.01\alpha)^{10}$$
$$+ 9(0.01 + 0.01\alpha)^2(0.99 - 0.01\alpha)^{19}$$

$$FAOQ^U[\alpha] = 0.95(0.03 - 0.01\alpha)(0.97 + 0.01\alpha)^{10}$$
$$+ 9(0.03 - 0.01\alpha)^2(0.97 + 0.01\alpha)^{19}$$

Under $\alpha = 0$ we obtain $FAOQ[0] = [0.0093, 0.0256]$, that is, it is expected that for every 50 lots in such a process, 93 to 256 products will be defective items. And under $\alpha = 1$ we obtain $FAOQ[1] = [0.0178, 0.0178]$. Figure 1 shows the FAOQ in the comparison of the input quality process has improved. FAOQ is the function of lot' s quality and with the changing it, FAOQ will change. If the FAOQ draw in terms of the proportion of defective items of input lot, then the diagram will be as a band which has down and up bounds and it is called FAOQ band. To achieve this aim we consider the structure of $\tilde{p}$ as follows:

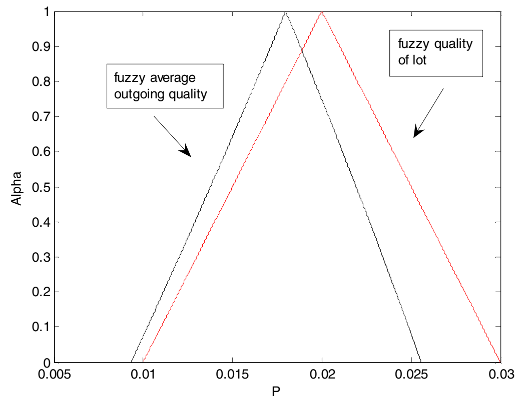

Fig.1 Fuzzy average outgoing quality for a double sampling plan of N=200, n_1=n_2=10, c_1=0, c_2=1, p=(0.01, 0.02, 0.03)

$$p \in \tilde{p}[1], q \in \tilde{q}[1], p + q = 1$$

Which with variation of k in the domain of $[0, 1 - a_3]$ and

$$\tilde{p}[\alpha] = [p_1(\alpha), p_2(\alpha)] = [k + a_2\alpha, a_3 + k - (a_3 - a_2)\alpha]$$

$$\tilde{p}_{a,k}^{\ I} = \tilde{P}_k(d_1 \le c_1)[\alpha]$$

$$\tilde{p}_{a,k}^{\ II} = \tilde{P}_k(c_1 \langle d_1 \le c_2, d_1 + d_2 \le c_2)[\alpha]$$

and

$$FAOQ[\alpha] = [FAOQ^L[\alpha], FAOQ^U[\alpha]] =$$
$$\left\{ \frac{[p_{a,k}^{\ I}(N-n_1) + p_{a,k}^{\ II}(N-n_1-n_2)]p}{N} \middle| p \in \tilde{p}[\alpha] \right\}$$

That

$$FAOQ^L[\alpha] =$$
$$\min \left\{ \frac{[p_{a,k}^I(N-n_1) + p_{a,k}^{II}(N-n_1-n_2)]p}{N} \middle| p \in \tilde{p}[\alpha] \right\}$$

and

$$FAOQ^U[\alpha] =$$
$$\max \left\{ \frac{[p_{a,k}^I(N-n_1) + p_{a,k}^{II}(N-n_1-n_2)]p}{N} \middle| p \in \tilde{p}[\alpha] \right\}$$

Knowing the uncertainty degree of proportion parameter (given a_1, a_2, a_3) and variation of its position on horizontal axis, we have different fuzzy number ($\tilde{p}$) and hence we will have different proportion (p) which the FAOQ bands are plotted in terms of it.

Example2: Suppose that $N = 200, a_2 = 0.01$, $a_3 = 0.02$, $n_1 = n_2 = 20, c_1 = 0, c_2 = 1$ then we have

$$\tilde{p}[\alpha] = [k + 0.01\alpha, k + 0.02 - 0.01\alpha]$$

$$FAOQ[\alpha]=$$
$$\left\{0.9p(1-p)^{20}+16p^2(1-p)^{39}\,\middle|\,p\in\tilde{p}[\alpha]\right\}$$

$$\mathrm{FAOQ}=\begin{cases}[FAOQ^*,FAOQ^{**}] & ,0\leq k<0.028078\\ [FAOQ^*,0.0215649] & ,0.028078\leq k<0.03875\\ [FAOQ^{**},0.0215649] & ,0.03875\leq k<0.048078\\ [FAOQ^{**},FAOQ^*] & ,0.048078\leq k<0.98\end{cases}$$

$$FAOQ^*=0.9k(1-k)^{20}+16k^2(1-k)^{39}$$
$$FAOQ^{**}=0.9(k+0.02)(0.98-k)^{20}$$
$$+16(k+0.02)^2(0.98-k)^{39}$$

Figure 2 shows the FAOQ band for the sampling plan with fuzzy parameter. We observe, if the proportion of defective items of input lot be very good or very bad, the FAOQ will be very good. If the incoming quality of lot is good, then a large proportion of the lots will be accepted by the sampling plan and only a smaller fraction will be screened and hence the outgoing quality will be small. Similarly, when the incoming quality is not good, a large proportion of the lots will go for 100% inspected and in this case also, the outgoing quality will be good since defective items will be replaced. Only for intermediate quality levels, lot acceptance will be at a moderate rate and hence the AOQ will rise.

One measure of how sampling plans perform is the average outgoing quality limit (AOQL). The AOQL is the maximum percentage of defective items that can be expected in the lots examined by the plan [14]. The maximum amount of FAOQ is the worse amount of FAOQ which will be earned in terms of an amount like in terms of an $\tilde{p}^*$ that will be called FAOQL. In example 2 is as:

$$\tilde{p}^*=[0.03875,0.05875],$$
$$FAOQL=[0.02096,0.02156]$$

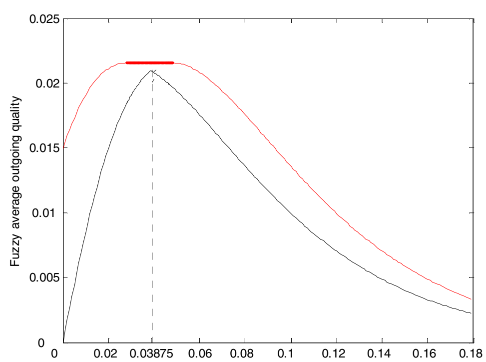

Fig.2 FAOQ band with N=200, n_1=n_2=20, c_1=0, c_2=1

IV. FUZZY AVERAGE TOTAL INSPECTION

Fuzzy average total inspection is an important in the rectifying inspection for sampling plan with fuzzy parameter. If the lot is accepted in the first stage (the fuzzy probability being $\tilde{p}_a^I$), the number of inspection items equal to n_1, and if the lot is accepted in the secondary stage, the number of inspected items is equal n_1+n_2 (the fuzzy probability being $\tilde{p}_a^{II}$) otherwise, equal to N (the fuzzy probability being $(1-\tilde{p}_a^I-\tilde{p}_a^{II})$). Consequently FATI according to the definition of fuzzy mean is as following:

$$FATI[\alpha]=\tilde{\mu}_{TI}[\alpha]=\left\{n_1p_a^I+(n_1+n_2)p_a^{II}+Np_a^{III}\,\middle|\,s\right\}$$
$$=\left\{N-(N-n_1)p_a^I-(N-n_1-n_2)p_a^{II}\,\middle|\,s\right\}$$
$$=\left[FATI^L[\alpha],FATI^U[\alpha]\right] \tag{8}$$

That

$$FATI^l[\alpha]=\min\left\{N-(N-n_1)p_a^I-(N-n_1-n_2)p_a^{II}\,\middle|\,s\right\}$$

$$FATI^U[\alpha]=\max\left\{N-(N-n_1)p_a^I-(N-n_1-n_2)p_a^{II}\,\middle|\,s\right\}$$

For $0\leq\alpha\leq1$, where S stands for the statement "$p_a^i\in\tilde{p}_a^i, i=I,II,III; p_a^I+p_a^{II}+p_a^{III}=1$"

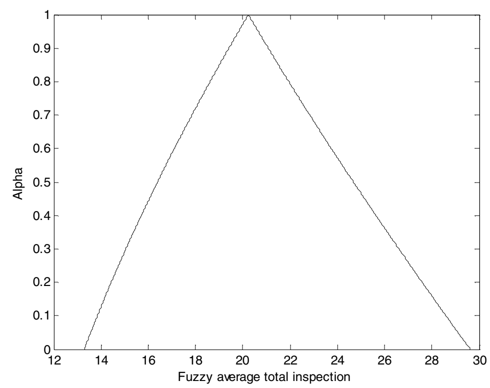

Fig.3 Fuzzy average total inspection for a double sampling plan of N=200, n_1=n_2=10, c_1=0, c_2=1, p=(0.01,0.02,0.03)

Fuzzy average total inspection in the example 1is as follows:

$$FATI=$$
$$\left\{200-190(1-p)^{10}-1800p(1-p)^{19}\,\middle|\,p\in\tilde{p}[\alpha]\right\}$$

$\alpha-$ cut FATI is as follows:

$$FATI^L[\alpha]=[200-190(0.99-0.01\alpha)^{10}-$$
$$(18+18\alpha)(0.99-0.01\alpha)^{19}]$$
$$FATI^U[\alpha]=[200-190(0.97+0.01\alpha)^{10}-$$
$$(54+18\alpha)(0.97+0.01\alpha)^{19}]$$

Under $\alpha=0$ we obtain FATI [0] $\cong$ [13, 30] and under $\alpha=1$, we obtain FATI [1] $\cong$ 20. Figure 3 illustrates a fuzzy average total inspection in the example 1.

According to the defined structure for $\tilde{p}$ in section III we can draw FATI band in terms of $\tilde{p}$. Then this diagram is as a band which as up and down bounds. The uncertainty degree of a proportion parameter is one of the factors that bandwidth of FAOQ and FATI depend on that. The less uncertainty value results in less bandwidth, and if proportion parameter gets a crisp value, lower and upper bounds will become equal, which that AOQ and ATI curve is in classic state. FATI band is the increasing function of proportion of defective items of input lot. FATI band in the example 1 with a_2=0.01 and a_3=0.02 is as follows:

$$\widetilde{p}[\alpha]=[k+0.01\alpha, k+0.02-0.01\alpha]$$

$$FATI_k[\alpha]=$$

$$\left\{200-190(1-p)^{10}-1800p(1-p)^{19} \middle| p\in \widetilde{p}[\alpha]\right\}$$

Under $\alpha=0$ we obtain

$$FATI_k^L[0]=200-190(1-k)^{10}-1800k(1-k)^{19}$$

$$FATI_k^U[0]=200-190(0.98-k)^{10}$$
$$-1800(k+0.02)(0.98-k)^{19}$$

Figure 4 shows three FATI bands for N=100, N=200, N=300. Figure 4 shows that FATI is increasing in term proportion of defective items. These figure represents that when the process quality decrease, then the FATI band will be narrower, and we observer when the quality of process is high, the FATI is near to the size of sample and if the quality of process is very low then the most of the lots will rejected and the FATI will be near to the size of lot. Table1shows FAOQ and FATI in terms of different $\widetilde{p}$ and $N=200, n_1=n_2=20, c_1=0, c_2=1$.

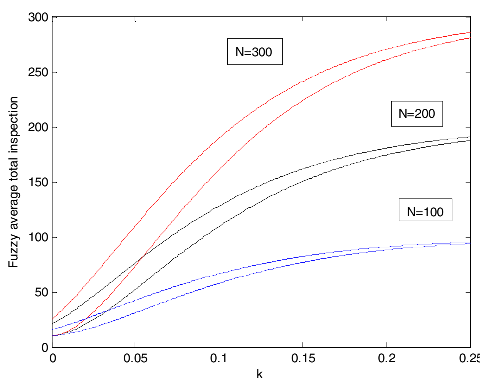

Fig .4 FATI bands with n_1=n_2=10, c_1=0, c_2=1

V. CONCLUSION

In the present paper we have proposed a method for definition and calculation AOQ and ATI for acceptance double sampling plans with fuzzy quality characteristic. This method is well defined since if the fraction of defective items is crisp they reduce to classical plans. As it was shown that AOQ and ATI curves of the plan is like a band having a high and low bounds.

REFERENCES

[1] E. Baloui Jamkhaneh, B. Sadeghpour Gildeh, GH. Yari, “Acceptance double sampling plan with fuzzy parameter,” proceedings of the 11th joint conference on information science, Shenzhen, China, 2008.

[2] E. Baloui Jamkhaneh, B. Sadeghpour Gildeh, GH. Yari, “Acceptance single sampling plan with fuzzy parameter with the using Poisson distribution,” proceedings world Academy of Science, Engineering and technology, volume 37, Dubai, 2009.

[3] I. Beaing, K. E., “A wide variety of AOQ and ATI performance measures with and without inspection error,” *Journal of quality technology*, vol. 13, no. 1, 1981, pp. 1-9.

[4] J.J. Buckley, Fuzzy probability: new approach and application, physica-velage, Heidelberg, Germany, 2003.

[5] J.J. Buckley, fuzzy probability and statistics, Springer-verlage Berlin Heidelberg, 2006.

[6] H. F. Dodge, H. G.Romig,, Sampling inspection tables, John Wiley Co., New York, U.S.A , 1956.

[7] B.P.M. Duate, P.M. Saraiva, “An optimization baesd approach for designing attribute acceptance sampling plans,” Int. journal of quality & reliability management, vol. 25, no.8, 2008.

[8] D. Dubis, H. Prade, “Operations of fuzzy number,” Int. J. syst, 1978.

[9] N. R. Farnum, “Closed-form approximation for the AOQL of attributes acceptance sample plans,” Communication in statistics-simulation and computation, 2006, 35:1057-1065.

[10] E. Grant, R.S. Leaven Worth., Statistical quality control, MC Graw-Hill, NewYork, 1996.

[11] P. Grzegorzewski, “Acceptance sampling plans by attributes with fuzzy risks and quality levels,” in: Frontiers in frontiers in statistical quality control. vol. 6, Eds. Wilrich P. Th. Lenz H. J. Springer, Heidelberg, 2001, pp. 36-46.

[12] P. Grzegorzewski, “A soft design of acceptance sampling plans by variables, ” in: Technologies for constructing intelligent systems, Eds, Springer, vol. 2, 2002, pp. 275-286.

[13] W.C. Guenther, “Determination of rectifying inspection plans for single sampling by attributes,” Journal *of quality technology*. vol. 17, no. 2, 1984, pp. 81-85.

[14] A. Kanagawa,H. Ohta, “A design for single sampling attribute plan based on fuzzy set theory, ” fuzzy sets and systems, 37, 1990, pp. 173-181.

[15] D. C. Montgomery, introduction to statistical quality control, Wiley New York, 1991.

[16] H. Ohta ,H. Ichihashi, “Determination of single-sampling attribute plans based on membership function, ” Int. J. Prod, Res 26, 1998, pp. 1477-1485.

[17] E.G. Schiling, Acceptance sampling quality control, Dekker, New York, 1982.

[18] K.K. Suresh, T.B. Ramkumar, “Selection of a sampling plan indexed with maximum allowable average outgoing quality,” Journal of applied statistics, 23(6), 1996, pp. 645-654.

[19] F. Tamaki , A. Kanagawa, Ohta H., “A fuzzy design of sampling inspection plans by attributes, ” Japanese journal of fuzzy theory and systems, no. 3, 1991, pp. 315-327.