Springer

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Journal of Mechanical Science and Technology 27 (12) (2013) 3645~3649
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-013-0907-z

Exact analysis of resonance frequency and mode shapes of isotropic and laminated composite cylindrical shells; Part II: Parametric studies †

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(Manuscript Received July 3, 2013; Revised July 7, 2013; Accepted July 7, 2013)

Abstract

In the second part of this study, the approach developed in Part I is used to analyze parameters which effect the natural frequencies and mode shapes of circular cylindrical shells. Therefore, amplitude ratios are determined analytically for shells of different geometries. The effects of circumferential and longitudinal wave numbers and geometrical parameters are studied on longitudinal, tangential and radial motions. Finally, numerical studies are conducted to investigate the effects of composite laminate parameters on resonance frequencies. Various laminate parameters such as stacking sequence and fiber angle are considered in the study.

Keywords: Amplitude ration; Circular cylindrical shell; Fiber orientation; Natural frequency; Stacking sequence

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1. Introduction

Unlike beams or plates which normally vibrate in one or two dimensions, shells can freely vibrate in three directions. These combined fluctuations will cause complicated motions at the resonance frequencies of the shell. Thus, apart from the frequency behavior, modal and amplitude identification of cylindrical shells has always been of great importance. Fields such as engineering design, acoustics and sound radiation are very dependent on the amplitude ratios of cylindrical shells. Furthermore, the use of composite laminates instead of isotropic material has also attracted a lot of attention. Composite cylindrical shells are currently being widely used in the aerospace, automotive, marine and civil industries.

Soedel [1] calculated modal amplitude ratios using the approximate beam function technique. Farshidinfar et al. [2] used Soedel's theory to investigate the modal amplitudes of a long circular cylindrical shell. Although modal amplitudes play an important role in vibration and sound radiation of shells, there are no detailed studies available in the literature, expressing the variations of these amplitudes with respect to the shell parameters.

Composite materials are similar to functionally graded materials. They both can be modified and designed for desired mechanical properties. Useful research has been reported on shells with functionally graded material [3-5]. The correct and effective use of composite materials requires abundant analyses. Studying vibration behavior of composite shells will lead us to a good understanding of the system response characteristics. Hence, knowledge of resonance frequencies is a critical component in the design process.

Soldatos [6] studied the free vibration problem of a thin elastic cross-ply laminated circular cylindrical panels, considering the most commonly used thin shell theories (Donnell's, Love's, Sander's and Flugge's theories). Soldatos and Messina [7, 8] continued their work using the higher order theory. They also investigated the vibration of angle-ply laminated plates and cylindrical panels. Sivadas and Genesan [9, 10] studied the effect of thickness variation of thick composite circular cylindrical shells on natural frequencies. Lam and Qian [11] presented the frequency characteristics for thick symmetric angle-ply laminated composite shells. Dogan and Arslan [12] presented effects of curvature on free vibration characteristics of cross-ply laminated composite shallow shells. To the author's knowledge, there are no numerical solutions available in the literature that study the vibration of angle-ply laminated composite cylindrical shells.

In this paper, the effects of longitudinal and circumferential wave numbers are studied on the modal amplitudes and mode shapes. Moreover, the behavior of cylindrical shells with various aspect and thickness ratios is investigated. Observations show that, variations in aspect and thickness ratios change the dominant motion of the shell, in longitudinal, tangential and radial directions. In addition, the influence of various parameters is investigated on natural frequencies of a composite shell.

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The effects of stacking sequence and fiber angle are studied on natural frequencies of a circular cylindrical shell for graphite/epoxy composite laminates. The proposed method is applied to various laminate configurations: cross-ply, angle-ply, sub-laminate and ply-level.

2. Exact analysis of resonance frequency

In Part I of this paper a method was developed to obtain the exact resonance frequencies of a cylindrical shell. Instead of using beam functions as approximations in the longitudinal direction, four boundary conditions on each end of the shell were used to solve the equation of motion. Thus, solving the displacement coefficient matrix and the eight boundary condition equations simultaneously, the exact resonance frequencies and modal amplitudes of a shell were obtained. The method was compared to the more conventional approximate solutions available in the literature. Results showed that the exact method was more accurate than the approximate technique in defining resonance frequencies of a simply supported shell. In the following sections, the exact procedure is used to study the mode shapes and modal amplitudes of a circular cylindrical shell. The method is also applied to a composite cylindrical shell in order to investigate its dynamic behavior.

3. Parametric studies on mode shapes and modal amplitude ratios

Due to their complex shapes and curvature effects, mode shape and modal amplitude identification of shells has always been of great interest. Modal amplitude identification is especially important in acoustical engineering. As pointed out in the previous sections, three types of motion occur for each resonance frequency: 1) radial, 2) circumferential and 3) tangential. Moreover, in a cylindrical shell, each mode shape is repeated at three distinct frequencies. Understanding the importance of each motion in each of the three resonance frequencies is crucial. For example, in acoustics only modes with a dominant radial motion radiate sound easily. In this section, variations of the amplitude ratios (A/C and *B*/*C*) are studied with respect to parameters of *m, n, L/mR* and *h/R*.

In Fig. 1, the amplitude ratios (A/C and *B*/*C*) are plotted for the lowest frequencies as a function of circumferential wave parameter *n*, for *m* = 1-6 and *L/R* = 3 according to the Soedel theory. As illustrated in Fig. 1, for $(m,n)=\{(1,0)\}$ the longitudinal and tangential motions are much stronger than the radial motion. In this mode, the longitudinal motion is dominant. Thus, at mode $(m,n)=\{(1,0)\}$, the shell experiences a dominant longitudinal motion with a rotation around the *x*-axis. However, in Fig. 1, for all modes except the *n* = 0, amplitude ratios gain values of A/C <1 and *B*/*C* <1. Therefore, for *m* = 1 modes (beam-like modes), as *n* increases (the frequency increases), the motion tends to become more dominant in the radial direction.

A similar behavior is also observed in modes with *m* = 2.

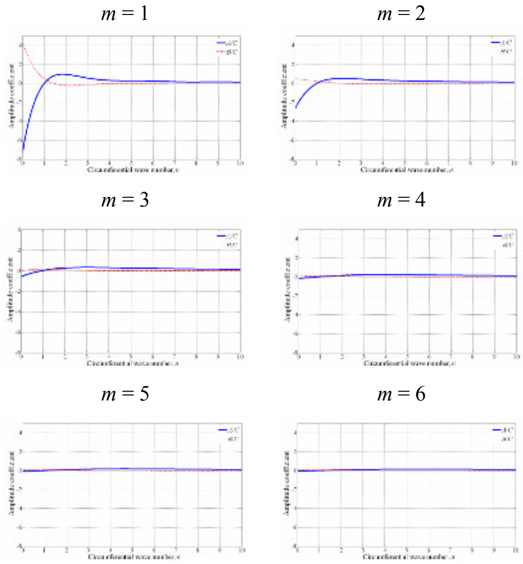


Fig. 1. Amplitude ratio versus circumferential wave number for *L/R* = 3, *h/R* = 1/20 and *m* = 1-6.

However, for mode $(m,n)=\{(2,0)\}$ only the longitudinal motion is dominant, and the tangential motion is weaker than the radial. Thus, at $(m,n)=\{(2,0)\}$ the shell is dominantly moving only in the axial direction. Furthermore, except at *n* = 0, all other *m* = 2 modes move dominantly in the radial direction with a behavior similar to that of the *m* = 1 modes.

For all other axial mode groups with *m* = 3, 4, 5 and 6, the dominancy of the radial motion is observed throughout all circumferential mode shapes *n*. According to Fig. 1 as each combination of *m* and *n* increases, the displacements of the shell tend to a more dominant radial motion. Therefore, at high frequencies the shell is dominantly excited in the radial direction, except at low mode numbers of *n* such as $(m,n)=\{(1,0)\text{ and }(2,0)\}$.

Let us now study the effects of length and radius on amplitude ratios and motions of the mode shapes. In Fig. 2, amplitude ratios are reported as a function of *L/mR* for various circumferential wave parameters of *n* = 1-6.

According to Fig. 2, for all circumferential wave parameters *n*, both amplitude ratios increase asymptotically with *L/mR*. Consequently, long shells or shells with small radius yield constant amplitude ratios at low axial mode numbers of *m*. On the other hand, as *m* increases, the amplitude ratios move away from the asymptotes, obtaining different values. Therefore, at higher axial wave parameters *m*, the radial motion is dominant. Moreover, as shown in Fig. 2 for *n* = 1, shells of longer length and smaller radius have no certain dominant motion, since A/C ≈1. Thus, the axial and circumferential motions are equally excited, which is completely understandable since long shells such as pipes behave more like beams and therefore create strong axial motions. Generally, according to Fig. 2, as *n* increases, both amplitude ratios tend to zero.

Hence, at high mode numbers regardless of the length and

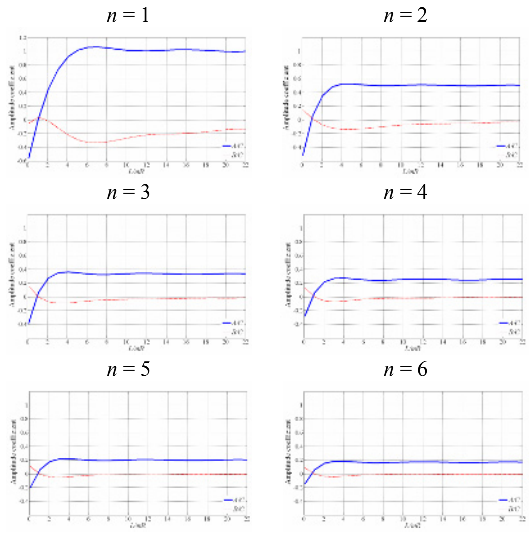


Fig. 2. Amplitude ratio versus *L/mR* for *h/R* = 1/20 and *n* = 1-6.

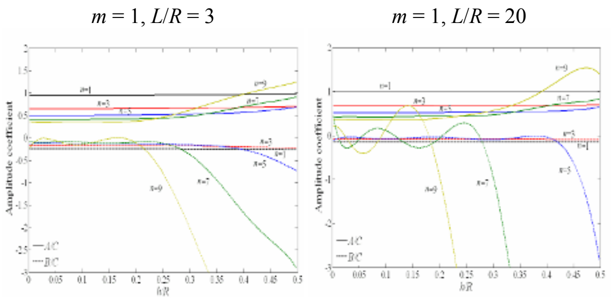


Fig. 3. Amplitude ratio versus *h/R* for *m* = 1: (a) *L/R* = 3; (b) *L/R* = 20.

radius of the shell, the motions are strongest in the radial direction. The exceptional behavior of cylindrical shells is noteworthy when *L/mR* = 1. In Fig. 2, amplitude coefficients tend to zero at *L/mR* = 1 for all circumferential wavenumbers. Thus, at *L/mR* = 1, the shell undergoes strong radial motions.

In Figs. 3-6, amplitude ratios are plotted as functions of the thickness ratio *h/R*, for shells with two aspect ratios *L/R* = 3 and 20. According to Figs. 3-6 for shells of *L/R* = 3 and *h/R* < 0.25, all modes up to *m* = 3 have dominant radial motions. This finding is true except for $(m,n)=\{(1,1)\}$, which also has an equally axial motion. It is observed that for shells of *h/R* > 0.25, the axial and tangential motions become stronger; however, these types of shells are categorized into thick-walled shells. According to Love's first approximation for thin-walled shells, only shells of small thickness to radius and length, are categorized into thin-walled theories. Thus, it may not be correct to calculate shells of *h/R* > 0.25 with thin-walled theories. However, the general vibration behavior and its response can be shown using these theories. Although, one would expect dominant radial motions for higher mode numbers of *m*, for *m* > 3 modes and *L/R* = 3, both amplitude ratios behave in a quasi-sinusoidal and irregular manner. Therefore, shells with small *L/R* ratios have a completely irregular amplitude behavior at high axial mode numbers.

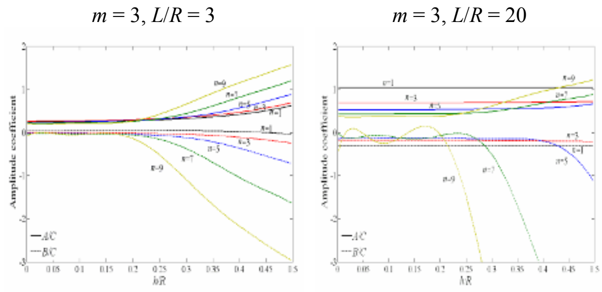


Fig. 4. Amplitude ratio versus *h/R* for *m* = 3: (a) *L/R* = 3; (b) *L/R* = 20.

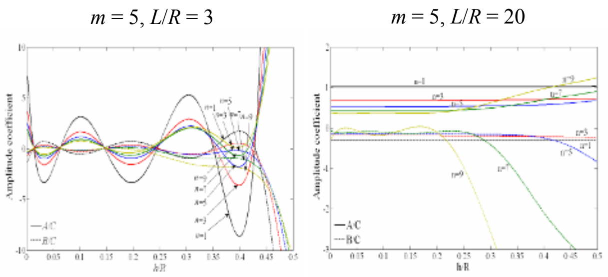


Fig. 5. Amplitude ratio versus *h/R* for *m* = 5: (a) *L/R* = 3; (b) *L/R* = 20.

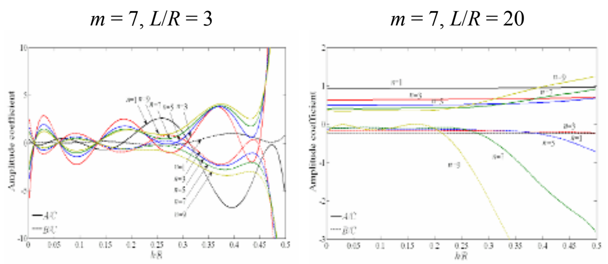


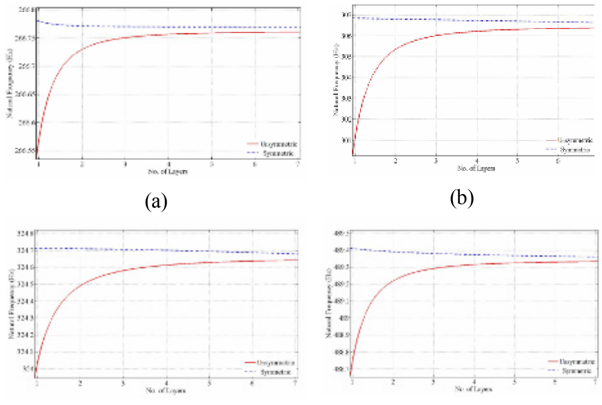
Fig. 6. Amplitude ratio versus *h/R* for *m* = 7: (a) *L/R* = 3; (b) *L/R* = 20.

On the other hand, for *L/R* = 20, a completely opposite pattern is observed compared to *L/R* = 3. As illustrated in Figs. 3-6, for shells of *L/R* = 20, at low mode numbers the tangential ratio behaves similar to a quasi-sinusoidal wave. However, the magnitude of the tangential and axial ratios does not exceed unity. Thus, the motions are always dominantly radial for small-radius long shells, regardless of the thickness. Nevertheless, at higher axial modes, the amplitude ratio behaves more consistently, having constant values for low *h/R* ratios.

It is concluded from Figs. 3-6 that the thickness is a crucial parameter when confronting short shells of small radius. Because these types of shells vibrate more like a ring rather than a beam.

4. Parametric studies on natural frequencies of laminated shells

Cylindrical shells have broad ranges of applications in the industry. Defining resonance frequencies is a major task in all these applications. Composite materials can be the best solu-



(a) (b)

(c) (d)

Fig. 7. Effect of sub-laminate configuration on natural frequency: (a) ω = 266 Hz; (b) ω = 301 Hz; (c) ω = 324 Hz; (d) ω = 488 Hz.

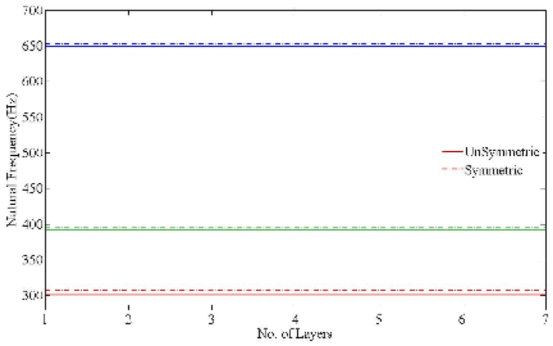


Fig. 8. Effect of ply-level configuration on natural frequency.

tion for this problem. Since a large number of design parameters are associated with these materials, the enormous design flexibility can be helpful in achieving certain design criterions. Changing laminate parameters such as fiber angle (α) and stacking sequence modifies the natural frequency. Hence, it is important to analyze different configurations of fiber orientation, stacking sequences, and numbers of layers. The various configurations for a laminate can be categorized into two groups according to fiber angle: 1) cross-ply and 2) angle-ply. Each of these can be considered as ply-level or sub-laminate related to the stacking sequence.

Boron/epoxy and graphite/epoxy have been the main composite materials used for bonding to metallic structures with epoxy-based adhesive [13]. In this study, graphite/epoxy (T300/976) laminate was used as an alternative to the aluminum material. The thickness of all composite patches with different number of layers is 1.47 mm.

Fig. 7 presents a variation of natural frequency when the cross-ply/sub-laminate is used. Different sub-laminate configurations are achieved by increasing the number of layers, $(0/90)_N$, N = 1,2,…,7. As shown in Fig. 7, increasing the number of layers, the natural frequency for asymmetric case, $(0/90)_N$, increases; however, it converges to a constant value when the number of layers rises to six. For symmetric case

Table 1. Natural frequency of a shell with angle-ply/sub-laminate configuration.

α	n = 1s	n = 2s	n = 3s	n = 4s	n = 5s	n = 6s	n = 7s
10°	352.18	352.08	352.02	351.99	351.97	351.95	351.94
20°	339.06	338.92	339.00	339.06	339.08	339.08	339.09
30°	336.11	337.69	337.80	337.80	337.79	337.77	337.76
40°	286.64	286.64	286.64	286.64	286.64	286.64	286.64
50°	253.65	253.65	253.66	253.66	253.66	253.66	253.66
60°	246.58	246.59	246.59	246.59	246.59	246.59	246.59
70°	265.38	265.51	265.53	265.53	265.53	265.54	265.54
80°	266.43	266.44	266.44	266.44	266.44	266.44	266.44
90°	264.39	264.39	264.39	264.39	264.39	264.39	264.39

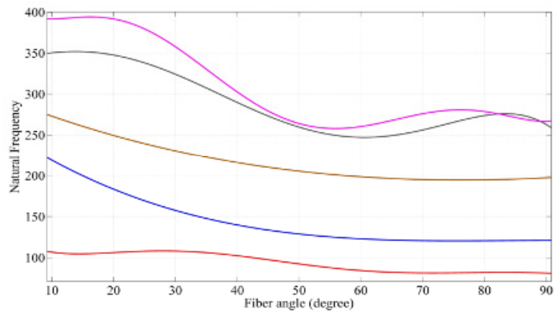


Fig. 9. Variation of natural frequency with fiber angle.

$(0/90)_{Ns}$, the trend is different. As the number of layers increases, the natural frequency decreases, but the rate of reduction is very slow compared to that of the asymmetric case, which has a high rate of increase for up to three layers.

Fig. 8 shows variation of natural frequency when cross-ply/ply-level laminates are used. Different ply-level configurations are achieved by repeating each ply $(0_N/90_N)$, N = 1,2,…,7. Of note, increasing number of layers has no effects on resonance frequencies of both the symmetric $(0_N/90_N)_s$ and asymmetric $(0_N/90_N)$ cases. It can be concluded from Figs. 7 and 8 that the natural frequency for symmetric case is always higher than the asymmetric one.

The effects of fiber angle for a constant value of layers (N = 1) is also studied. Symmetric laminate configuration is considered, $(\pm\alpha)_{1s}$. Fig. 9 presents natural frequencies of the thin circular cylindrical shell versus the fiber angle (α). The trend varies for different natural frequencies. At low frequencies, the resonance frequency is almost constant, but at medium frequencies as the fiber angle of plies increases, the resonance frequency decreases, gaining its relative minimum value at α = 60°. At high frequencies, the decreasing and increasing rates are high, compared to medium and low frequencies. Comparing Figs. 7 and 9, it can be concluded that by modifying the fiber angle from cross-ply (α = 0°, 90°) to angle-ply (α = 10°, 20°,…, 90°), the natural frequency reduces significantly.

Table 1 shows the variation of natural frequency for angle-ply/sub-laminate configuration $(\pm\alpha)_{Ns}$, N = 1,2,…,7. It is observed that the changing numbers of layers has minor effects on natural frequency, and for $N \geq 5$, this variation tends to zero.

5. Conclusions

Modal amplitudes, which are important parameters in acoustical engineering were analyzed in detail. The significance of each longitudinal, tangential and radial motion was studied through different wave numbers. Moreover, the effects of length, radius and thickness are completely studied on amplitude ratios and mode shapes. Results show that both the aspect and thickness ratios are crucial parameters that can change the behavior of a shell from beam type motion to ring type.

The influence of various composite laminate parameters on natural frequency was also investigated. The results presented herein indicate that sub-laminate level configuration has more influence on free vibration of a cylindrical shell than ply-level configuration; indeed, the ply-level configuration has no effects on the natural frequency. However, the number of layers modifies the natural frequency. For an asymmetric configuration, the natural frequency increases with the number of layers, whereas, for a symmetric configuration, it decreases. In both cases, the natural frequency converges to a constant value for six layers or more. Finally, changing the fiber orientation from cross-ply to angle-ply, a significant reduction in natural frequency was observed.

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