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Entanglement dynamics and decoherence of an atom coupled to a dissipative cavity field

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Abstract. In this paper, we investigate the entanglement dynamics and decoherence in the interacting system of a strongly driven two-level atom and a single mode vacuum field in the presence of dissipation for the cavity field. Starting with an initial product state with the atom in a general pure state and the field in a vacuum state, we show that the final density matrix is supported on $\mathbb{C}^2 \otimes \mathbb{C}^2$ space, and therefore, the concurrence can be used as a measure of entanglement between the atom and the field. The influences of the cavity decay on the quantum entanglement of the system are also discussed. We also examine the Bell-CHSH violation between the atom and the field and show that there are entangled states for which the Bell-BCSH inequality is not violated. Using the above system as a quantum channel, we also investigate the quantum teleportation of a generic qubit state and also a two-qubit entangled state, and show that in both cases the atom-field entangled state can be useful to teleport an unknown state with fidelity better than any classical channel.

1 Introduction

Quantum entanglement is one of the most prominent non-classical properties of quantum mechanics which has recently attracted much attention in view of its connection with the theory of quantum information and computation. The rapidly increasing in quantum information processing has stimulated the interest of studying the quantum entanglement. It has been recognized that entanglement provides a fundamental potential resource for communication and information processing [1–3] and it is, therefore, essential to create and manipulate entangled states for quantum information application. Entanglement is usually arising from quantum correlations between separated subsystems which can not be created by local actions on each subsystems. A pure quantum state of two or more subsystems is said to be entangled if it is not a product of states of each components. On the other hands, a bipartite mixed state ρ is said to be entangled if it can not be expressed as a convex combination of pure product states [4], otherwise, the state is separable or classically correlated.

Entangled states are very fragile when they are exposed to environment. Actually, the biggest enemy of entanglement is decoherence which is believed to be the responsible mechanism for emergence of the classical behavior in quantum systems [5,6]. Since the maintenance and control of entangled states is essential to realization of quantum information processing systems, the study of deteriorating effect of decoherence in entangled states would be of considerable importance from theoretical as well as experimental point of view [7–9].

Entanglement dynamics and decoherence have been studied in the frame of various models. The interaction of a two-level atom with a single mode of the electromagnetic field, described by the Jaynes-Cummings model [10], is one of the simplest and most fundamental quantum systems. The Jaynes-Cummings model and related models with dissipation have more recently attracted interest in studies of quantum entanglement [11–22]. Solano et al. [23] have shown that multipartite entanglement can be generated by putting several two-level atoms in a cavity of high quality factor with a strong classical driving field. The resonance interaction of a cavity mode with a two-level atom that is driven by a coherent field have considered by Casagrande and Lulli [24]. They have shown that the system can reach the maximum entanglement after a unitary evolution for long enough interaction times. Lougovski et al. [25] have proposed the implementation of a strongly driven one-atom laser, based on the off-resonant interaction of a three-level atom in Λ configuration with a single cavity mode and three laser fields. They have shown that the system can be well approximated by a two-level atom resonantly coupled to the cavity mode and driven by a strongly effective coherent field. They have also studied

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the entanglement properties of the atom-field system on a time scale much shorter than the cavity decay time, where the atom-field system is almost a pure state. The entanglement of an open tripartite system where a cavity field mode in thermal equilibrium is off-resonantly coupled with two atoms that are simultaneously driven by a resonant coherent field have investigated in [26]. Bina et al. [27] have studied entanglement between two strongly driven atoms resonantly coupled to a dissipative cavity field mode. They have shown that for this system, the master equation is analytically solvable. In reference [28], the authors have studied the dynamics of an open quantum system where N strongly driven two-level atoms are equally coupled on resonance to a dissipative cavity mode, and have shown that also in this case the master equation is analytically solvable. Very recently, Zhang and Xu [29] have investigated entanglement dynamics and purity of a two-level atom, driven by a classical field, and interacting with a coherent field in a dissipative environment.

The aim of our paper is to analyze the dynamics of the Jaynes-Cummings model in order to find relation between the entanglement of the atomic and the filed degrees of freedom and the decoherence in the presence of dissipation for the field. The system considered here consists of a strongly driven two-level atom resonantly coupled to a dissipative cavity field mode [25]. We start initially with the atom in a general pure state and the field in a vacuum state and show that the final density matrix is supported on $\mathbb{C}^2 \otimes \mathbb{C}^2$ space, and therefore the concurrence can be used as a measure of the degree of entanglement between the atom and the field. The influences of the cavity decay on the quantum entanglement of the system are investigated, and find that the dissipation suppresses the entanglement. We also examine the Bell-CHSH violation between the atom and the field and show that there are entangled states for which the Bell-BCSH inequality is not violated. The decoherence induced by the cavity is also studied and it is shown that the coherence properties of the atom and also the field are affected by the cavity. The possibility of writing the atom-field density matrix as a two-qubit system enables us to use the atom-field system as a quantum channel for teleportation. The one-qubit teleportation and also the two-qubit entanglement teleportation via the quantum channel constructed by the atom-field system are also investigated and the fidelity of the teleportation and also the entanglement of the replica are also discussed. We show that in both cases the atom-field entangled state can be useful to teleport an unknown state with fidelity better than any classical channel.

The paper is organized as follows: in Section 2, we introduce the Hamiltonian of an atom interacting with a single mode vacuum field in the presence of dissipation. We also give the solution of the master equation in this section. In Section 3, we study entanglement of the atom-field system by using the concurrence, and investigate the effect of dissipation on the concurrence. We also examine the possible violation of the Bell-CHSH inequality. Section 4 is devoted to investigating the effect of dissipation on the purity of the system and its corresponding subsystems. The possibility of using the entanglement between the atom and the field as a resource to teleport the one-qubit and two-qubit states is also considered in Section 5. The paper is concluded in Section 6 with a brief conclusion.

2 Master equation and solution

The starting point for our analysis is the following Hamiltonian for the atom-field interaction [27]

$$\hat{H}(t) = \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\omega_f \hat{a}^\dagger \hat{a} + \hbar\Omega\left(e^{-i\omega_D t}\hat{\sigma}^\dagger + e^{i\omega_D t}\hat{\sigma}\right) + \hbar g\left(\hat{\sigma}^\dagger \hat{a} + \hat{\sigma}\hat{a}^\dagger\right). \tag{1}$$

This Hamiltonian describes a driven two-level atom interacting with a cavity field. Here g is the atom-field coupling constant, Ω is the Rabi frequency associated with the coherent driving field amplitude, $\omega_a = (\epsilon_e - \epsilon_g)/\hbar$ is the atomic transition frequency, ω_f denotes the field frequency, and ω_D is the frequency of the classical field. The atomic "spin-flip" operators $\hat{\sigma} = |g\rangle\langle e|(\hat{\sigma}^\dagger = |e\rangle\langle g|)$, and the atomic inversion operator $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ act on the atom Hilbert space $\mathcal{H}^A = \mathbb{C}^2$ spanned by the excited state $|e\rangle \to (1,0)^T$ and the ground state $|g\rangle \to (0,1)^T$. The field annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ satisfy the commutation relation $[\hat{a}, \hat{a}^\dagger] = 1$ and act on the field Hilbert space $\mathcal{H}^F$ spanned by the photon-number states $\left\{|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n!}}|0\rangle\right\}_{n=0}^{\infty}$.

In the following, we consider the dissipative dynamics for the cavity field when it is in contact with the environment, but we neglect atomic decays. The dynamics of the atom-field density operator $\hat{\rho}'$ is described by the master equation

$$\dot{\hat{\rho}}' = -\frac{i}{\hbar}[\hat{H}, \hat{\rho}'] + \hat{\mathcal{L}}_f \hat{\rho}', \tag{2}$$

where the super-operator $\hat{\mathcal{L}}_f$ describes the losses inside the cavity, and at zero temperature it is written as follows

$$\hat{\mathcal{L}}_f \hat{\rho}' = \frac{k}{2}\left[2\hat{a}\hat{\rho}'\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}' - \hat{\rho}'\hat{a}^\dagger\hat{a}\right], \tag{3}$$

where k is the cavity decay rate. In the interaction picture the master equation (2) can be written as

$$\dot{\hat{\rho}}_I = -\frac{i}{\hbar}[\hat{H}_I, \hat{\rho}_I] + \hat{\mathcal{L}}_f \hat{\rho}_I. \tag{4}$$

The dissipative term remains unchanged, and the time-independent Hamiltonian is given by $\hat{H}_I = \hat{H}_0 + \hat{H}_1$ with

$$\hat{H}_0 = -\hbar\delta\hat{a}^\dagger\hat{a} + \hbar\Omega(\hat{\sigma}^\dagger + \hat{\sigma}), \qquad \hat{H}_1 = \hbar g(\hat{\sigma}^\dagger\hat{a} + \hat{\sigma}\hat{a}^\dagger), \tag{5}$$

where we introduced the atom-cavity field detuning parameter $\delta = \omega_a - \omega_f$. Employing the unitary transformation $\hat{U}(t) = \exp\left\{\frac{i}{\hbar}\hat{H}_0 t\right\}$, we arrive at the following master equation for the density operator $\hat{\rho}(t) = \hat{U}(t)\hat{\rho}_I(t)\hat{U}^\dagger(t)$

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{U}\hat{H}_1\hat{U}^\dagger, \hat{\rho}] + \hat{\mathcal{L}}_f \hat{\rho}, \tag{6}$$

with

$$\hat{U}\hat{H}_1\hat{U}^\dagger = \frac{\hbar g}{2}\left[|+\rangle\langle+| - |-\rangle\langle-| + e^{2i\Omega t}|+\rangle\langle-| \right. \\ \left. -e^{-2i\Omega t}|-\rangle\langle+|\right]\hat{a}e^{i\delta t} + \text{H.C.}, \quad (7)$$

where H.C. stands for Hermitian conjugate and the rotated basis $\{|+\rangle, |-\rangle\}$ is defined by $|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$. On resonance ($\delta = 0$), and in the strong-driving regime for interaction between the atom and the external field, $\Omega \gg g$, and in the rotating-wave approximation, the following effective master equation is obtained

$$\dot{\hat{\rho}} = -\frac{i}{\hbar}[\hat{H}_{\text{eff}}(t), \hat{\rho}] + \hat{\mathcal{L}}_f\hat{\rho}, \quad (8)$$

with the effective Hamiltonian

$$\hat{H}_{\text{eff}} = \frac{\hbar g}{2}(\hat{\sigma}^\dagger + \hat{\sigma})(\hat{a} + \hat{a}^\dagger). \quad (9)$$

Hamiltonian (9) contains both the Jaynes-Cummings term $(\hat{\sigma}^\dagger\hat{a} + \hat{\sigma}\hat{a}^\dagger)$ and the anti-Jaynes-Cummings term $(\hat{\sigma}^\dagger\hat{a}^\dagger + \hat{\sigma}\hat{a})$ [25]. In the following we will describe the solution of the above effective master equation.

In order to solve the master equation (8), we follow the method introduced in [25,27]. Let us first introduce the following decomposition for the density operator $\hat{\rho}(t)$ of the whole system

$$\hat{\rho}(t) = \sum_{i,j=1}^{2}\langle i|\hat{\rho}(t)|j\rangle|i\rangle\langle j| = \sum_{i,j=1}^{2}\hat{\rho}_{ij}|i\rangle\langle j|, \quad (10)$$

where $\{|i\rangle\}_{i=1,2} = \{|+\rangle, |-\rangle\}$ is the rotated basis of the atom, and $\hat{\rho}_{ij}(t) = \langle i|\hat{\rho}(t)|j\rangle$ are operators acting on the field Hilbert space. With this definition, the master equation (8) is equivalent to the following set of uncoupled equations for the field operators $\hat{\rho}_{ij}(t)$

$$\begin{aligned}
\dot{\hat{\rho}}_{11} &= -\frac{ig}{2}\left[\hat{a}^\dagger + \hat{a}, \hat{\rho}_{11}\right] + \frac{k}{2}\left(2\hat{a}\hat{\rho}_{11}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}_{11} - \hat{\rho}_{11}\hat{a}^\dagger\hat{a}\right), \\
\dot{\hat{\rho}}_{12} &= -\frac{ig}{2}\left\{\hat{a}^\dagger + \hat{a}, \hat{\rho}_{12}\right\} + \frac{k}{2}\left(2\hat{a}\hat{\rho}_{12}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}_{12} - \hat{\rho}_{12}\hat{a}^\dagger\hat{a}\right), \\
\dot{\hat{\rho}}_{22} &= \frac{ig}{2}\left[\hat{a}^\dagger + \hat{a}, \hat{\rho}_{22}\right] + \frac{k}{2}\left(2\hat{a}\hat{\rho}_{22}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho}_{22} - \hat{\rho}_{22}\hat{a}^\dagger\hat{a}\right),
\end{aligned} \quad (11)$$

where $\{,\}$ denotes anti-commutator symbol and $\dot{\hat{\rho}}_{21}(t) = [\dot{\hat{\rho}}_{12}(t)]^\dagger$. Now following the method of reference [27], let us first define the following functions in the phase space associated with the field

$$\chi_{ij}(\beta, t) = \text{Tr}_f\left[\hat{\rho}_{ij}(t)\hat{D}(\beta)\right], \quad \hat{D}(\beta) = \exp\left[\beta\hat{a}^\dagger - \beta^*\hat{a}\right]. \quad (12)$$

In this representation, equations (11) take the following form

$$\begin{aligned}
\dot{\chi}_{11}(\beta, t) &= \frac{ig}{2}(\beta + \beta^*)\chi_{11}(\beta, t) \\
&\quad - \frac{k}{2}\left(\beta\frac{\partial}{\partial\beta} + \beta^*\frac{\partial}{\partial\beta^*} + |\beta|^2\right)\chi_{11}(\beta, t), \\
\dot{\chi}_{12}(\beta, t) &= -ig\left[\frac{\partial}{\partial\beta} - \frac{\partial}{\partial\beta^*}\right]\chi_{12}(\beta, t) \\
&\quad - \frac{k}{2}\left(\beta\frac{\partial}{\partial\beta} + \beta^*\frac{\partial}{\partial\beta^*} + |\beta|^2\right)\chi_{12}(\beta, t), \\
\dot{\chi}_{22}(\beta, t) &= -\frac{ig}{2}(\beta + \beta^*)\chi_{22}(\beta, t) \\
&\quad - \frac{k}{2}\left(\beta\frac{\partial}{\partial\beta} + \beta^*\frac{\partial}{\partial\beta^*} + |\beta|^2\right)\chi_{22}(\beta, t).
\end{aligned} \quad (13)$$

We now assume that at $t = 0$ the atom is described by the pure state $|\psi_a(0)\rangle = \cos\theta/2|+\rangle + \text{e}^{i\phi}\sin\theta/2|-\rangle$, with $0 \leq \theta \leq \pi$, $0 \leq \phi < 2\pi$, and the cavity field is in the the vacuum state $|\psi_f(0)\rangle = |0\rangle$. In the representation given by (12), the above initial state takes the following form

$$\begin{aligned}
\chi_{11}(\beta, 0) &= \cos^2\theta/2\,\exp\left(-|\beta|^2/2\right), \\
\chi_{12}(\beta, 0) &= \frac{1}{2}\text{e}^{-i\phi}\sin\theta\,\exp\left(-|\beta|^2/2\right), \\
\chi_{22}(\beta, 0) &= \sin^2\theta/2\,\exp\left(-|\beta|^2/2\right),
\end{aligned} \quad (14)$$

and $\chi_{21}(\beta, 0) = \chi_{12}^*(\beta, 0)$. Now under the above initial conditions, equations (13) can be solved by using the method of characteristics [30], and we get

$$\begin{aligned}
\chi_{11}(\beta, t) &= \cos^2\theta/2\,\exp\left(-\frac{|\beta|^2}{2} - \alpha^*(t)\beta + \alpha(t)\beta^*\right), \\
\chi_{12}(\beta, t) &= \frac{1}{2}\text{e}^{-i\phi}\sin\theta\, f(t) \\
&\quad\times\exp\left(-\frac{|\beta|^2}{2} + \alpha^*(t)\beta + \alpha(t)\beta^*\right), \\
\chi_{22}(\beta, t) &= \sin^2\theta/2\,\exp\left(-\frac{|\beta|^2}{2} + \alpha^*(t)\beta - \alpha(t)\beta^*\right),
\end{aligned} \quad (15)$$

and $\chi_{21}(\beta, t) = \chi_{12}^*(\beta, t)$. In the above equations we have defined the time dependent coherent field amplitude $\alpha(t)$ and the function $f(t)$ as

$$\begin{aligned}
\alpha(t) &= i\frac{g}{k}\left(1 - e^{-kt/2}\right), \\
f(t) &= \exp\left(-2\left(\frac{g}{k}\right)^2 kt + 4\left(\frac{g}{k}\right)^2\left(1 - e^{-kt/2}\right)\right).
\end{aligned} \quad (16)$$

From the above expressions we find

$$\begin{aligned}
\hat{\rho}_{11}(t) &= \cos^2\theta/2\,|-\alpha(t)\rangle\langle-\alpha(t)|, \\
\hat{\rho}_{12}(t) &= \frac{1}{2}\,\text{e}^{-i\phi}\sin\theta\, f(t)\text{e}^{2|\alpha(t)|^2}\,|-\alpha(t)\rangle\langle\alpha(t)|, \\
\hat{\rho}_{22}(t) &= \sin^2\theta/2\,|\alpha(t)\rangle\langle\alpha(t)|,
\end{aligned} \quad (17)$$

$$\hat{\rho}(t)=\begin{pmatrix} x^2(t)\cos^2\theta/2 & x(t)\sqrt{1-x^2(t)}\cos^2\theta/2 & \frac{1}{2}f(t)\sin\theta\,\mathrm{e}^{-i\phi} & 0\\ x(t)\sqrt{1-x^2(t)}\cos^2\theta/2 & (1-x^2(t))\cos^2\theta/2 & \frac{\sqrt{1-x^2(t)}}{2x(t)}f(t)\sin\theta\,\mathrm{e}^{-i\phi} & 0\\ \frac{1}{2}f(t)\sin\theta\,\mathrm{e}^{i\phi} & \frac{\sqrt{1-x^2(t)}}{2x(t)}f(t)\sin\theta\,\mathrm{e}^{i\phi} & \sin^2\theta/2 & 0\\ 0 & 0 & 0 & 0\end{pmatrix} \tag{22}$$

and $\hat{\rho}_{21}(t)=\hat{\rho}_{12}^{\dagger}(t)$. As a matter of fact, by choosing the initial state of the atom as $\theta=\pi/2$, $\phi=0$, the above density matrix reduces to the relation (31) of reference [25]. However our objective here is to study the effect of dissipation on the entanglement of the atom-field system and also the possibility of using this system as a quantum channel for efficient quantum teleportation.

3 Quantum entanglement

In what follows, we will study entanglement dynamics of the above state. To some extent, the dynamics of entanglement is the time evolution of entanglement measures. Many entanglement measures have been introduced and analyzed in the literature, but the one most relevant to this work is entanglement of formation, which in fact intends to quantify the resources needed to create a given entangled state [3]. Remarkably, Wootters [31] has shown that entanglement of formation of a two-qubit state $\hat{\rho}$ is related to a quantity called concurrence as

$$E(\hat{\rho})=\Xi[C(\hat{\rho})]=h\left(\frac{1}{2}+\frac{1}{2}\sqrt{1-C^2}\right), \tag{18}$$

where $h(x)=-x\log_2 x-(1-x)\log_2(1-x)$ is the binary entropy function and $C(\hat{\rho})$ is the concurrence of the state $\hat{\rho}$, defined by

$$C(\hat{\rho})=\max\left\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\right\}, \tag{19}$$

where the λ_i are the non-negative eigenvalues, in decreasing order, of the non-Hermitian matrix $\hat{\rho}\tilde{\hat{\rho}}$. Here $\tilde{\hat{\rho}}$ is the matrix given by $\tilde{\hat{\rho}}=(\sigma_y\otimes\sigma_y)\,\hat{\rho}^*\,(\sigma_y\otimes\sigma_y)$ where $\hat{\rho}^*$ is the complex conjugate of $\hat{\rho}$ when it is expressed in a standard basis such as $\{|11\rangle,|12\rangle,|21\rangle,|22\rangle\}$ and σ_y represents Pauli matrix in the local basis $\{|1\rangle,|2\rangle\}$. Furthermore, the function Ξ is a monotonically increasing function of the concurrence $C(\hat{\rho})$, and ranges from 0 to 1 as $C(\hat{\rho})$ goes from 0 to 1, so that one can take the concurrence as a measure of entanglement in its own right.

Equation (17) shows that the operators $\hat{\rho}_{ij}(t)$ act on a field subspace spanned by two vectors $|\alpha(t)\rangle$ and $|-\alpha(t)\rangle$. Now it is easy to see that the determinant obtained from the inner product of these two vectors is equal to $1-\mathrm{e}^{-4|\alpha(t)|^2}$ which is nonzero provided that $|\alpha(t)|\neq 0$. This means that two vectors $|\alpha(t)\rangle$ and $|-\alpha(t)\rangle$ are linearly independent provided that $t\neq 0$ and $g\neq 0$. Therefore the final density matrix $\hat{\rho}(t)$ is supported at most on $\mathbb{C}^2\otimes\mathbb{C}^2$ space, and thus, one can use the concurrence as a measure of entanglement between the atom and the field. Now in order to calculate the concurrence for the atom-field density matrix given in equation (17), we must first write the density matrix in an orthonormal product basis. To this aim, we use the Gram-Schmidt procedure [32] to construct two orthonormal vectors $|v_1\rangle$ and $|v_2\rangle$ as

$$|v_1\rangle=|\alpha(t)\rangle\,,\qquad |v_2\rangle=\frac{|-\alpha(t)\rangle-x(t)\,|\alpha(t)\rangle}{\sqrt{1-x^2(t)}}, \tag{20}$$

where

$$x(t)=\langle\alpha(t)|-\alpha(t)\rangle=\exp\left(-2\,|\alpha(t)|^2\right). \tag{21}$$

Two vectors $|v_1\rangle$ and $|v_2\rangle$ span, effectively, the space of the field and constitute the field qubit states. Therefore, in our model, the atom-field system constitute a two-qubit system. Now in the orthonormal basis $\{|+\rangle|v_1\rangle,|+\rangle|v_2\rangle,|-\rangle|v_1\rangle,|-\rangle|v_2\rangle\}$, the atom-field density matrix can be represented by

see equation (22) above.

Now we can use the concurrence as a measure of entanglement between the atom and the field. For the atom-field state defined in equation (22) we obtain

$$\begin{aligned}\lambda_1&=\frac{(1-x^2(t))}{4x^2(t)}(x(t)+f(t))^2\sin^2\theta,\\ \lambda_2&=\frac{(1-x^2(t))}{4x^2(t)}(x(t)-f(t))^2\sin^2\theta,\\ \lambda_3&=\lambda_4=0,\end{aligned} \tag{23}$$

and therefore the concurrence between the atom and the field is given by

$$C(t)=\max\left\{0,\frac{\sqrt{1-x^2(t)}}{x(t)}f(t)\sin\theta\right\}. \tag{24}$$

It is clear that for $\theta=0$ the concurrence is zero for all times, i.e. the atom described by the initial state $|+\rangle=\frac{1}{\sqrt{2}}(|g\rangle+|e\rangle)$ does not get entangled with the field. Indeed, in this case the final state of the system is described by the pure state $\rho(t)=|\psi(t)\rangle\langle\psi(t)|$, where $|\psi(t)\rangle=|+\rangle|-\alpha(t)\rangle$. This means that the initial state $|+\rangle|0\rangle$ of the system defines a decoherence-free subspace in which the time evolution of the system is unitary. Thus the state $|+\rangle|0\rangle$ does not become entangled with the environment. But in the absence of dissipation, i.e. $k=0$, the unitary evolution operator $\hat{U}=\exp\{-i\hat{H}_{\text{eff}}t/\hbar\}$ of the Hamiltonian (9) can be written as [33]

$$\begin{aligned}&\hat{U}(\xi(t))=|+\rangle\langle+|\hat{D}(-\xi(t))+|-\rangle\langle-|\hat{D}(\xi(t)),\\ &\xi(t)=igt/2\end{aligned} \tag{25}$$

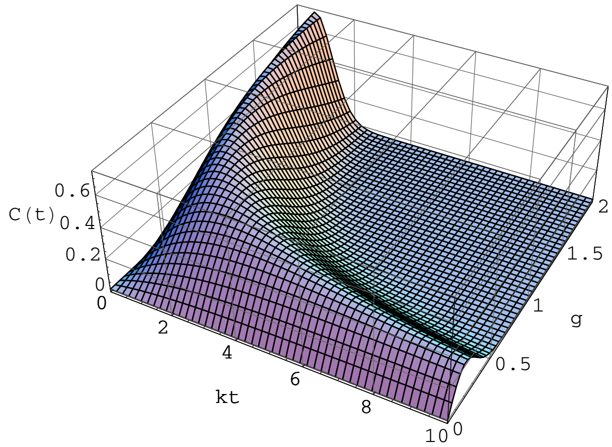

Fig. 1. (Color online) Concurrence $C(t)$ is plotted as a function of kt and coupling constant g with $\theta = \pi/2$.

where $\hat{D}$ is the displacement operator defined in equation (12). The evolution of the initial state $|+\rangle|0\rangle$ by the above unitary operator leads to the product state $|+\rangle|-\xi(t)\rangle$. Motivated by this we can say that in the decoherence-free subspace the unitary evolution of the system is governed by the unitary operator $\hat{U}(\alpha(t))$ where $\alpha(t)$ is the coherent field amplitude defined in equation (16). Obviously, for $k = 0$ we have $\alpha(t) = \xi(t)$.

In order to show the effect of dissipation rate k and coupling constant g on the entanglement of the system, we plot the concurrence as a function of kt and the coupling constant g in Figure 1. It shows that the entanglement of the system increases with increase of the coupling constant g, and decreases with increase of the dissipation rate k. Furthermore, the asymptotic long time density matrix is separable and has the following form

$$\hat{\rho}(\infty) = \cos^2\theta/2|+\rangle\langle+| \otimes |-\alpha(t)\rangle\langle-\alpha(t)| + \sin^2\theta/2|-\rangle\langle-| \otimes |\alpha(t)\rangle\langle\alpha(t)|. \quad (26)$$

Now in the following, we attempt to discuss nonlocality of the atom and the field. The most commonly discussed Bell inequality is the Clauser-Horne-Shimony-Holt (CHSH) inequality [34]. The Bell-CHSH operator formulated for two-qubit systems has the following form [34]

$$\mathcal{B} = \mathbf{a}\cdot\sigma \otimes (\mathbf{b}+\mathbf{b}')\cdot\sigma + \mathbf{a}'\cdot\sigma \otimes (\mathbf{b}-\mathbf{b}')\cdot\sigma, \quad (27)$$

where $\mathbf{a}, \mathbf{a}', \mathbf{b}, \mathbf{b}'$ are unite vectors in $\mathbb{R}^3$ and $\{\sigma_i\}_{i=1}^3$ are the standard Pauli matrices. The Bell-CHSH inequality states that within any local model the expectation value $\langle\mathcal{B}\rangle_{\hat{\rho}} \equiv \mathrm{Tr}(\hat{\rho}\mathcal{B})$ of the Bell-CHSH operator has to be bounded by 2, i.e.

$$|\langle\mathcal{B}\rangle_{\hat{\rho}}| \leq 2. \quad (28)$$

Horodecki et al. have presented an effective criterion for violating the Bell-CHSH inequality by an arbitrary mixed two-qubit state [35]. They have shown that the maximum amount of Bell violation of a two-qubit state $\hat{\rho}$, i.e. $\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}} = \max_{\mathcal{B}}|\langle\mathcal{B}\rangle_{\hat{\rho}}|$, is given by $2\sqrt{\mu+\tilde{\mu}}$ where $\mu, \tilde{\mu}$ are two greater eigenvalues of the matrix $T_{\hat{\rho}}^{\dagger}T_{\hat{\rho}}$. Here the matrix $T_{\hat{\rho}}$ is a 3×3 matrix whose elements are $[T_{\hat{\rho}}]_{ij} = \mathrm{Tr}(\hat{\rho}\sigma_i \otimes \sigma_j)$, and is responsible for correlations. It follows, therefore, from this maximal violation measure that a state shows Bell violation when $\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}} > 2$ and the maximal violation when $\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}} = 2\sqrt{2}$.

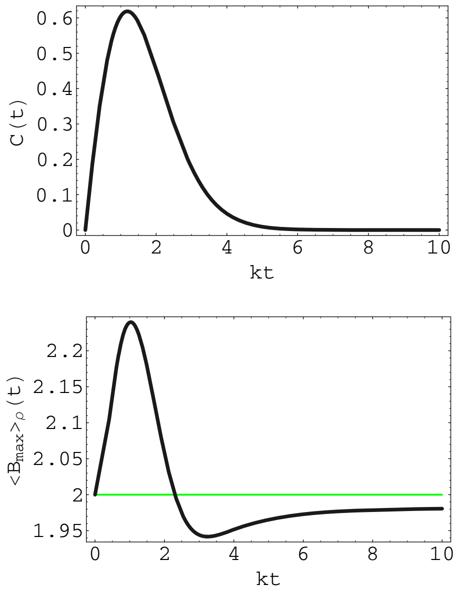

Fig. 2. (Color online) Concurrence $C(t)$ (upper panel) and the maximal violation measure $\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}}(t)$ (lower panel) are plotted as a function of kt with $\theta = \pi/2$ and $g = 1$. The horizontal line in the lower figure shows the minimum violation.

Now, it is not difficult to see that for the atom-field density operator $\hat{\rho}(t)$ given by equation (22), the maximal violation measure can be written as

$$\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}}(t) = 2\sqrt{1 + \left(\frac{f^2(t)}{x^2(t)} - x^2(t)\right)\sin^2\theta}. \quad (29)$$

In Figure 2 we plot the concurrence $C(t)$ (upper panel) and the maximal violation measure $\langle\mathcal{B}_{\max}\rangle_{\hat{\rho}}(t)$ (lower panel) as a function of kt with $\theta = \pi/2$ and $g = 1$. The horizontal line in the lower figure shows the boundary value 2 in equation (28), i.e. the minimum violation. There we can clearly see that there are entangled states for which the Bell-BCSH inequality is not violated. It is worth noting that although the atom-field entanglement disappears asymptotically, but the nonlocality defined by the Bell-CHSH inequality disappears at a finite time.

4 Decoherence

In quantum information processing, decoherence is another essential problem that deserves some attention. Generally, decoherence is used to estimate the deviation from an ideal state and can be considered as a symbol to express the reduction of purity and, therefore, one can use the linear entropy $S(\hat{\rho}) = 1 - \mathrm{Tr}[\hat{\rho}^2]$ as a measure of decoherence. The linear entropy has the limiting values 0 and $1 - 1/N$, respectively, for pure and maximally mixed states, where N is the dimension of the space that the density matrix $\hat{\rho}$ is supported on. The linear entropy of the atom-field system is given by

$$S(\hat{\rho}) = 1 - \mathrm{Tr}\,[\,\hat{\rho}^2\,] = \frac{1}{2}\left(1 - \frac{f^2(t)}{x^2(t)}\right)\sin^2\theta. \qquad (30)$$

On the other hand, the reduced density matrix of the atom can be obtained by tracing out over the field degrees of freedom where we get

$$\hat{\rho}_a(t) = \mathrm{Tr}_f[\,\hat{\rho}(t)\,] = \begin{pmatrix} \cos^2\theta/2 & \frac{1}{2}\,\mathrm{e}^{-i\phi}\sin\theta\, f(t) \\ \frac{1}{2}\,\mathrm{e}^{i\phi}\sin\theta\, f(t) & \sin^2\theta/2 \end{pmatrix}. \qquad (31)$$

The linear entropy of the atom is given by

$$S(\hat{\rho}_a) = 1 - \mathrm{Tr}\,[\,\hat{\rho}_a^2\,] = \frac{1}{2}\,(1 - f^2(t))\sin^2\theta. \qquad (32)$$

Similarly, we can obtain the reduced density matrix of the field by tracing out over the atom degrees of freedom and get

$$\hat{\rho}_f(t) = \mathrm{Tr}_a[\,\hat{\rho}(t)\,] = \begin{pmatrix} x^2(t)\cos^2\theta/2 + \sin^2\theta/2 & x(t)\sqrt{1-x^2(t)}\cos^2\theta/2 \\ x(t)\sqrt{1-x^2(t)}\cos^2\theta/2 & (1-x^2(t))\cos^2\theta/2 \end{pmatrix}, \qquad (33)$$

where, clearly, shows that the field reduced density matrix does not depend on the decoherence function $f(t)$. This matrix can be used to calculate the linear entropy of the field as

$$S(\hat{\rho}_f) = 1 - \mathrm{Tr}\,[\,\hat{\rho}_f^2\,] = \frac{1}{2}\,(1 - x^2(t))\sin^2\theta. \qquad (34)$$

It is clear that all of the three linear entropies obtained above are proportional to $\sin\theta$, and therefore when $\theta = 0$ the atom-field system and its corresponding subsystems have zero entropies. As we mentioned already this is because of the fact that in this particular case the time evolution of the system is unitary and the state remains separable as well as pure. In Figure 3 we plot the linear entropy of the atom-field system and its two reduced subsystems. It is clear that all three linear entropy have asymptotic values near 1/2. Although this asymptotic value for the atom subsystem is the maximum value that the atom can gain, i.e. 1/2, but for the field subsystem it is equal to $(1 - \mathrm{e}^{-4})/2$. Peixoto de Faria and Nemes [11] have employed the Jaynes-Cummings model in the dispersive approximation for a dissipative cavity at zero temperature and showed that the cavity has practically no influence on the coherence properties of the field from the qualitatively point of view, but the atom's coherence properties are strongly influenced by dissipation both qualitatively and quantitatively, although it is not directly coupled to the cavity. But our results show that the coherence properties of the field are also affected by the cavity.

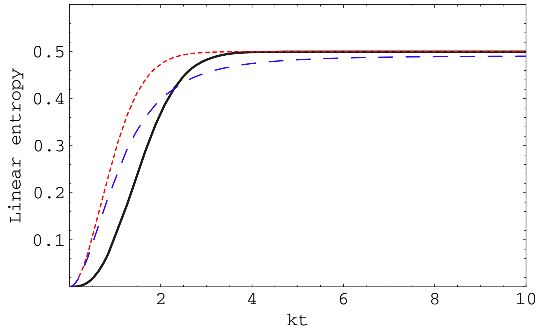

Fig. 3. (Color online) Linear entropy of atom-field (solid line), atom (dotted line) and field (dashed line) are plotted as a function of kt with $\theta = \pi/2$ and $g = 1$.

5 Teleportation

An important aspect of quantum nonseparability is the quantum teleportation, discovered by Bennett et al. [2]. Bennett et al have shown that two spin-$\frac{1}{2}$ particles, separated in space and entangled in a singlet state, can be used for teleportation. Popescu [36] noticed that the pairs in a mixed state could still be useful for (imperfect) teleportation, but they reduce the fidelity of teleportation. It has been shown [36–38] that the purely classical channel can give at most fidelity $F = \frac{2}{3}$. The possibility of using the entanglement between the atom and the field as a resource for the standard teleportation protocol $\mathcal{P}_0$ is considered in the next subsection.

5-1 One qubit teleportation

The standard teleportation $\mathcal{P}_0$ [2] involves two particle sources producing pairs in a given mixed state $\hat{\rho}_{\mathrm{ch}}$ which forms the quantum channel. This quantum channel is equivalent to a generalized depolarizing channel $\Lambda^{\hat{\rho}_{\mathrm{ch}},\mathcal{P}_0}$, with probabilities given by the maximally entangled components of the resources [39,40]. Now we look at the standard protocol $\mathcal{P}_0$, using the atom-field state $\hat{\rho}(t)$, i.e. a two-qubit mixed state, as resource. We consider as an input state a one-qubit system in an unknown pure state $|\psi_{\mathrm{in}}\rangle = \cos\vartheta/2|+\rangle + e^{i\varphi}\sin\vartheta/2|-\rangle$ with $0 \le \vartheta \le \pi$, $0 \le \varphi < 2\pi$. The density matrix related to $|\psi_{\mathrm{in}}\rangle$ is in the form

$$\hat{\rho}_{\mathrm{in}} = \begin{pmatrix} \cos^2\vartheta/2 & \frac{1}{2}e^{-i\varphi}\sin\vartheta \\ \frac{1}{2}e^{i\varphi}\sin\vartheta & \sin^2\vartheta/2 \end{pmatrix}. \qquad (35)$$

The output state $\hat{\rho}_{\mathrm{out}}$ can be obtained by applying a joint measurement and local unitary transformation on the input state $\hat{\rho}_{\mathrm{in}}$ [39]

$$\hat{\rho}_{\mathrm{out}} = \Lambda^{\hat{\rho}_{\mathrm{ch}},\mathcal{P}_0}(\hat{\rho}_{\mathrm{in}}) = \sum_{i=0}^{3} p_i \sigma^i \hat{\rho}_{\mathrm{in}} \sigma^i, \qquad (36)$$

where $p_i = \mathrm{Tr}(E^i\hat{\rho}_{\mathrm{ch}})$ such that $\sum_i p_i = 1$. Here $E^i = |\Psi^i_{\mathrm{Bell}}\rangle\langle\Psi^i_{\mathrm{Bell}}|$ where $|\Psi^i_{\mathrm{Bell}}\rangle$ are the four maximally entangled Bell states associated with the Pauli

$$\hat{\rho}_{\rm out} = \begin{pmatrix} (p_0+p_3)\cos^2\vartheta/2 + 2p_1\sin^2\vartheta/2 & \frac{1}{2}(p_0-p_3)\,e^{-i\varphi}\sin\vartheta \\ \frac{1}{2}(p_0-p_3)\,e^{i\varphi}\sin\vartheta & (p_0+p_3)\sin^2\vartheta/2 + 2p_1\cos^2\vartheta/2 \end{pmatrix}$$

matrices σ^i, i.e. $E^i = (\sigma^i \otimes \sigma^0)E^0(\sigma^i \otimes \sigma^0)$, where $\sigma^0 = I$, $\sigma^1 = \sigma_x$, $\sigma^2 = \sigma_y$ and $\sigma^3 = \sigma_z$. Furthermore for optimal utilization of a given entangled state as resource, one must choose local basis states such that p_0 is maximum, i.e. $p_0 = \max\{p_i\}$. We therefore find that $|\Psi^0_{\rm Bell}\rangle = \frac{1}{\sqrt{2}}(|+\rangle|v_2\rangle + |-\rangle|v_1\rangle)$, $|\Psi^1_{\rm Bell}\rangle = \frac{1}{\sqrt{2}}(|+\rangle|v_1\rangle + |-\rangle|v_2\rangle)$, $|\Psi^2_{\rm Bell}\rangle = \frac{1}{\sqrt{2}}(|+\rangle|v_1\rangle - |-\rangle|v_2\rangle)$, $|\Psi^3_{\rm Bell}\rangle = \frac{1}{\sqrt{2}}(|+\rangle|v_2\rangle - |-\rangle|v_1\rangle)$, and

$$\begin{aligned} p_0 &= \frac{1}{2}\left(1 - x^2(t)\cos^2\theta/2 + \frac{f(t)\sqrt{1-x^2(t)}}{x(t)}\sin\theta\cos\phi\right), \\ p_1 &= p_2 = \frac{1}{2}x^2(t)\cos^2\theta/2, \\ p_3 &= \frac{1}{2}\left(1 - x^2(t)\cos^2\theta/2 - \frac{f(t)\sqrt{1-x^2(t)}}{x(t)}\sin\theta\cos\phi\right). \end{aligned} \tag{37}$$

Therefore according to equation (36), for the output we get

see equation above.

To characterize the quality of the teleported state $\hat{\rho}_{\rm out}$, it is often quite useful to look at the fidelity between $\hat{\rho}_{\rm in}$ and $\hat{\rho}_{\rm out}$ defined as $F(\hat{\rho}_{\rm in},\hat{\rho}_{\rm out}) = \left[{\rm Tr}\left(\sqrt{\sqrt{\hat{\rho}_{\rm in}}\hat{\rho}_{\rm out}\sqrt{\hat{\rho}_{\rm in}}}\right)\right]^2$, [41,42]. For our system that the input state is pure, the fidelity $F(\hat{\rho}_{\rm in},\hat{\rho}_{\rm out})$ can be easily calculated as

$$F(\hat{\rho}_{\rm in},\hat{\rho}_{\rm out}) = \langle\psi_{\rm in}|\hat{\rho}_{\rm out}|\psi_{in}\rangle = (p_0+p_3) + (p_1-p_3)\sin^2\vartheta. \tag{38}$$

The average fidelity is another useful concept for characterizing the quality of teleportation and can be obtained by averaging the fidelity $F(\hat{\rho}_{in},\hat{\rho}_{out})$ over all possible input states

$$\overline{F}(\Lambda^{\hat{\rho}_{\rm ch},\mathcal{P}_0}) = \frac{1}{4\pi}\int_0^{2\pi}{\rm d}\varphi\int_0^{\pi}F(\hat{\rho}_{\rm in},\hat{\rho}_{\rm out})\sin\vartheta{\rm d}\vartheta. \tag{39}$$

For our system we get

$$\overline{F}(\Lambda^{\hat{\rho}_{\rm ch},\mathcal{P}_0}) = \frac{2}{3} + \frac{1}{3}\left(\frac{f(t)\sqrt{1-x^2(t)}}{x(t)}\sin\theta\cos\phi - x^2(t)\cos^2\theta/2\right). \tag{40}$$

We, therefore, see that the atom-field entangled state $\hat{\rho}(t)$ can be useful to transmit $|\psi_{\rm in}\rangle$ with fidelity better than any classical communication protocol, i.e. fidelity better that $2/3$, if we require that the second term in the above equation be strictly positive.

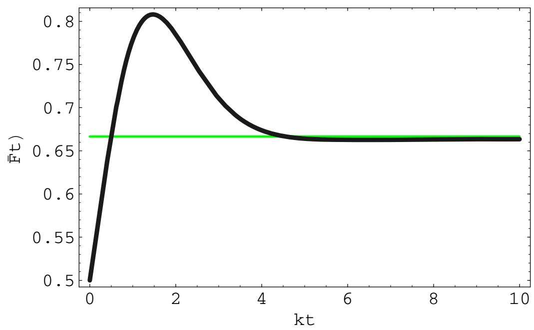

Fig. 4. (Color online) Optimal fidelity is plotted as a function of kt with $\theta = \pi/2$, $\phi = 0$ and $g = 1$. The horizontal line shows the classical capacity $2/3$.

Horodecki et al. have presented a beautiful formula relating the optimal fidelity of teleportation and the maximal entangled fraction [43]. They have shown that for a given bipartite state acting on $\mathbb{C}^d\otimes\mathbb{C}^d$, the optimal fidelity of teleportation is given by

$$F_{\max}(\Lambda^{\hat{\rho}_{\rm ch},\mathcal{P}_0}) = \frac{f_{\max}(\Lambda^{\hat{\rho}_{\rm ch}})\,d+1}{d+1}, \tag{41}$$

where $f_{\max}(\Lambda^{\hat{\rho}_{\rm ch}})$ is the maximal entangled fraction of the channel. Simple calculation shows that in our model, i.e. $d = 2$ and $f_{\max}(\Lambda^{\hat{\rho}_{\rm ch}}) = \max\{p_0,p_1,p_2,p_3\} = p_0$, equation (41) gives the same result as equation (40). In Figure 4 we plot the optimal fidelity of teleportation as a function of kt with $\theta = \pi/2$, $\phi = 0$ and $g = 1$. The horizontal line shows the best classical fidelity $2/3$.

5-2 Entanglement teleportation

We now consider the atom-field state as a quantum channel for entanglement teleportation of a two-qubit state. We will consider Lee and Kim's [44] two-qubit teleportation protocol, and use two copies of the above atom-field state as resource. In this protocol, the joint measurement is decomposable into two independent Bell measurements and the unitary operation into local one-qubit Pauli rotations. Accordingly, for the output state we get

$$\hat{\rho}_{\rm out} = \Lambda^{\hat{\rho}_{\rm ch},\mathcal{P}_1}(\hat{\rho}_{\rm in}) = \sum_{i,j=0}^{3}p_{ij}(\sigma_i\otimes\sigma_j)\hat{\rho}_{\rm in}(\sigma_i\otimes\sigma_j), \tag{42}$$

where $p_{ij} = {\rm Tr}(E^i\hat{\rho}_{\rm ch}){\rm Tr}(E^j\hat{\rho}_{\rm ch}) = p_ip_j$. Here E^i are projection on the Bell states, defined in the last section. We consider as input a two-qubit state in the following pure state $|\psi_{\rm in}\rangle = \cos\vartheta/2|+-\rangle + e^{i\varphi}\sin\vartheta/2|-+\rangle$ with $0 \le \vartheta \le \pi$, $0 \le \varphi < 2\pi$. The density matrix related to

$$\hat{\rho}_{\rm out} = \begin{pmatrix} 2p_1(p_0+p_3) & 0 & 0 & 0 \\ 0 & (p_0+p_3)^2 a + 4p_1^2 b & (p_0-p_3)^2 c & 0 \\ 0 & (p_0-p_3)^2 c^* & (p_0+p_3)^2 b + 4p_1^2 a & 0 \\ 0 & 0 & 0 & 2p_1(p_0+p_3) \end{pmatrix}$$

$|\psi_{in}\rangle$ is in the form

$$\hat{\rho}_{\rm in} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a & c & 0 \\ 0 & c^* & b & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

where we have defined $a = \cos^2\vartheta/2$, $b = \sin^2\vartheta/2$ and $c = \frac{1}{2}e^{-i\varphi}\sin\vartheta$. The concurrence of this state is $C(\hat{\rho}_{\rm in}) = \sin\vartheta$. For the output we get

see equation above.

Now in order to calculate the concurrence of $\hat{\rho}_{\rm out}$, we first calculate the eigenvalues of the operator $\hat{\rho}_{\rm out}\tilde{\hat{\rho}}_{\rm out}$ as

$$\begin{aligned} \lambda_1 &= \left((p_0+p_3)^2 a + 4p_1^2 b\right)\left((p_0+p_3)^2 b + 4p_1^2 a\right) \\ &\quad + (p_0-p_3)^4 |c|^2 + 2(p_0-p_3)^2 |c| \\ &\quad \times \sqrt{\left((p_0+p_3)^2 a + 4p_1^2 b\right)\left((p_0+p_3)^2 b + 4p_1^2 a\right)}, \\ \lambda_2 &= \lambda_3 = 4(p_0+p_3)^2 p_1^2, \\ \lambda_4 &= \left((p_0+p_3)^2 a + 4p_1^2 b\right)\left((p_0+p_3)^2 b + 4p_1^2 a\right) \\ &\quad + (p_0-p_3)^4 |c|^2 - 2(p_0-p_3)^2 |c| \\ &\quad \times \sqrt{\left((p_0+p_3)^2 a + 4p_1^2 b\right)\left((p_0+p_3)^2 b + 4p_1^2 a\right)}. \end{aligned} \quad (43)$$

Then by using equation (19), we obtain the concurrence of the teleported state $\hat{\rho}_{\rm out}$ as

$$\begin{aligned} C(\hat{\rho}_{\rm out}) &= \max\left\{0, (p_0-p_3)^2 \sin\vartheta - 4(p_0+p_3)p_1\right\} \\ &= \max\left\{0, \frac{(1-x^2)f^2}{x^2}\sin^2\theta\cos^2\phi\ \sin\vartheta \right. \\ &\quad \left. -2x^2\left(1-x^2\cos^2\theta/2\right)\cos^2\theta/2\right\}. \end{aligned} \quad (44)$$

In Figure 5 we plot the concurrence of the output state as a function of kt and θ. It is clear from the figure that teleportation of a maximally Bell state (concurrence 1) via this channel, give an output state with concurrence less than 0.1. We can also calculate the fidelity of $\hat{\rho}_{\rm in}$ and $\hat{\rho}_{\rm out}$ as

$$F(\hat{\rho}_{\rm in}, \hat{\rho}_{\rm out}) = (p_0+p_3)^2 + 2(p_1^2 - p_0 p_3)\sin^2\vartheta. \quad (45)$$

Now using equation (41) with $d = 4$ and $f_{\max}(\Lambda^{\hat{\rho}_{\rm ch}}) = p_0^2$, the optimal teleportation fidelity achievable is given by

$$\begin{aligned} \overline{F}(\Lambda^{\hat{\rho}_{\rm ch}, \mathcal{P}_1}) &= \frac{2}{5} + \frac{1}{5}\left(\frac{f^2(1-x^2)}{x^2}\sin^2\theta\cos^2\phi + x^4\cos^4\theta/2 \right. \\ &\quad \left. + 2(1-x^2\cos^2\theta/2)\frac{f\sqrt{1-x^2}}{x}\sin\theta\cos\phi - 2x^2\cos^2\theta/2\right). \end{aligned} \quad (46)$$

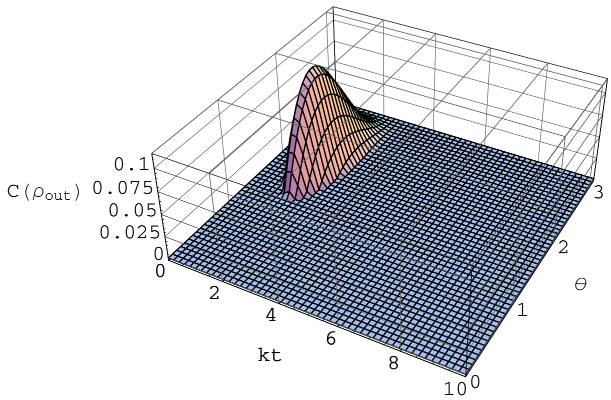

Fig. 5. (Color online) Concurrence of the output state is plotted as a function of kt and θ, with $g = 1$, $\phi = 0$ and $\vartheta = \pi/2$.

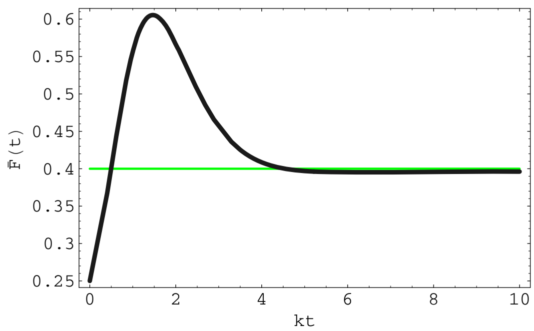

Fig. 6. (Color online) Optimal fidelity is plotted as a function of kt with $\theta = \pi/2$, $\phi = 0$ and $g = 1$. The horizontal line shows the classical fidelity 2/5.

In Figure 6 we plot optimal fidelity of teleportation as a function of kt with $\theta = \pi/2$, $\phi = 0$ and $g = 1$. The horizontal line shows the classical fidelity 2/5.

6 Conclusion

We have investigated the quantum entanglement and decoherence in the interacting system of a two-level atom and a single mode vacuum field in the presence of field dissipation. Starting from the cavity field in a vacuum state and the atom in a general pure state, it is shown that the final density matrix has support on $\mathbb{C}^2 \otimes \mathbb{C}^2$ space, i.e. the atom-field system constitute a two-qubit system. We have, therefore, used the concurrence as a relevant measure of entanglement between the atom and the field. The effect of the atomic initial pure state on the entanglement of the system is studied and it is shown that when the atom is initially in the state $|+\rangle = \frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$, the atom-field entanglement is zero for all times. In this case we have shown that the evolution of the system is unitary and therefore, the system initial state $|+\rangle|0\rangle$ defines a decoherence-free subspace. The influence of the cavity decay on the quantum entanglement of the system has also been discussed and we have found that the dissipation suppresses the entanglement. We have also examined the Bell-CHSH violation between the atom and the field and have shown that there are entangled states for which the Bell-BCSH inequality is not violated. The decoherence induced by the cavity decay is also studied and it is shown that the coherence properties of the atom and also the field are affected by cavity decay. The one-qubit teleportation via the quantum channel constructed by the atom-field system is also investigated. We have shown that the atom-field entangled state can be useful to transmit a generic one-qubit state $|\psi_{in}\rangle$ with fidelity better than any classical communication protocol, i.e. fidelity better that 2/3. We have also studied the two-qubit entanglement teleportation via two copies of the atom-field system. The fidelity of the teleportation and also the entanglement of the replica are also discussed and it is shown that the atom-field entangled state is still superior to classical channel in performing the two-qubit teleportation.

References

1. C.H. Bennett, S.J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992)
2. C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, W.K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993)
3. C.H. Bennett, D.P. Divincenzo, J.A. Smolin, W.K. Wootters, Phys. Rev. A **54**, 3824 (1996)
4. R.F. Werner, Phys. Rev. A **40**, 4277 (1989)
5. J. Kofler, C. Brukner, Phys. Rev. Lett. **99**, 180403 (2007)
6. E. Joos, H.D. Zeh, C. Kiefer, D. Giulini, J. Kupsch, I.O. Stamatescu, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer-Verlag, 2003)
7. W. Dur, H.J. Briegel, Phys. Rev. Lett. **92**, 180403 (2004)
8. A.R.R. Carvalho, F. Minter, A. Buchleitner, Phys. Rev. Lett. **93**, 230501 (2004)
9. C. Simon, J. Kempe, Phys. Rev. A **65**, 052327 (2002)
10. E.T. Jaynes, F.W. Cummings, Proc. IEEE **51**, 89 (1963)
11. J.G. Peixoto de Faria, M.C. Nemes, Phys. Rev. A **59**, 3918 (1999)
12. L. Zhou, H.S. Song, Y.X. Luo, C. Li, Phys. Lett. A **284**, 156 (2001)
13. L. Sanz, K. Furuya, J. Opt. B: Quantum Semiclass. Opt. **4**, S184 (2002)
14. R.W. Rendel, Rajagopal, Phys. Rev. A **67**, 062110 (2003)
15. V.V. Dodonov, W.D. José, S.S. Mizrahi, J. Opt. B: Quantum Semiclass. Opt. **5**, S567 (2003)
16. G.-X. Li, K. Allaart, D. Lenstra, Phys. Rev. A **69**, 055802 (2004)
17. R. Tanas, Z. Ficek, J. Opt. B: Quantum Semiclass. Opt. **6**, S90 (2004)
18. M.A. Can, Ö. Çakir, A. Klyachko, A. Shumovsky, J. Opt. B: Quantum Semiclass. Opt. **6**, S13 (2004)
19. J. Gea-Banacloche, T.C. Burt, P.R. Rice, L.A. Orozco, Phys. Rev. Lett. **94**, 053603 (2005)
20. M. Janowicz, A. Orlowski, J. Phys. B: At. Mol. Opt. Phys. **39**, 1763 (2006)
21. M.H. Naderi, M. Soltanolkotabi, Eur. Phys. J. D **39**, 471 (2006)
22. L. Li, J. Zou, J.-G. Li, H.-P. Cui, B. Shao, J. Phys. B: At. Mol. Opt. Phys. **41**, 085507 (2008)
23. E. Solano, G.S. Agarwal, H. Walther, Phys. Rev. Lett. **90**, 027903 (2003)
24. F. Casagrande, A. Lulli, Open Syst. Inf. Dyn. **13**, 437 (2006)
25. P. Lougovski, F. Casagrande, A. Lulli, E. Solano, Phys. Rev. A **76**, 033802 (2007)
26. F. Casagrande, A. Lulli, Eur. Phys. J. D **46**, 165 (2008)
27. M. Bina, F. Casagrannde, A. Lulli, E. Solano, Phys. Rev. A **77**, 033839 (2008)
28. M. Bina, F. Casagrande, A. Lulli, Eur. Phys. J. D **49**, 257 (2008)
29. J.-S. Zhang, J.-B. Xu, Opt. Commun. **282**, 2543 (2009)
30. S.M. Barnett, P.M. Radmore, *Methods in Theoretical Quantum Optics* (Clarendon Press, Oxford, 1997)
31. W.K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998)
32. M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, 2000)
33. P. Lougovski, F. Casagrande, A. Lulli, B.-G. Englert, E. Solano, H. Walther, Phys. Rev. A **69**, 023812 (2004)
34. J.F. Clauser, M.A. Horne, A. Shimony, R.A. Holt, Phys. Rev. Lett. **23**, 880 (1969)
35. R. Horodecki, P. Horodecki, M. Horodecki, Phys. Lett. A **200**, 340 (1995)
36. S. Popescu, Phys. Rev. Lett. **72**, 797 (1994)
37. S. Massar, S. Popescu, Phys. Rev. Lett. **74**, 1259 (1995)
38. N. Gisin, Phys. Lett. A **210**, 157 (1996)
39. G. Bowen, S. Bose, Phys. Rev. Lett. **87**, 265901 (2001)
40. S. Albeverio, S.M. Fei, W.L. Yang, Phys. Rev. A **66**, 012301 (2002)
41. A. Uhlmann, Rep. Math. Phys. **9**, 273 (1976)
42. R. Jozsa, J. Mod. Opt. **41**, 2315 (1994)
43. M. Horodecki, P. Horodecki, R. Horodecki, Phys. Rev. A **60**, 1888 (1999)
44. J. Lee, M.S. Kim, Phys. Rev. Lett. **84**, 4236 (2000)