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A new approach to calculate the gluon polarization

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Abstract We derive the Leading-Order (LO) master equation to extract the polarized gluon distribution $G(x, Q^2) = x\delta g(x, Q^2)$ from polarized proton structure function, $g_1^p(x, Q^2)$. By using a Laplace-transform technique, we solve the master equation and derive the polarized gluon distribution inside the proton. The test of accuracy which is based in our calculations on two different methods, confirms that we achieve to the correct solution for the polarized gluon distribution. To determine the polarized gluon distribution $x\delta g(x, Q^2)$ more accurately, we only need to have more experimental data on the polarized structure functions, $g_1^p(x, Q^2)$. Our result for polarized gluon distribution is in good agreement with some phenomenological models.

1 Introduction

The spin structure of the proton is one of the most challenging open puzzles in Quantum Chromo Dynamics (QCD). One of the main questions in the particle and nuclear physics is: How is the proton spin built up from its quark and gluon constituents? Now, we know that the quark contribution to the spin of the proton is about 0.30. It is determined precisely in a QCD fit to the polarized proton structure function data, $g_1^p(x, Q^2)$. Therefore at the present time, the role played by the gluons in the nucleon spin is the most challenging task.

The main goal of the experiments at HERMES, SMC, COMPASS and RHIC spin physics programs is the measurement of the helicity contribution of the gluons to the nucleon spin, Δg. Experimentally this value is mainly accessible via two processes in polarized DIS experiments. The first one is the production of high p_t hadron pairs with large transverse momentum, but these processes have large background contributions from QCD Compton processes and

fragmentation that one has to control. The second one is the open charmed meson production in photon-gluon fusion process. The cross section of these two processes is directly related to the ratio of the polarized gluon density to the unpolarized gluon density, $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$ [1, 2]. Then, to measure the gluon contribution to the nucleon spin with this approach, it is necessary to know well the unpolarized gluon distribution. Polarized gluon measurements from deep inelastic experiments are summarized in Table 1.

In line with experiments, many NLO QCD global fits to the inclusive $g_1^p(x, Q^2)$ data were used to extract the magnitude of Δg and the shape of $\delta g(x, Q^2)$ [8–11]. Unfortunately, these global fits are sensitive to the initial assumption for the polarized gluon distribution and also to the initial scale of Q_0^2. The shape of the polarized gluon distribution extracted from different groups are not identical because of their different assumptions for the initial polarized gluon distribution, initial Q_0^2 and also different approaches for global fits. We would like to know how big the gluon spin contribution is to the total spin of the proton and look for a simple way to calculate this value. In this paper we consider a proposal to calculate directly the polarized gluon distribution inside the proton so as to have as much as possible accuracy.

Recently an explicit expression for the unpolarized gluon distribution function $xg(x, Q^2)$ in the proton in terms of the proton structure function $F_2^p(x, Q^2)$ was derived in the LO approximation, using a Laplace-transform technique [12–14]. Here, a similar procedure is used to derive the polarized gluon distribution function, $G(x, Q^2) = x\delta g(x, Q^2)$, inside the proton. We obtain an analytical solution for the polarized gluon distribution in terms of the polarized proton structure function, $g_1^p(x, Q^2)$. Thus we can calculate the polarized gluon distribution inside the proton directly just by finding a global fit for $g_1^p(x, Q^2)$ over a large kinematic range of x and Q^2. This is the main theoretical result which we achieve here.

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Table 1 Polarized gluon measurements from deep inelastic experiments

Experiment	Process	$\langle x_g \rangle$	$\langle \mu^2 \rangle$	$\frac{\delta g(x_g,\mu^2)}{g(x_g,\mu^2)}$	Reference
HERMES	hadron pairs	0.17	~ 2	$0.41 \pm 0.18 \pm 0.03$	[3]
HERMES	inclusive hadrons	0.22	1.35	$0.071 \pm 0.034 \pm 0.105$	[2]
SMC	hadron pairs	0.07	–	$-0.20 \pm 0.28 \pm 0.10$	[4]
COMPASS	hadron pairs, $Q^2 < 1$	0.085	~ 3	$0.016 \pm 0.058 \pm 0.055$	[5, 6]
COMPASS	hadron pairs, $Q^2 > 1$	0.13	–	$0.06 \pm 0.31 \pm 0.06$	[5, 6]
COMPASS	open charm	0.15	13	$-0.57 \pm 0.41 \pm 0.17$	[7]

This paper is organized as follows. In Sect. 2, we derive the LO master equation to extract the polarized gluon distribution $G(x, Q^2) = x\delta g(x, Q^2)$ from polarized proton structure function, $g_1^p(x, Q^2)$. Section 3 is devoted to the solution of the master equation. In the last section, we first perform a global fit to parameterize the $g_1^p(x, Q^2)$, using all available experimental data and then we calculate analytically the polarized gluon distribution. Our conclusion is given in Sect. 5.

2 LO master equation for polarized gluon distribution

The LO DGLAP equation, [15–17] for the evolution of the polarized proton structure function, $g_1^p(x, Q^2)$ can be written as (see Appendix)

$$x\frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2)\delta K_{qq}\left(\frac{x}{z}\right)$$
$$= x\frac{\alpha_s}{2\pi}\sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2)\delta K_{qg}\left(\frac{x}{z}\right), \qquad (1)$$

where $\delta K_{qq}(x)$ and $\delta K_{qg}(x)$ are the LO polarized splitting functions and α_s is the renormalized running coupling constant. We introduce $\mathcal{G}_1^p(x, Q^2)$ as

$$\mathcal{G}_1^p(x, Q^2) = x\frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2}$$
$$- \frac{\alpha_s}{2\pi} x \int_x^1 \frac{dz}{z} g_1^p(z, Q^2)\delta K_{qq}\left(\frac{x}{z}\right). \qquad (2)$$

Now we can write the master equation, the DGLAP equation for the $g_1^p(x, Q^2)$, as follows:

$$gg(x, Q^2) = x \int_x^1 \frac{dz}{z^2} G(z, Q^2)\delta K_{qg}\left(\frac{x}{z}\right), \qquad (3)$$

where $gg(x, Q^2) = (\frac{\alpha_s}{2\pi}\sum e_i^2)^{-1}\mathcal{G}_1^p(x, Q^2)$. In the (2) and (3), the LO $q \to q$ and $g \to q$ polarized splitting function

are given by [18]

$$\delta K_{qq}(x) = \frac{4}{3}\left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(x-1)\right), \qquad (4)$$

$$\delta K_{qg}(x) = \frac{1}{2}(2x-1) = x - \frac{1}{2}. \qquad (5)$$

To extract the polarized gluon distribution, $G(x, Q^2) = x\delta g(x, Q^2)$ from the DGLAP equation, we should solve the master equation (3) and find the polarized gluon distribution inside the proton. This issue is the subject of the next section.

3 Solution of the master equation

To solve the master equation (3), we follow the procedure that was used by BDM [12] and employ the Laplace transformation to solve this equation. Now we use the coordinate transformation as

$$v \equiv \ln(1/x). \qquad (6)$$

Then the functions $\hat{G}$, $\hat{K}_{qg}$, and $\hat{\mathcal{G}}$ in v-space are given by

$$\hat{G}(v, Q^2) \equiv G(e^{-v}, Q^2), \qquad (7)$$

$$\delta\hat{K}_{qg}(v, Q^2) \equiv \delta K_{qq}(e^{-v}, Q^2), \qquad (8)$$

$$\hat{\mathcal{G}}(v, Q^2) \equiv gg(e^{-v}, Q^2). \qquad (9)$$

Explicitly from (5), we have

$$\delta\hat{K}_{qg}(v, Q^2) = e^{-v} - \frac{1}{2}. \qquad (10)$$

Therefore (3) in v space appears as

$$\hat{\mathcal{G}}(v, Q^2) = \int_0^v \hat{G}(w, Q^2)e^{-(v-w)}\delta\hat{K}_{qg}(v-w)\,dw$$
$$= \int_0^v \hat{G}(w, Q^2)\hat{H}(v-w)\,dw, \qquad (11)$$

where $w \equiv \ln(1/z)$ and $\hat{H}(v)$ is defined as

$$\hat{H}(v) \equiv e^{-v}\delta\hat{K}_{qg}(v) = e^{-2v} - \frac{1}{2}e^{-v}. \tag{12}$$

If we take the Laplace transform of (11), then we have

$$\mathcal{L}[\hat{\mathcal{G}}(v, Q^2); s]$$

$$= \mathcal{L}\left[\int_0^v \hat{G}(w, Q^2)\hat{H}(v - w)\,dw; s\right] \tag{13}$$

$$\implies \hat{\mathcal{G}}(s, Q^2) = \hat{G}(s, Q^2) \times h(s). \tag{14}$$

In the above equation we used the following property for Laplace transformation

$$\mathcal{L}^{-1}[F(s)G(s)] = \int_0^t f(t - \tau)g(\tau)\,d\tau$$

$$= \int_0^t f(\tau)g(t - \tau)\,d\tau. \tag{15}$$

So we have the polarized gluon distribution in s-space as

$$\hat{G}(s, Q^2) = \frac{\hat{\mathcal{G}}(s, Q^2)}{h(s)}. \tag{16}$$

Now the polarized gluon distribution in v-space is given by

$$\hat{G}(v, Q^2) = \mathcal{L}^{-1}[\hat{\mathcal{G}}(s, Q^2)h(s)^{-1}]$$

$$= \int_0^v \hat{\mathcal{G}}(w, Q^2)\hat{J}(v - w)\,dw, \tag{17}$$

where $\hat{J}(v) \equiv \mathcal{L}^{-1}[h(s)^{-1}, v]$. The calculation of $\hat{J}(v)$, using (12) and inverse Laplace transform of $h(s)^{-1}$ for LO is straightforward and given in terms of the Dirac delta function:

$$\hat{J}(v) = 4 + 6\delta(v) + 2\delta'(v). \tag{18}$$

Combining (18) with (17), the polarized gluon distribution in v-space is given by

$$\hat{G}(v, Q^2) = 4\int_0^v \hat{\mathcal{G}}(w, Q^2)\,dw$$

$$+ 6\int_0^v \delta(v - w)\hat{\mathcal{G}}(w, Q^2)\,dw$$

$$+ 2\int_0^v \delta'(v - w)\hat{\mathcal{G}}(w, Q^2)\,dw. \tag{19}$$

Using the properties of Dirac delta function, we will have

$$\hat{G}(v, Q^2) = 4\int_0^v \hat{\mathcal{G}}(w, Q^2)\,dw + 6\hat{\mathcal{G}}(v, Q^2)$$

$$+ 2\frac{\partial\hat{\mathcal{G}}(v, Q^2)}{\partial v}. \tag{20}$$

Finally we have the polarized gluon distribution

$$G(x, Q^2) = 4\int_x^1 gg(z, Q^2)\frac{dz}{z} + 6gg(x, Q^2)$$

$$- 2\left(x\frac{\partial gg(x, Q^2)}{\partial x}\right), \tag{21}$$

where $gg(x, Q^2) = (\frac{\alpha_s}{2\pi}\sum e_i^2)^{-1}\mathcal{G}_1^p(x, Q^2)$ and $\mathcal{G}_1^p(x, Q^2)$ is given by (2). We use four massless quarks (u, d, s, c) in our calculations so as $\sum e_i^2 = \frac{10}{9}$.

To calculate the right hand side of the (21), we have to have an expression for $g_1^p(x, Q^2)$. So in the next section we try to do a global fitting parametrization for the structure function, $g_1^p(x, Q^2)$, using all available experimental data to obtain this function. Then we are able to calculate numerically the polarized gluon distribution function.

4 Numerical results

In this section we intend to use (21) to extract the polarized gluon distribution inside the proton.

4.1 Global parametrization of $g_1^p(x, Q^2)$ using all available experimental data

We parameterize the polarized proton structure function, $g_1^p(x, Q^2)$ in $0 < x < 1$ as

$$xg_1^p(x, Q^2) = x^a(1 - x)^b\frac{x_p g_p}{(1 - x_p)^b x_p^a}$$

$$\times \left(1 + A(Q^2)\ln\left[\frac{x_p(1 - x)}{x(1 - x_p)}\right]\right). \tag{22}$$

Here $x_P = 0.234$ is an approximate fixed point observed in the data where the curves for different Q^2 cross each other. At that point, $\partial g_1^p(x_P, Q^2)/\partial \ln Q^2 \approx 0$ for all Q^2; $g_P = g_1^p(x_P, Q^2) = 0.241$ is the common value of g_1^p. We use all available experimental data for polarized proton structure function including E143, SMC, HERMES 2006 and COMPASS 2009 [22–27]. A more accurate global fit, using more data over large kinematic ranges of x and Q^2 will lead to the precisely determination of the polarized gluon distribution function. The Q^2 dependence of $g_1^p(x, Q^2)$ in our global fit is given by

$$A(Q^2) = a_0 + a_1\ln Q^2 + a_2\ln^2 Q^2. \tag{23}$$

The fitted parameters are tabulated in Table 2. We use the global fit in (22) and depict in Fig. 1 and Fig. 2 the polarized proton structure function for some values of Q^2 and compare them with available experimental data. The comparison confirms that the global fit is working well.

Fig. 1 Polarized proton structure function, g_1^p at some values of Q^2, based on our global fit over all available experimental data
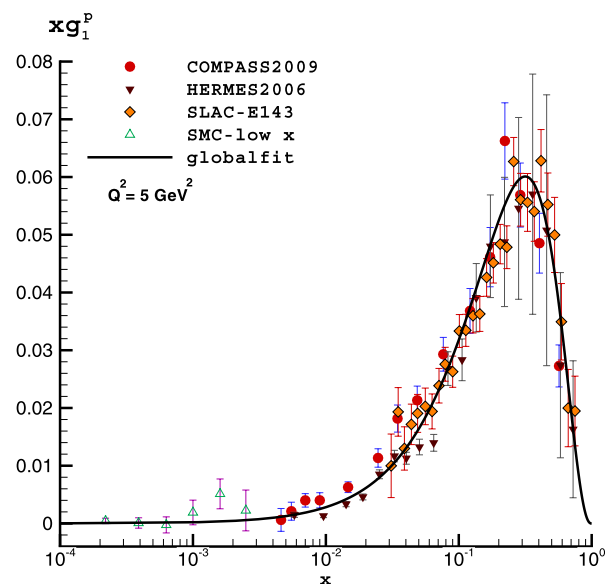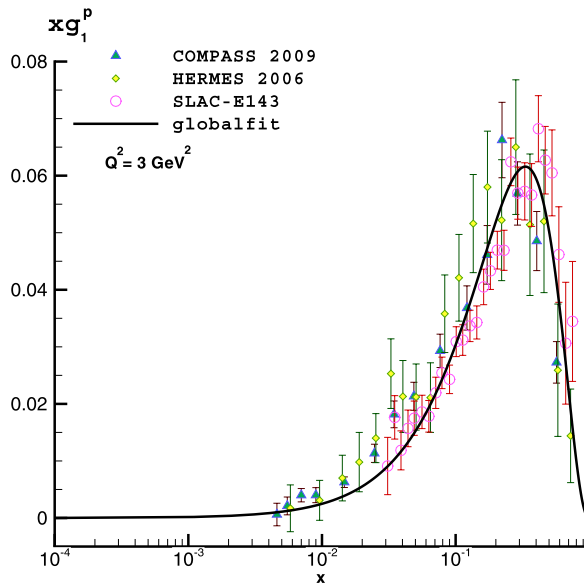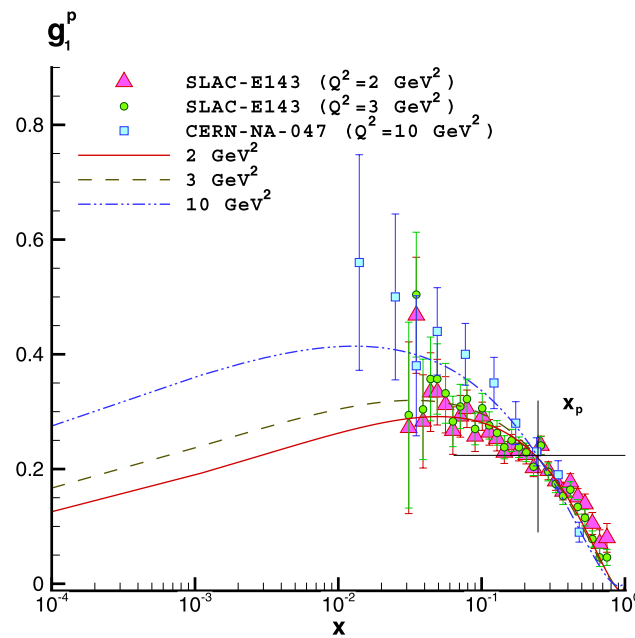
Fig. 2 Polarized proton structure function, xg_1^p at $Q^2 = 3$ GeV^2 and $Q^2 = 5$ GeV^2, resulting from a global fit over all available experimental data [22–27, 31]

Table 2 Parameters of (22), resulting from a global fit over all available experimental data

Parameters	Values
a_0	-0.0287 ± 0.1463
a_1	0.1253 ± 0.0671
a_2	-0.0092 ± 0.0145
a	1.2624 ± 0.0979
b	2.227 ± 0.4745
χ^2 (goodness of fit)	0.984

4.2 Numerical results for the polarized gluon distribution

In this section we will briefly describe the numerical results for polarized gluon distribution, using the analytical solution, (21). Finally we will compare our result for $x\delta g(x, Q^2)$ with those from *AAC*'08, *DSSV*'08 and *LSS*'06 global fits [28–30].

In this calculation, we use the LO approximation of $\alpha_s(Q^2)$ which is defined in [18] by

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)}, \tag{24}$$

$$\beta_0 = 11 - \frac{2}{3}n_f, \tag{25}$$

with $n_f = 5$ and $\Lambda_5 = 146$ MeV for $Q > 4.5$ GeV, $n_f = 4$ and $\Lambda_4 = 192$ MeV for 1.3 GeV $< Q \le 4.5$ GeV, and $n_f = 3$ and $\Lambda_3 = 221$ MeV for $Q < 1.3$ GeV. These numerical values have been used in *CTEQ5L* global fit [32] and also used to extract the analytical unpolarized gluon distribution in [12].

Now by following these two steps we calculate the polarized gluon distribution:

- Calculating $gg(x, Q^2)$ by using (2): $gg(x, Q^2) = (\frac{\alpha_s}{2\pi} \sum e_i^2)^{-1} \mathcal{G}_1^p(x, Q^2)$. The convolution integral in (2) which contains plus prescription, $()_+$, can be easily calculated by [18]

$$\int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right)_+ g(y) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right)\left[g(y) - \frac{x}{y}g(x)\right]$$
$$- g(x) \int_0^x dy\, f(y). \tag{26}$$

- Using (21), we can extract numerically the polarized gluon distribution inside the proton.

Our result for polarized gluon distribution inside the proton is shown in Fig. 3 (left) and compared with some global fits.

To make a rough estimate of the uncertainty in $x\delta g(x, Q^2)$ caused by the experimental uncertainties in the input structure function $g_1^p(x, Q^2)$, we follow Prytz's approach in [19] and find for gluon polarization at zeroth order

approximation with respect to the x expansion

$$x\delta g(x, Q^2) \approx \frac{9}{10} \frac{4\pi}{\alpha_s} \frac{\partial g_1^P(x, Q^2)}{\partial \ln Q^2}. \tag{27}$$

Numerical studies show that $\mathcal{G}_1^p(x, Q^2)$ is dominated at small x by $x\frac{\partial g_1^p(x,Q^2)}{\partial \ln Q^2}$. For a Q^2 in the available range 1.1–62 GeV^2 at $x = 0.0004$, we find that $\Delta[\frac{\partial g_1^p(x,Q^2)}{\partial \ln Q^2}]/\frac{\partial g_1^p(x,Q^2)}{\partial \ln Q^2} \approx 0.374$–$0.376$, leading to the rough estimate

$$\left.\frac{\Delta x\delta g(x, Q^2)}{x\delta g(x, Q^2)}\right|_{x=0.0004} \approx 37\% \tag{28}$$

for the error associated with the uncertainties in the fit to the $g_1^p(x, Q^2)$. What has been reported in [20, 21] for gluon polarization, implies also on large amount for the uncertainly of this quantity.

4.3 How to test the accuracy of the procedure?

As an independent method to check the numerical accuracy of the entire procedure for extracting the polarized gluon distribution, we go back to the original DGLAP equation, (3), from which we started, i.e.,

$$gg(x, Q^2) = x \int_x^1 \frac{dz}{z^2} G(z, Q^2) \delta K_{qg}\left(\frac{x}{z}\right),$$

and numerically integrate the right hand side of this equation, which depends on our numerical solution for $G(x, Q^2)$.
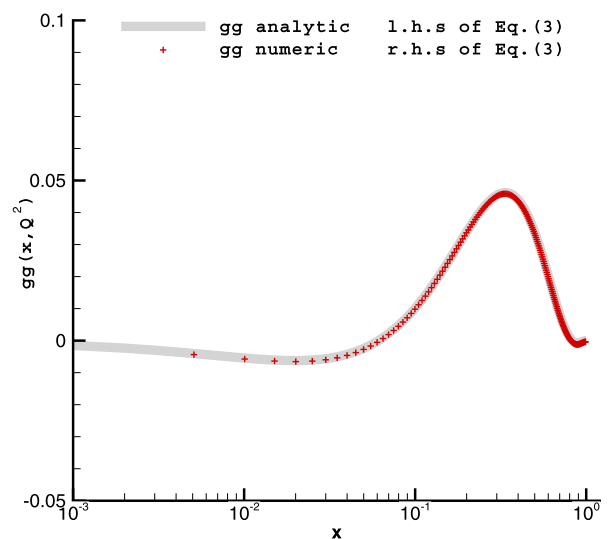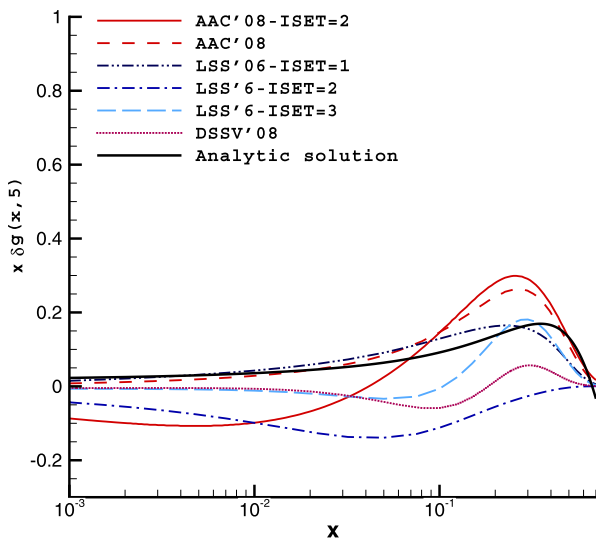
Fig. 3 *Left*: Polarized gluon distribution function, $x\delta g(x, Q^2)$ at $Q^2 = 5$ GeV^2, resulting from our calculation (21) and its comparison with other global fits. *Right*: *Solid* and *dotted* curves, resulting, respectively from the analytical and numerical solution of (3) to confirm the test of accuracy in our calculations

We then compare it with the $gg(x, Q^2)$, which is independently known and arising from our global fit for the polarized proton structure function, $g_1^p(x, Q^2)$, based on its relation to $\mathcal{G}_1^P(x, Q^2)$ in (2). In the right hand side of Fig. 3, the results of these two different methods are plotted. The accordance of the two solid and dotted curves in this part of Fig. 3, can be used as the test of accuracy in our calculations which confirms the correctness of our solution for polarized gluon distribution in (21).

In order to have more accurate solution for the polarized gluon distribution, we need to a best global fit for $g_1^p(x, Q^2)$, using all available experimental data. At the present time, there are not enough experimental data for small x ($x < 10^{-2}$) and also for large x values ($x > 0.7$). To achieve to a more trustful shape for polarized gluon distribution, we need to the experimental values for $g_1^p(x, Q^2)$ at these regions. This means that future experiments should be focused on measuring the polarized structure function in these two regions.

The gluon contribution to the spin of the proton in our calculation is

$$\Delta g\left(Q^2 = 5 \text{ GeV}^2\right) = \int_0^1 \delta g\left(x, Q^2 = 5 \text{ GeV}^2\right) dx$$
$$= 0.43. \qquad (29)$$

This value indicates that the gluon contribution to the proton spin is considerable and compatible with other phenomenological models [28–30].

5 Conclusions

In this paper, we have tried to find an analytical solution for the DGLAP equation for polarized structure function and derived the polarized gluon distribution inside the proton. Our solution is model independent and it is free of any parameters (initial input densities to solve the DGLAP equations at the initial scale of Q_0^2 and also Q_0^2 itself). It is only depended on finding a phenomenological function for $g_1^p(x, Q^2)$, using all available experimental data. Although in our calculations we have generalized the BDM [12] approach from unpolarized to polarized case, it contains important results as summarized here.

- A method to calculate directly the polarized gluon distribution inside the proton (an important part of the missing proton spin) is proposed. This method suggest the experimentalists to extract $\delta g(x, Q^2)$ instead of $\frac{\delta g(x, Q^2)}{g(x, Q^2)}$.
- It can be provided as a cross-check for the existence constrains of the polarized gluon distribution and it shows that standard current polarized fits are not too far off from actual values.

- It indicates that the gluon contribution to the proton spin is considerable.

Our results suggest that the precise measurement of experimental values for $g_1^p(x, Q^2)$ over a wide kinematic range of x and Q^2, can predict the polarized gluon distribution more accurately. The expansion of the results from LO to NLO approximation can be done as a new research job in future.

Appendix

The DGLAP equation for the polarized proton structure function is as follows:

$$x\frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi}x\int_x^1 \frac{dz}{z}g_1^p\left(z, Q^2\right)\delta K_{qq}\left(\frac{x}{z}\right)$$
$$= x\frac{\alpha_s}{2\pi}\sum e_i^2\int_x^1 \frac{dz}{z^2}G\left(z, Q^2\right)\delta K_{qg}\left(\frac{x}{z}\right). \qquad (30)$$

Here we are going to prove (1). We begin with the DGLAP equations for the polarized parton distribution functions for the quark and anti-quark sectors

$$\frac{\partial \delta q_i}{\partial t}(x, t) = \frac{\alpha_s}{2\pi}\left[\int_x^1 \frac{dz}{z}\delta q_i(z, t)\delta K_{qq}\left(\frac{x}{z}\right)\right.$$
$$\left. + \int_x^1 \frac{dz}{z}\delta g\left(z, Q^2\right)\delta K_{qg}\left(\frac{x}{z}\right)\right], \qquad (31)$$

$$\frac{\partial \delta \bar{q}_i}{\partial t}(x, t) = \frac{\alpha_s}{2\pi}\left[\int_x^1 \frac{dz}{z}\delta \bar{q}_i(z, t)\delta K_{\bar{q}\bar{q}}\left(\frac{x}{z}\right)\right.$$
$$\left. + \int_x^1 \frac{dz}{z}\delta g\left(z, Q^2\right)\delta K_{\bar{q}g}\left(\frac{x}{z}\right)\right], \qquad (32)$$

where $t = Ln Q^2$ and we used the following properties for the polarized splitting functions [18]:

$$\delta K_{qq} = \delta K_{\bar{q}\bar{q}}, \qquad (33)$$
$$\delta K_{qg} = \delta K_{\bar{q}g}. \qquad (34)$$

Adding (31) with the (32) and multiply both sides by $\frac{1}{2}\sum_{i=1}^{n_f} e_i^2 x$, we have

$$x\frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2}$$
$$= \frac{\alpha_s}{2\pi}x\int_x^1 \frac{dz}{z}g_1^p\left(z, Q^2\right)\delta K_{qq}\left(\frac{x}{z}\right)$$

$$+ x\frac{\alpha_s}{2\pi}\sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2)\delta K_{qg}\left(\frac{x}{z}\right). \qquad (35)$$

In (35) we have $G(z, Q^2) = z\delta g(z, Q^2)$ and $g_1^p(x, Q^2)$ is the LO polarized spin structure function for proton

$$g_1^p(x, Q^2) = \frac{1}{2}\sum_{i=1}^{n_f} e_i^2[\delta q_i(x, Q^2) + \delta\bar{q}_i(x, Q^2)]. \qquad (36)$$

Thus we achieved our desired result in (1):

$$x\frac{\partial g_1^p(x, Q^2)}{\partial \ln Q^2} - \frac{\alpha_s}{2\pi}x\int_x^1 \frac{dz}{z}g_1^p(z, Q^2)\delta K_{qq}\left(\frac{x}{z}\right)$$

$$= x\frac{\alpha_s}{2\pi}\sum e_i^2 \int_x^1 \frac{dz}{z^2} G(z, Q^2)\delta K_{qg}\left(\frac{x}{z}\right). \qquad (37)$$

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