



## **Partonic orbital angular momentum**

Firooz Arash, Fatemeh Taghavi-Shahri, and Abolfazl Shahveh

Citation: [AIP Conference Proceedings](#) **1523**, 166 (2013); doi: 10.1063/1.4802142

View online: <http://dx.doi.org/10.1063/1.4802142>

View Table of Contents: <http://scitation.aip.org/content/aip/proceeding/aipcp/1523?ver=pdfcov>

Published by the [AIP Publishing](#)

---

# Partonic Orbital Angular Momentum

Firooz Arash\*, Fatemeh Taghavi-Shahri† and Abolfazl Shahveh\*\*

\*Physics Department, Tafresh University, Tafresh, Iran

†Department of Physics, Ferdowsi University, Mashhad, Iran and School of Particles and Accelerators, Institute for Research in Fundamental Science (IPM), Tehran, Iran

\*\*School of Particles and Accelerators, IPM, Tehran, Iran

**Abstract.** Ji's decomposition of nucleon spin is used and the orbital angular momentum of quarks and gluon are calculated. We have utilized the so called valon model description of the nucleon in the next to leading order. It is found that the average orbital angular momentum of quarks is positive, but small, whereas that of gluon is negative and large. Individual quark flavor contributions are also calculated. Some regularities on the total angular momentum of the quarks and gluon are observed.

**Keywords:** Quark, gluon, spin, Orbital Angular momentum, Polarized structure Function

**PACS:** 13.60Hb, 13.88.+e

## INTRODUCTION

Spin content of the nucleon can be decomposed in terms of quarks  $\Delta\Sigma$ , gluon,  $\Delta G$  and overall angular momentum  $L_{q,g}$ . However, there is an ongoing debate about how to proceed with the proposed decomposition, because in the gauge theories the decomposition is not unique. Jaffe and Manohar [1] have used a light-like hypersurface and employed the light-cone framework to arrive at the following decomposition in the light-cone gauge

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \mathcal{L}_q^z + \frac{1}{2} \Delta G + \mathcal{L}_g^z \quad (1)$$

Each term is defined as the matrix element of the component of the orbital angular momentum tensor. The first and the third terms have a physical interpretation as the quark and gluon spin, respectively. The second and the fourth terms are identified as the quark and gluon orbital angular momentum. Unfortunately, except for the first term, other terms are not separately gauge invariant. An alternative decomposition is provided by Ji [2]

$$\frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L_q^z + J_g^z \quad (2)$$

where each term is separately gauge invariant, but the gluon total angular momentum is not decomposed into its spin and the orbital angular momentum in a gauge invariant way. In general, quark and gluon orbital angular momentum are defined differently in different decompositions.

Using Ji's decomposition, one can relate the quark orbital angular momentum to the Generalized Parton Distributions (GPD) [3] which in turn, can be measured in deeply

virtual Compton scattering.

$$L^q = \int dx \int d^2b (xH^q(x,b) + xE^q(x,b) - \tilde{H}^q(x,b)) \quad (3)$$

The GPDs describe the dynamics of partons in the transverse plane in position space. Complementary information on the dynamics of partons in the transverse plane, but in the momentum space, is obtained from Transverse Momentum Dependent parton distributions (TMD-PDF) [4, 5]. Therefore, one naturally expects that TMD and GPDs will teach us about partonic orbital angular momentum.

In the local limit, GPDs reduce to form factors, which are obtained from the matrix elements of the energy momentum tensor  $\Theta^{\mu\nu}$ . Since one can define  $\Theta^{\mu\nu}$  for each parton, one can identify the momentum fraction and contributions to the orbital angular momentum of each quark flavor and gluon in a hadron. Spin flip form factor  $B(q^2)$  provides a measure of the orbital angular momentum carried by each quark and gluon constituent of the nucleon at  $q^2 = 0$ . Similarly, the spin conserving form factor  $A(q^2)$ , allows one to measure the momentum fraction carried by each constituent. This is the underlying physics of Ji's sum rule.

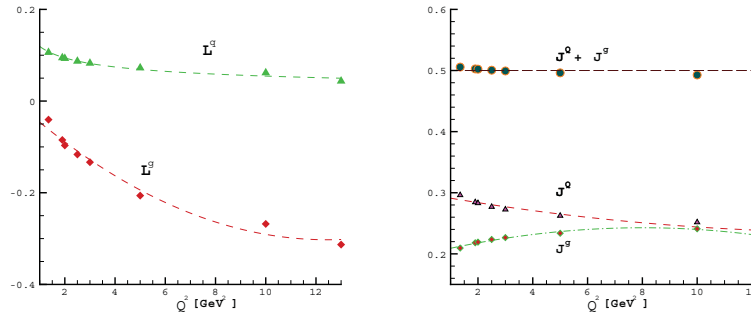
$$J_{q,g}^z = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)] \quad (4)$$

where,  $B_{q,g}$  are the second moments of unpolarized spin-flip GPD in the forward limit. It is subject to the constraint that  $B(0) = \sum_i B_i(0) = 0$ . That is, when summed over all partons, spin flip form factor vanishes. For the quark and the gluon sectors, equation [4] translates into

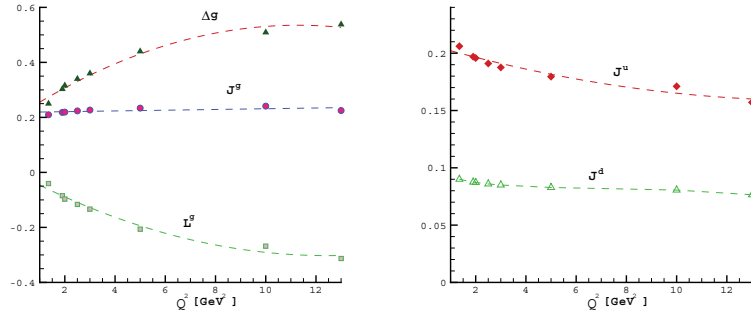
$$J_q^z(x) = \frac{1}{2} x [\langle q(x) \rangle + B_q(0)], \quad J_g^z(x) = \frac{1}{2} x [\langle g(x) \rangle + B_g(0)]. \quad (5)$$

Results from lattice calculations indicate that  $B_{q,g}$  is small and for the valence quarks a value between  $-0.077$  and  $0.015$  is obtained. It is believed that the sum of the contributions from the sea and the gluon must also be small. Therefore, we will set  $B_{q,g} = 0$  in the following analysis.

With a negligible value for  $B_{q,g}$ , the orbital angular momentum of partons,  $L^{q,g}$  can be determined entirely from the polarized and unpolarized parton distributions. Moreover, the evolution equation for the angular momentum distributions  $J^{q,g}(x)$  is exactly the same as that for the unpolarized quark and gluon distributions [6]. These distributions are evaluated in the valon model with good accuracy in a wide range of kinematics  $Q^2 = [0.4, 10^6]$  GeV<sup>2</sup> and  $x = [10^{-6}, 0.95]$ . The details can be found in [7]. In Figure 1 (left) we show the behavior of  $L^q(x)$ , and  $L^g(x)$  at several  $Q^2$  values. It is apparent that while the quark orbital angular momentum is small and positive, the gluon orbital angular momentum is negative and decreases as  $Q^2$  increases. We have checked to make sure that if our results reproduces  $J^p = \frac{1}{2} = J_q + J_g$ . The results are shown in Figure 1 (right). Evidently, this is the case and the sum of the total angular momentum of quarks and gluons is equal to  $\frac{1}{2}$ . In Figure 2 (Left) we present the gluon spin,  $\Delta g$ , gluon orbital angular momentum,  $L^g$  and the total angular momentum as a function of  $Q^2$ . This figure indicates that  $J^g$  is independent of  $Q^2$  and contributes an amount of

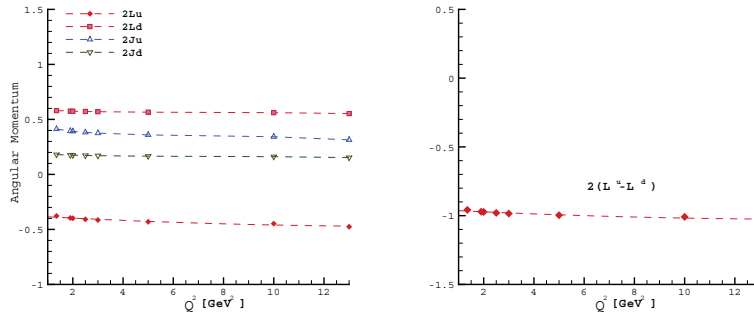


**FIGURE 1.** *Left:* Orbital Angular momentum of quarks and gluons in the valon model. *Right:* Total angular momentum of quarks and gluons in the valon model.



**FIGURE 2.** *Left:* Orbital Angular momentum of quarks and gluons in the valon model. *Right:* Total angular momentum of quarks and gluons in the valon model.

about 0.22 to the nucleon spin. On the right hand side panel of Figure 2 the total angular momentum of individual quark flavors,  $J^u$  and  $J^d$ , are shown. We see that the orbital angular momentum of u-quark,  $L^u$ , and d-quark,  $L^d$ , have opposite signs and largely cancel each other. Our results indicate that  $L^d$  is positive and  $L^u$  is negative. This is shown in Figure 3 (left). Their difference is shown in the right panel, and seems that its dependence on  $Q^2$  is marginal. In an interesting recent paper [8] the authors have derived a sum rule for spin-1 system through which they have obtained the total and the orbital angular momenta for  $u$  and  $d$  quarks in the proton at  $Q^2 = 4 \text{ GeV}^2$ . Our results on  $J^q = J^u + J^d$  agrees with their findings, amounting to 0.26 at  $Q^2 = 4 \text{ GeV}^2$ . This is interesting, because the two approaches are quite different. The two approaches also agree on the sign of  $L^u$  and within the errors, the numerical values are also compatible. Our findings, however, is different from those of [8] on the total and the orbital angular momenta of the  $d$  quark. Both approaches produce compatible values for the spin component of the the  $d$  quark. The total quark orbital angular momentum,  $L^Q = L^u + L^d$  in our model gives a value of 0.08 at  $Q^2 = 4 \text{ GeV}^2$ , whereas, the result of the Ref. [8] is  $-0.016 \pm 0.084$ . An earlier estimate of  $L^Q = 0.05 - 0.15$  is also given by Ji and



**FIGURE 3.** *Left:* Orbital Angular momentum of quarks and gluons in the valon model. *Right:* Total angular momentum of quarks and gluons in the valon model.

Tang [9]. It is evident that within the errors quoted, the two values are not far apart. In fact, except for  $L^d$ , within the errors, our results are fairly close to those of Ref. [8].

## CONCLUSION

We have investigated the orbital angular momentum contribution of quarks and gluons to the nucleon spin within the valon model. It shows that the quark orbital angular momentum contribution to nucleon's total angular momentum is positive and relatively small. However, that of the gluon is substantial. Thus, we conclude that the gluon is a major player in describing the spin structure of nucleon. On the one hand, while  $\frac{\delta g}{g}$  is small, but the first moment of the gluon polarization,  $\Delta G$  is large and increases as  $Q^2$  grows. On the other hand, gluon orbital angular momentum is large and negative. Thus, compensating the growth of  $\Delta G$ .

Some regularities have also emerged from our study: both orbital and total angular momenta of the u- and the d-quark seems to be independent of  $Q^2$ , though, some  $Q^2$  dependence for  $J_u$  is observed at low  $Q^2$ , but rapidly disappears. The same is true for the total angular momentum of the gluon. yet, its orbital angular momentum varies.

## REFERENCES

1. R.L. Jaffe and A. Manohar, Nucl. Phys. B **337**, 509 (1990).
2. X. D. Ji, Phys. Rev. Lett. **78**, 610 (1997).
3. X. D. Ji, Phys. Rev. D **55**, 7114 (1997).
4. D. Boer and P. J. Mulders, Phys. Rev. D **57**, 5780 (1998).
5. M. Anselmino, *et al.*, Phys. Rev. D **73**, 014020 (2006); hep-ph/0509035.
6. P. Hoodbhoy, X. Ji, and W. Lu, Phys. Rev. D **59**, 014013 (1998).
7. F. Arash and A. N. Khorramian, Phys. Rev. C **67**, 045201 (2003); F. Arash and F. Taghavi-Shahri, JHEP **0707**, 071 (2007); F. Arash and F. Taghavi-Shahri, Phys. Lett. B **668**, 193 (2008).
8. S. K. Taneja, K. Kathuria, S. Liuti, and G. R. Goldstein, arXiv:1101.0581 [hep-ph].
9. Xiangdong Ji and Jian Tang, Phys. Rev. Lett. **76**, 740 (1996).