

Influence of dephasing on the entanglement teleportation via a two-qubit Heisenberg XYZ system

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Abstract. We study the entanglement dynamics of an anisotropic two-qubit Heisenberg XYZ system in the presence of intrinsic decoherence. The usefulness of such a system for performance of the quantum teleportation protocol \mathcal{T}_0 and entanglement teleportation protocol \mathcal{T}_1 is also investigated. The results depend on the initial conditions and the parameters of the system. The roles of system parameters such as the inhomogeneity of the magnetic field b and the spin-orbit interaction parameter D , in entanglement dynamics and fidelity of teleportation, are studied for both product and maximally entangled initial states of the resource. We show that for the product and maximally entangled initial states, increasing D amplifies the effects of dephasing and hence decreases the asymptotic entanglement and fidelity of the teleportation. For a product initial state and specific interval of the magnetic field B , the asymptotic entanglement and hence the fidelity of teleportation can be improved by increasing B . The XY and XYZ Heisenberg systems provide a minimal resource entanglement, required for realizing efficient teleportation. Also, in the absence of the magnetic field, the degree of entanglement is preserved for the maximally entangled initial states $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$. The same is true for the maximally entangled initial states $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, in the absence of spin-orbit interaction D and the inhomogeneity parameter b . Therefore, it is possible to perform quantum teleportation protocol \mathcal{T}_0 and entanglement teleportation \mathcal{T}_1 , with perfect quality, by choosing a proper set of parameters and employing one of these maximally entangled robust states as the initial state of the resource.

1 Introduction

Entanglement is a central theme in quantum information processing as a uniquely quantum mechanical resource that plays a key role in many of the most interesting applications of quantum computation and quantum information [1–3]. Thus a great deal of efforts have been devoted to study and characterizing entanglement in the recent years. The central task of quantum information theory is to characterize and quantify entanglement of a given system. A pure state of a pair of quantum systems is called entangled if it is unfactorizable, e.g., singlet state of two half-spin systems. A mixed state ρ of a bipartite system is said to be separable or classically correlated if it can be expressed as a convex combination of uncorrelated states ρ_A and ρ_B of each subsystems i.e. $\rho = \sum_i \omega_i \rho_A^i \otimes \rho_B^i$ such that $\omega_i \geq 0$ and $\sum_i \omega_i = 1$, otherwise ρ is entangled [2,4]. Many measures of entanglement have been introduced and analyzed [5]. Here we use the negativity as a measure of entanglement [5]. For the $\mathbb{C}^2 \otimes \mathbb{C}^2$ bipartite systems, the negativity of a state ρ is defined as

$$E_N(\rho) = 2 \max\{-\lambda_{min}, 0\}, \quad (1)$$

where λ_{min} is the minimum eigenvalue of ρ^{TA} , and T_A denotes the partial transpose with respect to the part A of the bipartite system.

It is well known that a two-qubit entangled system can be used to perform the quantum teleportation protocols [6]. The pioneering authors of quantum information theory have shown that the mixed quantum channels which allow to transfer the quantum information with fidelity larger than $\frac{2}{3}$ are worthwhile [7]. By using the isomorphism between quantum channels and a class of bipartite states and twirling operations, Horodecki et al. have shown that the optimal fidelity of teleportation for a bipartite state acting on $\mathbb{C}^2 \otimes \mathbb{C}^2$ [8] is $\mathcal{F}_{max} = \frac{d\mathcal{F}_{max}+1}{d+1}$ where \mathcal{F}_{max} is the fully entangled fraction of the resource. Then, Bowen and Bose have shown that in the standard teleportation protocol \mathcal{T}_0 , with an arbitrary mixed state resource, the teleportation process can be considered as a general depolarizing channel with the probabilities given by the maximally entangled component of the resource and without additional twirling operations [9]. Using the property of the linearity of teleportation process, Lee and Kim [10] have shown that the quantum teleportation preserves the nature of quantum correlation in the unknown entangled states if the channel is quantum mechanically correlated.

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They have considered entanglement teleportation of an entangled state via two independent, equally entangled, noisy channels, represented by Werner states. In this teleportation protocol \mathcal{T}_1 , the joint measurement is decomposable into two independent Bell measurements and the unitary operation is also decomposable into two local Pauli rotations. In other word, \mathcal{T}_1 is a straightforward generalization of the standard teleportation protocol \mathcal{T}_0 just doubling the setup. Lee and Kim found that the quantum correlation of the input state will be lost during the teleportation, even if the channel has nonzero entanglement. They also found that in order to achieve a favorite teleportation fidelity, the quantum channel should possess a minimal entanglement. Hence, in comparison with the quantum teleportation, entanglement teleportation imposes more stringent conditions on the quantum channel [10].

Unfortunately, decoherence destroys the quantumness of the system and hence will decrease the useful entanglement between parts of the system [11–13]. There are several approaches to consider decoherence in a quantum system which is responsible for quantum-classical transition. One of these approaches is based on modifying Schrödinger equation in such a way that the quantum coherence be automatically destroyed as the system evolves. This mechanism is called *intrinsic decoherence* and has been studied in the framework of several models (see [14] and references therein). In particular, Milburn has proposed a simple modification of the standard quantum mechanics based on the assumption that for sufficiently short time steps the system evolution is governed by a stochastic sequence of identical unitary transformations rather than a continuous unitary evolution [15]. This assumption leads to a modification of Schrödinger equation which includes a term corresponding to the decay of quantum coherence in the energy basis. Using a *Poisson model* for the stochastic time step, Milburn obtained the following dynamical master equation in the first order approximation

$$\frac{d}{dt}\rho(t) = -i[H, \rho] - \frac{1}{2\gamma}[H, [H, \rho(t)]], \quad (2)$$

where H is Hamiltonian of the system, $\rho(t)$ indicates the state of the system and γ is the mean frequency of the unitary steps and determines the rate of decoherence [15]. In the limit $\gamma \rightarrow \infty$, Schrödinger's equation is recovered. Note that, in this mechanism of decoherence, the decay of quantum coherence is a result of phase relaxation process, so in the following we will only deal with the dephasing processes without the usual energy dissipation associated with normal decay. The first order correction to the equation (2) leads to diagonalization of the density operator in the energy eigenstate basis

$$\begin{aligned} \frac{\partial}{\partial t}\langle \varepsilon' | \rho(t) | \varepsilon \rangle &= -i(\varepsilon - \varepsilon')\langle \varepsilon' | \rho(t) | \varepsilon \rangle \\ &\quad - \frac{1}{2\gamma}(\varepsilon - \varepsilon')^2 \langle \varepsilon' | \rho(t) | \varepsilon \rangle. \end{aligned} \quad (3)$$

Note that the rate of diagonalization (dephasing) in the energy basis depends on the square of the energy separa-

tion of the superposed states. Thus the coherence between states that are widely separated in energy, decays rapidly. A formal solution of Milburn's dynamical master equation (2) can be written as [14,16,17]

$$\rho(t) = \sum_{k=0}^{\infty} M_k(t)\rho(0)M_k^\dagger(t), \quad (4)$$

where

$$M_k(t) = \sqrt{\frac{1}{k!} \left(\frac{t}{\gamma}\right)^k} H^k \exp(-iHt) \exp\left(-\frac{t}{2\gamma}H^2\right). \quad (5)$$

It is evident from the master equation (2) that the state of the system does not change with time, if the initial state of the system commutes with H . Thus all density matrices which can be written as a classical mixture of the eigenstates $|\psi_i\rangle$ of the Hamiltonian, i.e. $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ with $\sum_i p_i = 1$, are immune to intrinsic decoherence. One important case is the thermal state with $p_i = \frac{e^{-\beta\varepsilon_i}}{\text{tr}[e^{-\beta H}]}$, where $T = \frac{1}{k_B\beta}$ is the temperature and k_B is the Boltzmann constant [18]. In other words, the set of all thermal states with different temperatures span an intrinsic decoherence free subspace. A decoherence free subspace is a Hilbert space such that each state of this space is immune to decoherence [11].

The effects of intrinsic decoherence on the dynamics of entanglement, quantum teleportation, and entanglement teleportation of the Heisenberg systems have been studied in a number of works [17,19–24]. For example Yeo et al. [19] have shown that in an anisotropic two-qubit XY Heisenberg system, the nonzero thermal entanglement produced by adjusting the external magnetic field beyond some critical strength is a useful resource for teleportation via \mathcal{T}_0 and \mathcal{T}_1 protocols. The authors of reference [20] have shown that adjusting the magnetic field can reduce the effects of the intrinsic decoherence and accordingly one can obtain the ideal fidelity of teleportation via XYZ Heisenberg systems. Also, the results of reference [21] show that for an initial pure state of the resource, which is the entangled state of a two-qubit XXZ Heisenberg chain, an inhomogeneous magnetic field can reduce the effects of intrinsic decoherence. The authors of reference [17] argued that if the initial state is an unentangled state, one can improve the fidelity of teleportation protocol \mathcal{T}_0 via two-qubit Heisenberg XXX systems in the absence of the magnetic field by introducing the spin-orbit (SO) interaction, arising from Dzyaloshinski-Moriya (DM) interaction. However, the dynamics of entanglement and entanglement teleportation of more involved spin systems have not been discussed, yet.

In this paper, we investigate the influence of the intrinsic decoherence (dephasing) on the entanglement dynamics and teleportation scheme of a two qubit anisotropic Heisenberg XYZ system under the influence of an inhomogeneous magnetic field and in the presence of SO interaction. This system is suitable for modeling of a system

which is realized by the spin of two electrons confined in two coupled quantum dots [25–27]. Because of weak vertical or lateral confinement, electrons can tunnel from one dot to the other and spin-spin and spin-orbit interactions between the two qubits exist. In summary, we show that the dynamical and asymptotical behavior of the entanglement, the quality of the quantum teleportation and the entanglement teleportation and also the entanglement of the replica state, are dependent on the initial conditions and the parameters of the model. We discuss the problem for some special initial states and investigate the role of the parameters of model (such as coupling coefficients J_μ , magnetic field B , inhomogeneity of magnetic field b , SO interaction parameter D ...) on the entanglement properties of the system. The results show that for the maximally entangled initial states $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and the product initial states $|01\rangle$ and $|10\rangle$, the asymptotic value of the entanglement can be controlled by the parameters D and b . Also for the maximally entangled initial states $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ and the product initial states $|00\rangle$ and $|11\rangle$, the parameters B and χ determine the asymptotic value of the entanglement. The fidelity of the teleportation approaches $\frac{2}{3}$ from above for large values of D , for both product and maximally entangled initial states of the resource. Furthermore the results show that, it is possible to perform the teleportation protocols \mathcal{T}_0 and \mathcal{T}_1 with perfect quality in the XY and XYZ Heisenberg systems if the initial conditions and system parameters are set properly.

The paper is organized as follows. In Section 2, we introduce the Hamiltonian of the Heisenberg system under the influence of an inhomogeneous magnetic field in the presence of the SO interaction. For some initial states, the density matrix of the system at a later time is derived exactly by solving the Milburn's dynamical equation. The effects of initial conditions and system parameters on the dynamics of entanglement, as measured by negativity, are also studied in this section. The quantum teleportation and entanglement teleportation processes via the above system are investigated in Sections 3 and 4, respectively. Finally, in Section 5 a discussion concludes the paper.

2 Theoretical treatment

The Hamiltonian of a two-qubit anisotropic Heisenberg XYZ-model in the presence of inhomogeneous magnetic field and spin-orbit interaction is defined by [18]

$$H = \frac{1}{2}(J_x \sigma_1^x \sigma_2^x + J_y \sigma_1^y \sigma_2^y + J_z \sigma_1^z \sigma_2^z + \mathbf{B}_1 \cdot \boldsymbol{\sigma}_1 + \mathbf{B}_2 \cdot \boldsymbol{\sigma}_2 + \mathbf{D} \cdot (\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) + \delta \boldsymbol{\sigma}_1 \cdot \bar{\boldsymbol{\Gamma}} \cdot \boldsymbol{\sigma}_2), \quad (6)$$

where $\boldsymbol{\sigma}_j = (\sigma_j^x, \sigma_j^y, \sigma_j^z)$ is the vector of Pauli matrices, \mathbf{B}_j ($j = 1, 2$) is the magnetic field on site j , J_μ ($\mu = x, y, z$) are the real coupling coefficients (the chain is antiferromagnetic (AFM) for $J_\mu > 0$ and ferromagnetic (FM) for $J_\mu < 0$) and \mathbf{D} is Dzyaloshinski-Moriya vector, which is of first order in spin-orbit coupling and is proportional

to the coupling coefficients (J_μ) and $\bar{\boldsymbol{\Gamma}}$ is a symmetric tensor which is of second order in spin-orbit coupling [28–31]. For simplicity, we assume $\mathbf{B}_j = B_j \hat{\mathbf{z}}$ such that $B_1 = B + b$ and $B_2 = B - b$, where b indicates the amount of inhomogeneity of magnetic field. If we take $\mathbf{D} = J_z D \hat{\mathbf{z}}$ and ignore the second order spin-orbit coupling, then the above Hamiltonian can be written as¹:

$$H = J\chi(\sigma_1^+ \sigma_2^+ + \sigma_1^- \sigma_2^-) + (J + iJ_z D)\sigma_1^+ \sigma_2^- + (J - iJ_z D)\sigma_1^- \sigma_2^+ + \frac{J_z}{2}\sigma_1^z \sigma_2^z + \left(\frac{B+b}{2}\right)\sigma_1^z + \left(\frac{B-b}{2}\right)\sigma_2^z, \quad (7)$$

where $J := \frac{J_x + J_y}{2}$, is the mean coupling coefficient in the XY-plane, $\chi := \frac{J_x - J_y}{J_x + J_y}$ specifies the amount of anisotropy in the XY-plane (partial anisotropy, $-1 \leq \chi \leq 1$) and $\sigma^\pm = \frac{1}{2}(\sigma^x \pm i\sigma^y)$ are lowering and raising operators. The spectrum of H is easily obtained as

$$H|\Gamma^\pm\rangle = \varepsilon_{1,2}|\psi^\pm\rangle, \\ H|\Sigma^\pm\rangle = \varepsilon_{3,4}|\Sigma^\pm\rangle, \quad (8)$$

where the eigenstates expressed in the standard basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ are

$$|\Gamma^\pm\rangle = N^\pm \left(\left(\frac{b \pm \xi}{J - iJ_z D} \right) |01\rangle + |10\rangle \right), \\ |\Sigma^\pm\rangle = M^\pm \left(\left(\frac{B \pm \eta}{J\chi} \right) |00\rangle + |11\rangle \right), \quad (9)$$

with the eigenvalues

$$\varepsilon_{1,2} = -\frac{1}{2}J_z \pm \xi, \\ \varepsilon_{3,4} = \frac{1}{2}J_z \pm \eta, \quad (10)$$

respectively. In the above equations $N^\pm = \frac{1}{\sqrt{1 + \frac{(b \pm \xi)^2}{J^2 + (J_z D)^2}}}$

and $M^\pm = \frac{1}{\sqrt{1 + \frac{(B \pm \eta)^2}{(J\chi)^2}}}$ are the normalization constants.

Here we have defined, $\xi := \sqrt{b^2 + J^2 + (J_z D)^2}$ and $\eta := \sqrt{B^2 + (J\chi)^2}$, for later convenience.

According to equation (4) it is easy to show that, under intrinsic decoherence, the dynamics of the density operator $\rho(t)$ for the above mentioned two qubit Heisenberg system which is initially in the state $\rho(0)$ is given by

$$\rho(t) = \sum_{m,n=1}^4 \exp \left[-\frac{t}{2\gamma}(\varepsilon_m - \varepsilon_n)^2 - i(\varepsilon_m - \varepsilon_n)t \right] \\ \times \langle \phi_m | \rho(0) | \phi_n \rangle | \phi_m \rangle \langle \phi_n |, \quad (11)$$

¹ The parameters D and δ are dimensionless. In systems like coupled GaAs quantum dots D is of order of a few percent, while the order of the last term is 10^{-4} which is negligible.

where eigenenergies $\varepsilon_{m,n}$ and the corresponding eigenstates $|\phi_{1,2}\rangle = |I^\pm\rangle$ and $|\phi_{3,4}\rangle = |\Sigma^\pm\rangle$ are given in equations (9) and (10), and γ is the phase decoherence rate.

In the following we will examine the entanglement dynamics of a class of bipartite density matrices having the standard form

$$\begin{aligned} \rho(0) = & \mu_+|00\rangle\langle 00| + \mu_-|11\rangle\langle 11| + \nu(|00\rangle\langle 11| + |11\rangle\langle 00|) \\ & + w_1|01\rangle\langle 01| + w_2|10\rangle\langle 10| + z|01\rangle\langle 10| + z^*|10\rangle\langle 01|, \end{aligned} \quad (12)$$

which is called X states class and arises naturally in a wide variety of physical situations. If the initial state belongs to the set of X states (12), then the Hamiltonian (7) guarantees that $\rho(t)$ given by equation (11) also belongs to the same set. Therefore, the only non-vanishing components of the density matrix in the standard basis are

$$\begin{aligned} \rho_{11}(t) = & \frac{\mu_+}{2\eta^2} [2B^2 + (J\chi)^2(1 + \Phi(t))] \\ & + \frac{\mu_-}{2\eta^2} [(J\chi)^2(1 - \Phi(t))] - \frac{\nu}{\eta^2} [BJ\chi(1 - \Phi(t))], \\ \rho_{22}(t) = & \frac{w_1}{2\xi^2} [2b^2 + (J^2 + (J_z D)^2)(1 + \Phi'(t))] \\ & + \frac{w_2}{2\xi^2} [(J^2 + (J_z D)^2)(1 - \Phi'(t))] \\ & - \left[\frac{z}{2\xi^2} [(J - iJ_z D)b(1 - \Psi'(t))] + C.C \right], \\ \rho_{33}(t) = & \frac{w_1}{2\xi^2} [(J^2 + (J_z D)^2)(1 - \Phi'(t))] \\ & + \frac{w_2}{2\xi^2} [2b^2 + (J^2 + (J_z D)^2)(1 + \Phi'(t))] \\ & + \left[\frac{z}{2\xi^2} [(J - iJ_z D)b(1 - \Psi'(t))] + C.C \right], \\ \rho_{44}(t) = & \frac{\mu_+}{2\eta^2} [(J\chi)^2(1 - \Phi(t))] \\ & + \frac{\mu_-}{2\eta^2} [2B^2 + (J\chi)^2(1 + \Phi(t))] \\ & + \frac{\nu}{\eta^2} [BJ\chi(1 - \Phi(t))], \\ \rho_{14}(t) = & \frac{\mu_+}{2\eta^2} [BJ\chi(-1 + \Psi(t))] \\ & + \frac{\mu_-}{2\eta^2} [BJ\chi(1 - \Psi(t))] + \frac{\nu}{\eta^2} [(J\chi)^2 + B^2\Psi(t)], \\ \rho_{23}(t) = & \frac{w_1}{2\xi^2} [(J + iJ_z D)b(-1 + \Psi'(t))] \\ & + \frac{w_2}{2\xi^2} [(J + iJ_z D)b(1 - \Psi'(t))] \\ & + \frac{z}{2\xi^2} [(J^2 + (J_z D)^2) + \Theta(t)] \\ & + \frac{z^*}{2\xi^2} [(J + iJ_z D)^2(1 - \Phi'(t))], \\ \rho_{41}(t) = & \rho_{14}(t)^*, \\ \rho_{32}(t) = & \rho_{23}(t)^*. \end{aligned} \quad (13)$$

Here, we have defined $\Phi(t) := e^{-2\eta^2 t/\gamma} \cos 2\eta t$, $\Psi(t) := [\cos 2\eta t - \frac{i\nu}{B} \sin 2\eta t]e^{-2\eta^2 t/\gamma}$, $\Phi'(t) := e^{-2\xi^2 t/\gamma} \cos 2\xi t$, $\Psi'(t) := [\cos 2\xi t - \frac{i\xi}{b} \sin 2\xi t]e^{-2\xi^2 t/\gamma}$ and $\Theta(t) := [(\xi^2 + b^2) \cos 2\xi t - 2ib\xi \sin 2\xi t]e^{-2\xi^2 t/\gamma}$. For asymptotically large times, all of these functions vanish, and hence the state of the system reaches an entangled state ρ^∞ , asymptotically. The asymptotic value of entanglement depends on the values of the parameters and the initial conditions of the system. Knowing the density matrix $\rho(t)$, we can calculate the entanglement by negativity:

$$N(\rho) = \max\{-2 \min\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}, 0\} \quad (14)$$

where

$$\begin{aligned} \lambda_{1,2} = & \frac{1}{2} \left((w_1 + w_2) \pm \sqrt{(\rho_{22}(t) - \rho_{33}(t))^2 + 4|\rho_{14}(t)|^2} \right) \\ \lambda_{3,4} = & \frac{1}{2} \left((\mu_+ + \mu_-) \pm \sqrt{(\rho_{11}(t) - \rho_{44}(t))^2 + 4|\rho_{23}(t)|^2} \right), \end{aligned} \quad (15)$$

are eigenvalues of the partially transposed matrix $\rho^{TA}(t)$. The negativity is a function of the model parameters and the initial conditions. In the following let us examine some important initial states:

- (i) If the system is prepared in the maximally entangled initial state $|\psi(0)\rangle = |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, the negativity of the system at later time can be obtained as

$$\begin{aligned} N(\rho(t)) = & \frac{1}{\xi^2} |J(J + iJ_z D)(1 - \Phi'(t)) \\ & + (\xi^2 - b^2)\Phi'(t) + b^2\Psi'(t)|. \end{aligned} \quad (16)$$

This function approach the value

$$N^\infty = N(\rho^\infty) = \frac{1}{\xi^2} [J^2(J^2 + (J_z D)^2)]^{\frac{1}{2}}, \quad (17)$$

for large time limit. These formulae reveal that the dynamics and asymptotic value of the entanglement of the system can be controlled by the parameter ξ (or equally by D , b , J and J_z). The entanglement reaches a steady state value N^∞ , after some coherent oscillations at the large time limit. To maximize N^∞ , by setting equal to zero its partial derivatives with respect to D , we observe that the expression has two maxima at $D = 0$ and $D = \sqrt{b^2 - J^2}/|J_z|$ for fixed values of b , J and J_z . Note that for the values $b < |J|$ the second maximum disappears and hence N^∞ is a decreasing function of D , in this region. The values $b > |J|$ are not desirable since the amount of asymptotic entanglement decreases for large values of b , this is because the maximum value of the function N^∞ with respect to b occurs at the point $b = 0$ (for fixed values of other parameters) and hence N^∞ is a decreasing function of b . Consequently, we can adjust the parameters of the system to achieve the

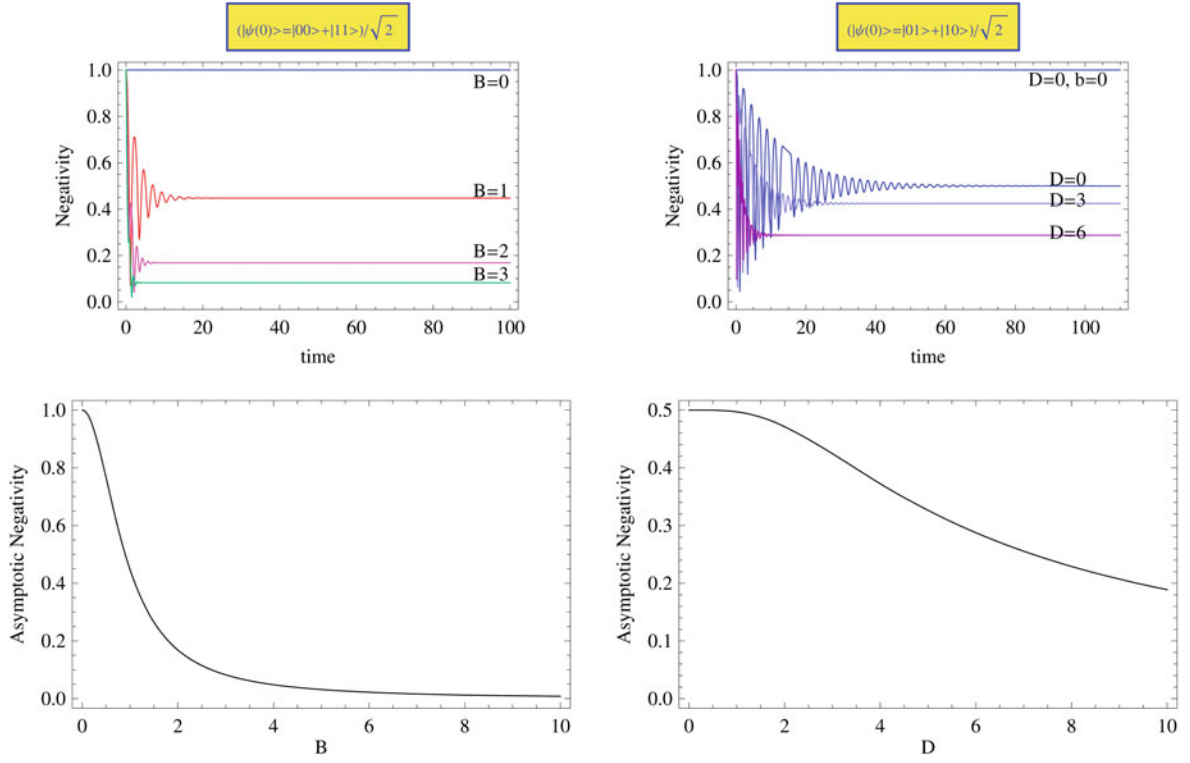


Fig. 1. (Color online) Dynamical and asymptotical behavior of the negativity vs. the model parameters and for the maximally entangled initial states, $|\Phi^+\rangle$ (left graph) and $|\Psi^+\rangle$ (right graph). The parameters of the model are chosen to be $J = 1$, $\chi = 0.9$, $J_z = 0.5$ and $b = 1$. We have set $D = 0$ and $\gamma^{-1} = 0.09$ for the left and $B = 3$ and $\gamma^{-1} = 0.02$ for the right graph. All parameters are dimensionless.

maximum asymptotic entanglement. For more clarifying, the time variation and asymptotical behavior of the function $N(t)$ versus D are depicted in Figure 1, (note that in this figure we set $b = J = 1$ and hence two maxima coincide at $D = 0$). Furthermore in this case and for the special value $D = b = 0$, we find $\rho(t) = \rho(0)$ for all times. Thus, in the absence of SO interaction and homogeneous magnetic field, the states $|\Phi^\pm\rangle$ are immune to intrinsic decoherence and hence has robust entanglement.

- (ii) If the system was initially in the the maximally entangled state $|\psi(0)\rangle = |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, the negativity of the system at later times can be written as

$$N(\rho(t)) = \frac{1}{\eta^2} |(J\chi)^2 + B^2\Psi(t)|, \quad (18)$$

and for large time limit reaches the value

$$N^\infty = N(\rho^\infty) = \frac{1}{\eta^2} |J\chi|. \quad (19)$$

In this case the parameter η (or equally B , J and χ) controls the dynamical and asymptotical behavior of the entanglement. The maximum value of the asymptotic entanglement N^∞ occurs for $B = 0$. This is because, in the absence of magnetic field, state of system remain untouched during the evolution i.e.

$\rho(t) = \rho(0)$ for $B = 0$. Thus in the absence of magnetic field the states $|\Phi^\pm\rangle$ are immune to dephasing processes and save its maximum entanglement during the evolution. The results also are shown in Figure 1.

- (iii) If the system being in the product state $|\psi(0)\rangle = |01\rangle$ (or $|10\rangle$), at early time of evolution, the negativity of $\rho(t)$ can be written as

$$N(\rho(t)) = \frac{b}{\xi^2} |(J + iJ_z D)(1 - \Psi'(t))|. \quad (20)$$

For large time limit, this function approaches the value

$$N^\infty = N(\rho^\infty) = \frac{b}{\xi^2} [(J^2 + (J_z D)^2)]^{\frac{1}{2}}. \quad (21)$$

According to these formulae, the asymptotic entanglement becomes maximize at the points $D = 0$ and $D = \sqrt{b^2 - J^2}/|J_z|$ for fixed values of b , J and J_z and also at the point $b = \sqrt{J^2 + (J_z D)^2}$ for fixed values of other parameters. Thus the size of asymptotic entanglement can be controlled by the parameter ξ (or equally by D , b , J and J_z). Also, equation (21) reveals that the asymptotic entanglement vanishes if $b = 0$, i.e. for the homogeneous magnetic field. It is in contrast with the case (i) where the asymptotic entanglement, becomes maximum at $b = 0$ (see Eq. (17)).

(iv) If $|\psi(0)\rangle = |00\rangle$ (or $|11\rangle$), the negativity of the system at time t can be expressed as

$$N(\rho(t)) = \frac{1}{\eta^2} |BJ\chi(-1 + \Psi(t))|, \quad (22)$$

and for asymptotically large time we obtain

$$N^\infty = N(\rho^\infty) = \frac{B}{\eta^2} |J\chi|. \quad (23)$$

Maximum value of the function N^∞ occurs for $B = |J\chi|$. Therefore this function increases with magnetic field for the values $B \leq |J\chi|$ and then decreases for large values of B . This means that in this case, we can suppress the effects of dephasing by increasing B in the region $B \leq |J\chi|$.

In summary, competition between Heisenberg interaction and dephasing processes leads the state of the system evolves to an asymptotic entangled state. The amount of this asymptotic entanglement can be controlled by the system parameters and initial conditions. From geometrical point of view, in the presence of the Heisenberg interaction and dephasing processes, both the product and the entangled initial states evolve to an asymptotic entangled state such that the start point, the trajectory and the end point of the evolution in state space depend on the initial conditions and the system parameters [32,33].

3 Quantum teleportation

According to the results of Bowen and Bose [9], the standard teleportation protocol \mathcal{T}_0 , when used with two-qubit mixed state of the Heisenberg spin chain $\rho(t)$ as a resource, acts as a generalized depolarizing channel $\Lambda_{\mathcal{T}_0}^{\rho(t)}[\rho_{in}]$. In the standard teleportation protocol an input state is destroyed and its replica (output) state appears at remote place after applying a local measurement and unitary transformation in the form of linear operators. Here we consider as input state an arbitrary pure state $|\psi_{in}\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$ ($0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$). The output (replica) state, ρ_{out} , can be obtained by applying joint measurement and local unitary transformation on the input state ρ_{in} . Thus the output state is given by

$$\rho_{out} = \Lambda_{\mathcal{T}_0}^{\rho(t)}[\rho_{in}] = \sum_{\mu=1}^4 p_\mu \sigma^\mu \rho_{in} \sigma^\mu, \quad (24)$$

where $\mu = 0, x, y, z$ ($\sigma^0 = I$), $p_\mu = \text{Tr}[E^\mu \rho(t)]$ represents the probabilities given by the maximally entangled fraction of the resource $\rho(t)$. Here $E^0 = |\Psi^+\rangle\langle\Psi^+|$, $E^1 = |\Phi^-\rangle\langle\Phi^-|$, $E^2 = |\Phi^+\rangle\langle\Phi^+|$ and $E^3 = |\Psi^-\rangle\langle\Psi^-|$ where $|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ and $|\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ are Bell states.

The quality of the teleportation is characterized by the concept of fidelity. The maximal teleportation fidelity

achievable in the standard teleportation protocol \mathcal{T}_0 is given by [8]

$$\Phi_{max} \left(\Lambda_{\mathcal{T}_0}^{\rho(t)} \right) = \frac{2\mathcal{F}(t) + 1}{3}, \quad (25)$$

where $\mathcal{F}(t) = \max_{\mu=0,1,2,3} \{p_\mu\}$ is the fully entangled fraction of the resource. For our model, the probabilities, p_μ s can be written as

$$\begin{aligned} p_{0,3} &= \frac{1}{2}(w_1 + w_2) \pm \Re[\rho_{23}(t)], \\ p_{1,2} &= \frac{1}{2}(\mu_+ + \mu_-) \pm \Re[\rho_{14}(t)]. \end{aligned} \quad (26)$$

Therefore, the maximum fidelity depends on both the initial conditions of the quantum channel and the parameters of the model. In the following we restrict our attention to the cases (i)–(iv), discussed in the previous section and choose them as the initial states of the resource.

(i) If $|\psi(0)\rangle = |\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$, then the maximum fidelity of \mathcal{T}_0 can be expressed as

$$\Phi_{max} \left(\Lambda_{\mathcal{T}_0}^{\rho(t)} \right) = \frac{2}{3} + \frac{1}{3} \frac{J^2 + (b^2 + (J_z D)^2) \Phi'(t)}{\xi^2}. \quad (27)$$

For the asymptotically large times $\Phi'(t)$ vanishes and we have

$$\Phi_{max}^\infty = \Phi_{max} \left(\Lambda_{\mathcal{T}_0}^{\rho^\infty} \right) = \frac{2}{3} + \frac{1}{3} \left(\frac{J}{\xi} \right)^2. \quad (28)$$

This equation states that, the maximum fidelity achievable at large time limit is always greater than $\frac{2}{3}$ i.e. this channel is superior to the classical channels. The function Φ_{max}^∞ is minimum ($\Phi_{max}^\infty = \frac{2}{3}$), if the interaction on the resource is Ising type in the z direction (i.e. $J = 0$) and reaches its maximum ($\Phi_{max}^\infty = 1$) for the case of $D = b = 0$. Note that in the later case the state of the channel is a maximally entangled state (see Eq. (17) and also Fig. 1).

(ii) If $|\psi(0)\rangle = |\Phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, then the maximum fidelity achievable for this quantum channel is

$$\Phi_{max} \left(\Lambda_{\mathcal{T}_0}^{\rho(t)} \right) = \frac{2}{3} + \frac{1}{3} \left(\frac{J\chi}{\eta} \right)^2 + \frac{1}{3} \left(\frac{B}{\eta} \right)^2 \Phi(t), \quad (29)$$

and hence,

$$\Phi_{max}^\infty = \Phi_{max} \left(\Lambda_{\mathcal{T}_0}^{\rho^\infty} \right) = \frac{2}{3} + \frac{1}{3} \left(\frac{J\chi}{\eta} \right)^2, \quad (30)$$

which means that, the XY and XYZ chains ($J, \chi \neq 0$) are more useful resources for performance of the teleportation protocol \mathcal{T}_0 . In this case, we have $\Phi_{max}^\infty = 1$ for $B = 0$. This result is compatible with the results of equation (19).

Table 1. The best Heisenberg model vs. resource/initial state.

Case	Initial state	Best Heisenberg model	Max. fidelity
(i)	$ \Psi^\pm\rangle$	XY model with $D = b = 0$	1
(ii)	$ \Phi^\pm\rangle$	XY(Z) model with $B = 0$	1
(iii)	$ 01\rangle/ 10\rangle$	XY model with $b = J $	$\frac{5}{6}$
(iv)	$ 00\rangle/ 11\rangle$	XY(Z) model with $B = J\chi $	$\frac{5}{6}$

(iii) $|\psi(0)\rangle_{channel} = |01\rangle$ (or $|10\rangle$), in this case just $w_1 = 1$ ($w_2 = 1$) is nonzero and hence

$$\Phi_{max} \left(\Lambda_{T_0}^{\rho(t)} \right) = \frac{2}{3} + \left| \left(\frac{bJ(1 - \Phi'(t))}{3\xi^2} - \frac{J_z D}{2\xi} \sin 2\xi t e^{-2\xi^2 t/\gamma} \right) \right|. \quad (31)$$

At the large time limit we can write

$$\Phi_{max}^\infty = \Phi_{max} \left(\Lambda_{T_0}^{\rho^\infty} \right) = \frac{2}{3} + \frac{|bJ|}{3\xi^2}. \quad (32)$$

In this case, we can adjust the quality of quantum teleportation by changing D , b , J_z and J . The asymptotic fidelity, Φ_{max}^∞ , tends to $\frac{2}{3}$ from above for large values of b , thus in the case of $J \neq 0$ our channel is superior to the classical communication. In this case, increasing $|J_z|$, decreases the quality of teleportation, and hence the XY chain is more suitable than XYZ chain. The maximum value of this function ($\text{Max}\{\Phi_{max}^\infty\} = \frac{5}{6}$) occurs for $b = |J|$. There is no way to reach the value $\Phi_{max}^\infty = 1$, since the equation $|bJ| = \xi^2$ has no real solution.

(iv) $|\psi(0)\rangle_{channel} = |00\rangle$ (or $|11\rangle$). In this case we have,

$$\Phi_{max} \left(\Lambda_{T_0}^{\rho(t)} \right) = \frac{2}{3} + \frac{B|J\chi|(1 - \Phi(t))}{3\eta^2}, \quad (33)$$

and for asymptotically large time we have

$$\Phi_{max}^\infty = \Phi_{max} \left(\Lambda_{T_0}^{\rho^\infty} \right) = \frac{2}{3} + \frac{B|J\chi|}{3\eta^2}. \quad (34)$$

According to equation (34), the presence of anisotropy in XY-plane ($\chi \neq 0$) provides the desirable fidelity ($\Phi^\infty > \frac{2}{3}$). Furthermore, the maximum value of this function ($\text{Max}\{\Phi_{max}^\infty\} = \frac{5}{6}$) occurs at the point $B = |J\chi|$, which is compatible with the previous results. In this case, the fidelity cannot take the maximum value $\Phi_{max}^\infty = 1$, because the equation $B|J\chi| = \eta^2$ has no solution in the domain of real numbers.

We summarize the results in Table 1. This table represents the best model related to the resource initial states and the maximum achievable fidelity of teleportation of an arbitrary state via T_0 protocol.

4 Entanglement teleportation

In this section, we consider Lee and Kim's teleportation protocol T_1 and use two copies of the above two-qubit

state, $\rho(t) \otimes \rho(t)$, as resource [10]. We consider the pure state $|\psi_{in}\rangle = \cos \frac{\theta}{2}|10\rangle + e^{i\phi} \sin \frac{\theta}{2}|01\rangle$ ($0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$) as the input state. The negativity associated with the input state, $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$ is $N(\rho_{in}) = N_{in} = |\sin \theta|$. By generalizing equation (24) the replica (output) state ρ_{out} can be written as

$$\rho_{out} = \Lambda_{T_1}^{\rho(t) \otimes \rho(t)} \rho_{in} = \sum_{\mu, \nu} p_{\mu\nu} (\sigma_\mu \otimes \sigma_\nu) \rho_{in} (\sigma_\mu \otimes \sigma_\nu), \quad (35)$$

where $\mu = 0, x, y, z$, $p_{\mu\nu} = p_\mu p_\nu$. By considering the two-qubit spin system as a quantum channel, the state of the channel is given by the equation (13) and hence one can obtain ρ_{out} as

$$\begin{aligned} \rho_{out} = & \alpha(|00\rangle\langle 00| + |11\rangle\langle 11|) + \kappa(t)(|00\rangle\langle 11| + |11\rangle\langle 00|) \\ & + a'|01\rangle\langle 01| + c'(t)|01\rangle\langle 10| \\ & + c'^*(t)|10\rangle\langle 01| + b'|10\rangle\langle 10| \end{aligned} \quad (36)$$

where

$$\begin{aligned} \alpha &= (w_1 + w_2)(\mu^+ + \mu^-), \\ \kappa(t) &= 4 \Re[\rho_{23}(t)] \Re[\rho_{14}(t)] \cos \phi \sin \theta, \\ a' &= (\mu^+ + \mu^-)^2 \cos^2 \frac{\theta}{2} + (w_1 + w_2)^2 \sin^2 \frac{\theta}{2}, \\ b' &= (w_1 + w_2)^2 \cos^2 \frac{\theta}{2} + (\mu^+ + \mu^-)^2 \sin^2 \frac{\theta}{2}, \\ c'(t) &= 2 e^{-i\phi} ((\Re[\rho_{23}(t)])^2 + e^{2i\phi} (\Re[\rho_{14}(t)])^2) \sin \theta. \end{aligned} \quad (37)$$

Now, we can determine the negativity of the output state as

$$N_{out} = N(\rho_{out}) = \max\{-2 \min\{\lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4\}, 0\} \quad (38)$$

where,

$$\begin{aligned} \lambda'_{1,2} &= \frac{1}{2} ((a' + b') \pm \sqrt{(a' - b')^2 + 4|\kappa(t)|^2}) \\ \lambda'_{3,4} &= \alpha \pm |c'(t)|, \end{aligned} \quad (39)$$

are the eigenvalues of $\rho_{out}^{T_A}(t)$. The function N_{out} is dependent on the entanglement of the input state N_{in} and the entanglement of the resource $N_{channel}$ (which is determined by the initial conditions and the parameters of the channel). It is easy to show that for the special cases (i)–(iv) of Section 2, the negativity of the replica state can be written as

$$N_{out}(t) = N(\rho_{out}(t)) = \cos^2 \Omega(t) [N_{channel}(t)]^2 N_{in}, \quad (40)$$

where $N_{channel}(t)$ refers to the entanglement of the channel given by equations (16), (18), (20), (22), respectively and

$$\Omega(t) = \begin{cases} \arctan \left[\frac{\Im[\rho_{23}(t)]}{\Re[\rho_{23}(t)]} \right] & \text{for the cases (i) and (iii),} \\ \arctan \left[\frac{\Im[\rho_{14}(t)]}{\Re[\rho_{14}(t)]} \right] & \text{for the cases (ii) and (iv).} \end{cases} \quad (41)$$

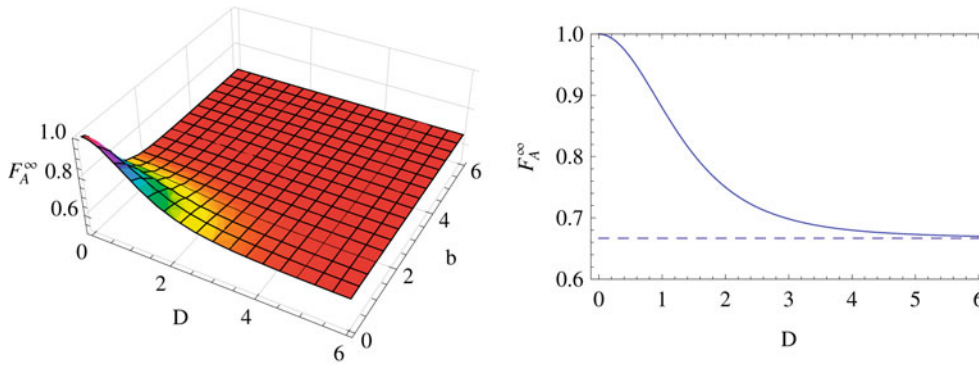


Fig. 2. (Color online) Asymptotic mean fidelity of the entanglement teleportation protocol \mathcal{T}_1 vs. D and b for maximally initial state, $\frac{|01\rangle+|10\rangle}{\sqrt{2}}$ of the resource. The parameters are the same as in Figure 1 and the value of $b = 0$ is chosen for the right graph.

For asymptotically large time limit we can write:

$$N_{out}^\infty = N(\rho_{out}^\infty) = \cos^2 \Omega^\infty [N_{channel}^\infty]^2 N_{in}, \quad (42)$$

where $N_{channel}^\infty$ is the asymptotic entanglement of the channel given by equations (17), (19), (21), (23), respectively and

$$\Omega^\infty = \begin{cases} \arctan[\frac{J_z D}{J}] & \text{for the cases (i) and (iii),} \\ 0 & \text{for the cases (ii) and (iv).} \end{cases} \quad (43)$$

Hence, we can manipulate the entanglement of replica state by manipulating the system parameters and by preparing suitable initial state of the resource. The results are compatible with the previous results. According to these formulae, we find that, more entangled channel state is, more input entanglement is preserved. This means that as the input entanglement increases, a more entangled quantum channel is required to realize efficient entanglement teleportation. Despite the fact that the above formula are valid only for the special cases under consideration, this result is general and is true for any pure or mixed initial state of the channel. To prove this generality let us study the fidelity of teleportation.

The fidelity between ρ_{in} and ρ_{out} in terms of the input negativity (N_{in}) is obtained as [34]

$$F(\rho_{in}, \rho_{out}; t) = |\langle \psi_{in} | \rho_{out} | \psi_{in} \rangle| = f_1(t) + f_2(t) N_{in}^2, \quad (44)$$

where $f_1(t) = (w_1 + w_2)^2$ and $f_2(t) = \frac{1}{2} - (w_1 + w_2) + 2((\Re[\rho_{14}(t)])^2 \cos 2\phi + (\Re[\rho_{23}(t)])^2)$. For $\phi = 0$, these functions depend only on the parameters of the channel. This formula has been also reported in reference [10] by Kim and Lee, but contrary to the Werner states, $f_2(t)$ can be a positive number for Heisenberg chains. This means that, there exists a channel which teleports more entangled initial states with more fidelity, but it should be noted that, if we choose the parameters of the channel such that $f_2(t) > 0$ then $f_1(t)$ decreases and ultimately $F(\rho_{in}, \rho_{out}; t)$ becomes smaller than $\frac{2}{3}$, which means that the entanglement teleportation of the mixed states is inferior to classical communication. Thus, to obtain the same proper fidelity, more entangled channels are needed for more entangled initial states.

The average fidelity F_A is another useful concept for characterizing the quality of teleportation. The average fidelity F_A of the teleportation can be obtained by averaging $F(\rho_{in}, \rho_{out}; t)$ over all possible initial states

$$\begin{aligned} F_A(t) &= \frac{\int_0^{2\pi} d\phi \int_0^\pi F(\rho_{in}, \rho_{out}; t) \sin \theta d\theta}{4\pi} \\ &= \frac{1}{3}(2(w_1 + w_2)^2 + (\mu_+ + \mu_-)^2 + 4(\Re[\rho_{23}(t)])^2). \end{aligned} \quad (45)$$

The function $F_A(t)$ depends on the initial conditions and parameters of the channel. For the special cases (ii) and (iv), this formula gives $F_A(t) = \frac{1}{3}$ which means these channels cannot teleport the input state $|\psi_{in}\rangle = \cos \frac{\theta}{2}|10\rangle + e^{i\phi} \sin \frac{\theta}{2}|01\rangle$, more efficiently than classical channels. In contrast, for the cases (i) and (iii), equation (45) implies that $F_A(t) = \frac{2}{3}(1 + \frac{1}{2} \cos^2 \Omega(t) (N_{channel}^\infty)^2)$ which always is greater than $\frac{2}{3}$. Thus these channels are superior to the classical ones for any given set of system parameters. According to this formula the asymptotic value of the average fidelity of entanglement teleportation becomes $F_A^\infty = \frac{2}{3}(1 + \frac{1}{2}(\frac{J}{\xi})^4)$, if the channel is initially in the state, $|\psi(0)\rangle_{channel} = |\Psi^+\rangle$ (case (i)). Figure 2 gives a plot of F_A^∞ in terms of the parameters D and b (which determine $N_{channel}^\infty$), in this case. The figure shows that, for fixed values of the other parameters and b , F_A^∞ decreases as D increases (or equally, $N_{channel}^\infty$ decreases), such that for the large values of D , F_A^∞ approaches $\frac{2}{3}$ from above. We can achieve perfect entanglement teleportation ($F_A^\infty = 1$) in the case of $D = b = 0$ which confirms the previous results. For the case of product initial state, $|\psi(0)\rangle_{channel} = |01\rangle$ of the resource (case (iii)), we have $F_A^\infty = \frac{2}{3}(1 + \frac{1}{2}(\frac{bJ}{\xi})^2)$ which tends to $\frac{2}{3}$ from above for large values of D , too. But never reaches the value 1 for any choice of the parameters.

5 Discussion

The effects of dephasing due to intrinsic decoherence on the entanglement dynamics of an anisotropic two-qubit Heisenberg XYZ system in the presence of an inhomogeneous magnetic field and SO interaction, are investigated.

The usefulness of such systems for performance of the quantum teleportation and entanglement teleportation protocols are also studied. Intrinsic decoherence destroys the quantum coherence (and hence quantum entanglement) of the system as the system evolves. For the case of noninteracting qubits, dephasing processes kill the quantum correlations (entanglement) of the system at a finite time and hence entanglement sudden death (ESD) phenomenon occurs (i.e. entanglement vanishes faster than local coherence of the system [32,33]). The results of this paper shows that, for interacting qubits, dephasing induced by intrinsic decoherence is competing with inter-qubit interaction terms to create a steady state level of entanglement after some coherence oscillation and hence the entanglement of the system reaches a stationary value, asymptotically. The dynamical and asymptotical behavior of the entanglement depend on the initial conditions and the system parameters. Indeed, the effects of dephasing can be amplified or weakened by adjusting the parameters of the model and initial conditions. We show that for the product and maximally entangled initial states, the asymptotic value of the entanglement decreases as D increases. This is because due to hermiticity of the Hamiltonian, we can express the state of the system as a superposition of the energy eigenstates. Increasing D , increases the energy separation of the superposed states (see Eq. (10)) and hence amplifies the effects of dephasing (see Eq. (3)). Consequently, for both product and maximally entangled initial states of the resource, the fidelity of the teleportation approaches $\frac{2}{3}$ form above for large values of D , this is the maximum fidelity for classical communication of a quantum state. Furthermore, our results show that, for product initial states and a specific interval of the magnetic field B , the asymptotic entanglement (and hence the fidelity of the teleportation) can be enhanced by increasing B . We have also argued that, a minimal entanglement of the resource is required to realize efficient entanglement teleportation. The results also show that, the XY and XYZ Heisenberg interaction can provide this minimal entanglement for the channel state.

We have also found that, in the absence of magnetic field ($B = 0$), the maximally entangled initial states, $|\Phi^\pm\rangle$, are immune to intrinsic decoherence and consequently, have robust entanglement. The same result is also true for the maximally entangled initial states $|\Psi^\pm\rangle$ in the absence of SO interaction and for homogeneous magnetic field (i.e. $D = b = 0$). Therefore choosing a proper set of parameters and employing one of these robust states as initial state of the resource, enable us to perform the quantum teleportation protocol \mathcal{T}_0 and the entanglement teleportation \mathcal{T}_1 with perfect quality ($\Phi_{max} = F_A = 1$).

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