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ORIGINAL PAPER

Head-on collision of electron-acoustic solitary waves in a plasma with superthermal hot electrons

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Abstract: The head-on collision between two electron-acoustic solitary waves in an unmagnetized plasma is investigated, including a cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions. By using extended Poincare′-Lighthill-Kuo perturbation method, the analytical phase shifts following head-on collision are derived. Effects of the ratio of number density of hot electrons to number density of cold electrons α and superthermal parameter k on phase shifts are studied. It is found that hot-to-cold electron density ratio significantly modifies the phase shifts.

Keywords: Nonlinear phenomena; Solitons; Physics

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1. Introduction

Plasma physics has a rich variety of nonlinear structure and modes. For instance, electron-acoustic waves (EAWs) are high frequency electrostatic modes (in comparison with the ion plasma frequency). EAWs exist in plasmas consisting of positive ions and two distinct electron species, one cool and one hot, at high frequencies, but below the plasma frequency, such that the ion dynamics plays no essential role [1, 2]. In such plasmas, cooler of the two electron components provides inertial effects needed to sustain wave, as ions do in usual description of ion-acoustic waves. Also its propagation has been invoked to explain emissions at frequencies between the ion and electron plasma frequencies in cusp of terrestrial magnetosphere, earth bow shock and the heliospheric termination shock [3–8]. Solitary electron acoustic structure has been studied by many authors [9, 10]. It has been found that the electron and ion distributions play crucial role in characterizing physics of nonlinear wave structures [11–15]. Studies of linear and nonlinear electron-acoustic waves in plasmas with nonthermal electrons have received a great deal of interest in recent years [16–18]. Negative potential solitary structures are shown to exist in a two-electron plasma, either for Maxwellian [16] or for nonthermal [17, 18] hot electrons. A linear analysis of electron-acoustic waves has been first carried out by assuming an unmagnetized Maxwellian homogeneous plasma, which exhibited a heavily damped acoustic-like solution in addition to Langmuir waves and ion-acoustic waves [19]. Those early results have been later extended to take into account the effect of excess suprathermal particles [20, 21], whose presence, in fact, results in an increase in the Landau damping at small wavenumbers, in particular when the hot electron component is dominant [20].

A recent investigation [22] has established the properties of modulated electron-acoustic wavepackets in kappa-distributed plasmas and has studied the effect of suprathermality on amplitude (modulational) stability. In a one-dimensional system, there are two distinct soliton interactions. One is the overtaking collision and the other is head-on collision [23]. The overtaking collision of solitary waves can be studied by inverse scattering transformation method [24]. For a head-on collision between two solitary waves, we must search for evolution of solitary waves propagating in opposite directions and hence we need to employ a suitable asymptotic expansion to solve the original fluid dynamics

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equations. What is most interesting us to is the phase shifts and the trajectories of the two solitary waves after a collision. Several authors [23–28] have investigated the head-on collision of two solitary waves in many plasma models using the extended Poincaré–Lighthill–Kuo (PLK) method. For example, Su and Mirie [23] have found analytically that the waves emerging from collision of two solitary waves preserve their original identities to third order of accuracy. Again, the collision generates secondary wave group (the dispersive tail) trailing behind their primary solitary waves. Huang and Velarde [25] have investigated the head-on collision of two concentric cylindrical ion-acoustic solitary waves and obtained the phase shifts of right and left going waves. Xue [26] has studied the head-on collision between two dust-ion acoustic solitary waves in a magnetized dusty plasma. It has been found that magnitude and obliqueness of the external magnetic field significantly modifies the phase shifts. Han et al. [27] have investigated the head-on collision of two ion-acoustic solitary waves in a weakly relativistic electron–positron-ion plasma. They have demonstrated that due to collision, each solitary wave has a negative phase shift in its traveling direction. Also, Han et al. [28] have studied the head-on collision of two ion acoustic solitary waves in an unmagnetized electron positron-ion plasma and illustrated that the ratio of electron temperature to positron temperature and the ratio of number density of positrons to that of electrons have a significant influence on the phase shifts of solitons. Eslami et al. [29] have investigeated the head-on collision of ion-acoustic solitary waves in plasma consisting of cold ions, nonextensive electrons and thermal electrons. It has been demonstrated that due to collision, the ratios of electron temperature to positron temperature, the ratio of the number density of positrons to that of electrons and the electron nonextensive distribution (q-parameter) have a significant influence on the phase shifts of solitons. Huang et al. [25, 30–33] have used PLK method to investigate the soliton interactions in optical fiber, to ion-acoustic waves and to ultracold atomic gases. In this paper, we have studied nonlinear dynamics of electron-acoustic waves in a plasma consisting of cold adiabatic electron and superthermal distributed hot electrons, in addition to stationary ions.

2. Basic equations and interaction between two EASWs

We consider a homogeneous, unmagnetized plasma consisting of a cold electron fluid, superthermal hot electrons and stationary ions. Nonlinear dynamics of the electron-acoustic waves (EAWs) in such a plasma system is governed by:

$$\frac{\partial n_c}{\partial t} + \frac{\partial (n_c u_c)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial u_c}{\partial t} + u_c \frac{\partial u_c}{\partial x} - \alpha \frac{\partial \varphi}{\partial x} = 0 \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x^2} - \frac{1}{\alpha} n_c - n_h + \left(1 + \frac{1}{\alpha}\right) = 0 \tag{3}$$

where the cold electron number density (n_c), the cold electron fluid velocity (u_c) and the electrostatic potential (φ) are normalized, respectively, by ion equilibrium density n_{c0}, ion fluid speed $C_e = \sqrt{k_B T_h / \alpha m_e}$ and the quantity $k_B T_h / e$, in which k_B is the Boltzmann's constant, e and m_e are electron charge and its mass, respectively. The space and time variables are normalized to the hot electron Debye radius $\lambda_{Dh} = \sqrt{T_h / 4\pi n_{h0} e^2}$ and the cold electron plasma frequency $\omega_{pc}^{-1} = \sqrt{m_e / 4\pi n_{c0} e^2}$. We have set $\alpha = n_{h0}/n_{c0}$, where n_{h0} and n_{c0} are the unperturbed number density of the hot electrons and cold electrons, respectively. To model the effects of superthermal electrons, we have the normalized number density of electron as

$$n_h = \left(1 - \frac{\varphi}{k - 1/2}\right)^{-k-\frac{1}{2}} \tag{4}$$

where k is a real parameter measuring deviation from Maxwellian equilibrium (recovered for k infinite), the normalization has been provided for any value of the $k > 1/2$ [34, 35]. We have assumed two solitons A and B in the plasma, which are, asymptotically, far apart in initial state and travel toward each other. After some time they interact, collide and then depart. We also assume that solitons have small amplitudes ε (where ε is a smallness formal perturbation parameter) and interactions between solitons are weak. Hence, we expect that the collision to be quasielastic, so it causes only shift of the postcollision trajectories (phase shift). In order to analyze the effects of collision, we have employed an extended *PLK* perturbation method [25, 36, 37]. This extended *PLK* method is a combination of the standard reductive perturbation method [38, 39] with the technique of strained coordinates. The main idea of this perturbation method is as follows. In the limit of the long wavelength approximation, asymptotic expansions for both the flow field variables and the spatial or time coordinates are used. This makes a uniformly valid asymptotic expansion (i.e., eliminates secular terms) and at the same time obtains the change of the trajectories (i.e., phase shifts) of the solitary waves after the collision. According to this method, we introduce the stretched coordinates

$$\begin{aligned}
\xi &= \varepsilon(r - ct) + \varepsilon^2 P_0(\eta, \tau) + \varepsilon^3 P_1(\eta, \xi, \tau) + \ldots \\
\eta &= \varepsilon(r + ct) + \varepsilon^2 Q_0(\xi, \tau) + \varepsilon^3 Q_1(\xi, \eta, \tau) + \ldots \\
\tau &= \varepsilon^3 t
\end{aligned} \tag{5}$$

Introducing the asymptotic expansion

$$\begin{aligned} n_c &= 1 + \varepsilon n^1 + \varepsilon^2 n^2 + \varepsilon^3 n^3 + \ldots \\ u_c &= \varepsilon u^1 + \varepsilon^2 u^2 + \varepsilon^3 u^3 + \ldots \\ \varphi &= \varepsilon \varphi^1 + \varepsilon^2 \varphi^2 + \varepsilon^3 \varphi^3 + \ldots \end{aligned} \tag{6}$$

where ξ and η denote the trajectories of the two solitary waves traveling to the right and left directions, respectively. The variable c is the unknown phase velocity of EAWs (to be determined later). The parameters $P_0(\eta,\tau)$ and $Q_0(\xi,\tau)$ are also to be calculated. Then, substituting Eq. (5) into Eqs. (1)–(3) and by equating the powers of ε, we obtain a hierarchy of coupled equations in different orders of ε. To the leading order, we have

$$-c\frac{\partial n^1}{\partial \xi} + c\frac{\partial n^1}{\partial \eta} + \frac{\partial u^1}{\partial \xi} + \frac{\partial u^1}{\partial \eta} = 0 \tag{7}$$

$$-c\frac{\partial u^1}{\partial \xi} + c\frac{\partial u^1}{\partial \eta} + \frac{\partial \varphi^1}{\partial \xi} + \frac{\partial \varphi^1}{\partial \eta} = 0 \tag{8}$$

and

$$n^1 = -\alpha\left[\frac{2k+1}{2k-1}\right]\varphi^1 \tag{9}$$

The solutions of Eqs. (7)–(9) read as

$$\varphi^1 = \varphi_1^1(\xi,\tau) + \varphi_2^1(\eta,\tau) \tag{10}$$

$$n^1 = -\alpha\left[\frac{2k+1}{2k-1}\right]\left[\varphi_1^1(\xi,\tau) + \varphi_2^1(\eta,\tau)\right] \tag{11}$$

$$u^1 = -\frac{\alpha}{\lambda}\varphi_1^1(\xi,\tau) + \frac{\alpha}{\lambda}\varphi_2^1(\eta,\tau) \tag{12}$$

And with the solvability condition [i.e., condition to obtain a uniquely defined n^1 and u^1 from Eqs. (7)–(9) when φ^1 is given by Eq. (10)], the phase velocity $c = \left[\frac{2k-1}{2k+1}\right]^{1/2}$ is also obtained.

The unknown functions φ_1^1 and φ_2^1 are determined from the next orders. Eqs. (10)–(12) imply that, at the leading order, we have two waves, one of which, $\varphi_1^2(\xi,\tau)$, is traveling to right and the other one $\varphi_2^2(\eta,\tau)$, is traveling to left. At next order, solutions also have the following shapes:

$$\varphi^2 = \varphi_1^2(\xi,\tau) + \varphi_2^2(\eta,\tau) \tag{13}$$

$$n^2 = -\alpha\left[\frac{2k+1}{2k-1}\right]\left[\varphi_1^2(\xi,\tau) + \varphi_2^2(\eta,\tau)\right] \tag{14}$$

$$u^2 = -\frac{\alpha}{\lambda}\varphi_1^2(\xi,\tau) + \frac{\alpha}{\lambda}\varphi_2^2(\eta,\tau) \tag{15}$$

To the next higher order, we can deduce

$$\begin{aligned} 2\left[\frac{2k-1}{2\mathrm{k}+1}\right]u^3 = &\int\left(\frac{\partial \varphi_1^1}{\partial \tau} - A\varphi_1^1\frac{\partial \varphi_1^1}{\partial \xi} + B\frac{\partial^3 \varphi_1^1}{\partial \xi^3}\right)d\eta \\ &+ \int\left(\frac{\partial \varphi_2^1}{\partial \tau} + A\varphi_1^1\frac{\partial \varphi_2^1}{\partial \eta} - B\frac{\partial^3 \varphi_2^1}{\partial \eta^3}\right)d\xi \\ &+ \iint\left(C\frac{\partial P_0}{\partial \eta} - D\varphi_1^2\right)\frac{\partial^2 \varphi_1^1}{\partial \xi^2}d\xi d\eta \\ &- \iint\left(C\frac{\partial Q_0}{\partial \xi} - D\varphi_1^1\right)\frac{\partial^2 \varphi_2^1}{\partial \eta^2}d\xi d\eta \end{aligned} \tag{16}$$

where

$$A = +\frac{1}{2\lambda^{3/2}}\left[3\alpha\lambda^2 + \frac{(2k+1)(2k+3)}{(2k-1)^2}\right]$$

$$B = \frac{1}{2\lambda^{3/2}}$$

$$C = \frac{2}{\lambda^{1/2}}$$

$$D = \frac{1}{2\lambda^{3/2}}\left[-\alpha\lambda^2 + \frac{(2k+1)(2k+3)}{(2k-1)^2}\right]$$

and

$$\lambda = \frac{(2k+1)}{(2k-1)}$$

The first (second) term in the right hand side of Eq. (16) is proportional to $\eta(\xi)$, because the integrated function is independent of $\eta(\xi)$. Thus, the first two terms of Eq. (16) are all secular terms, which must be eliminated in order to avoid spurious resonances. Hence, we have

$$\frac{\partial \varphi_1^1}{\partial \tau} - A\varphi_1^1\frac{\partial \varphi_1^1}{\partial \xi} + B\frac{\partial^3 \varphi_1^1}{\partial \xi^3} = 0 \tag{17}$$

$$\frac{\partial \varphi_2^1}{\partial \tau} + A\varphi_1^1\frac{\partial \varphi_2^1}{\partial \eta} - B\frac{\partial^3 \varphi_2^1}{\partial \eta^3} = 0 \tag{18}$$

The third and forth terms of Eq. (16) are not secular terms in this order, but they become secular in the next order. Hence we have [37, 40]

$$C\frac{\partial P_0}{\partial \eta} = D\varphi_2^1 \tag{19}$$

$$C\frac{\partial Q_0}{\partial \xi} = D\varphi_1^1 \tag{20}$$

Equations (19) and (20) are the two side traveling wave *KdV* equations in reference frames of ξ and η, respectively. Their corresponding solutions are

$$\varphi_1^1 = \varphi_A \sec h^2\left[\left(\frac{A\varphi_A}{12B}\right)^{1/2}\left(\xi - \frac{1}{3}A\varphi_A\tau\right)\right] \tag{21}$$

$$\varphi_2^1 = \varphi_B \sec h^2\left[\left(\frac{A\varphi_B}{12B}\right)^{1/2}\left(\eta + \frac{1}{3}A\varphi_B\tau\right)\right] \tag{22}$$

where φ_A and φ_B are the amplitudes of two solitons A and B in their initial positions. The leading phase changes due to the collision can be calculated from Eqs. (19) and (20) and are given by

$$P_0(\eta,\tau) = \frac{D}{C}\left(\frac{12B\varphi_B}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\varphi_B}{12B}\right)^{1/2}\left(\eta + \frac{1}{3}A\varphi_B\tau\right) + 1\right] \tag{23}$$

$$Q_0(\xi,\tau) = -\frac{D}{C}\left(\frac{12B\varphi_A}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\varphi_A}{12B}\right)^{1/2}\left(\xi - \frac{1}{3}A\varphi_A\tau\right) - 1\right] \tag{24}$$

Therefore, up to $O(\varepsilon^2)$, trajectories of the two solitary waves for weak head-on interactions are

$$\xi = \varepsilon(x - ct) - \varepsilon^2\frac{D}{C}\left(\frac{12B\varphi_B}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\varphi_B}{12B}\right)^{1/2}\left(\eta + \frac{1}{3}A\varphi_B\tau\right) + 1\right] + \cdots \tag{25}$$

$$\eta = \varepsilon(x + ct) + \varepsilon^2\frac{D}{C}\left(\frac{12B\varphi_A}{A}\right)^{1/2} \times \left[\tanh\left(\frac{A\varphi_A}{12B}\right)^{1/2}\left(\xi - \frac{1}{3}A\varphi_A\tau\right) - 1\right] + \cdots \tag{26}$$

To obtain phase shifts after a head-on collision of the two solitons, we have assumed that solitons A and B are, asymptotically, far from each other at the initial time ($t = -\infty$), i.e., soliton A is at $\xi = 0$, $\eta = -\infty$ and soliton B is at $\eta = 0$, $\xi = +\infty$, respectively. After the collision ($t = +\infty$), soliton A is far to the right of soliton B, i.e., soliton A is at $\xi = 0$, $\eta = +\infty$ and soliton B is at $\eta = 0$, $\xi = -\infty$. Using Eqs. (25) and (26), we obtain the corresponding phase shift ΔP_0 and ΔQ_0 as follows [41, 42]:

$$\Delta P_0 = -2\varepsilon^2\frac{D}{C}\left(\frac{12B\varphi_B}{A}\right)^{1/2} \tag{27}$$

$$\Delta Q_0 = 2\varepsilon^2\frac{D}{C}\left(\frac{12B\varphi_A}{A}\right)^{1/2} \tag{28}$$

3. Results and discussion

The present study describes propagation and interaction of two small but finite amplitude EASWs in an unmagnetized plasma composed of cold electrons, superthermal hot electrons and stationary ions using extended *PLK* perturbation method. The effects of the superthermal parameter k and hot-to-cold electron density ratio α on the phase shift and amplitudes φ_A and φ_B are studied. In our results $\varepsilon = 0.1$ and $\varphi_A = \varphi_B$=1 are used and all physical quantities are dimensionless.

Figure 1 indicates that the phase shift ΔQ_0 decreases with k for constant values of α. For a given value of α, the phase shift ΔQ_0decreases steeply with k, when $k < 2$ and changes smoothly with k when $k > 2$. It is seen that for a given value of k, the phase shift ΔQ_0decreases with an increase in value of α. Clearly an increase in the superthermal parameter leads to an increase in velocity of solitary pulses after interactions. The results obtained so far indicate that hot-to-cold electron density ratio α play a significant role in soliton collision. It is found from Fig. 1 that the sign of collision phase shifts is also determined by values of k and α. It is important to mention that the phase shift is positive or negative depending on coefficient D, because both coefficients C and A in Eq. (28) are positive. Thus, the phase shift (ΔQ_0) is positive when $D > 0$, i.e. $\alpha < (2k + 3)/(2k + 1)$, and also it appears with negative sign when $D < 0$, i.e. $\alpha > (2k + 3)/(2k + 1)$. A positive phase shift implies that post-collision parts of the soliton move ahead of the initial trajectory, whereas, a negative phase shift indicates that post-collision parts of the soliton lag behind the initial trajectory [43]. Similar effect has

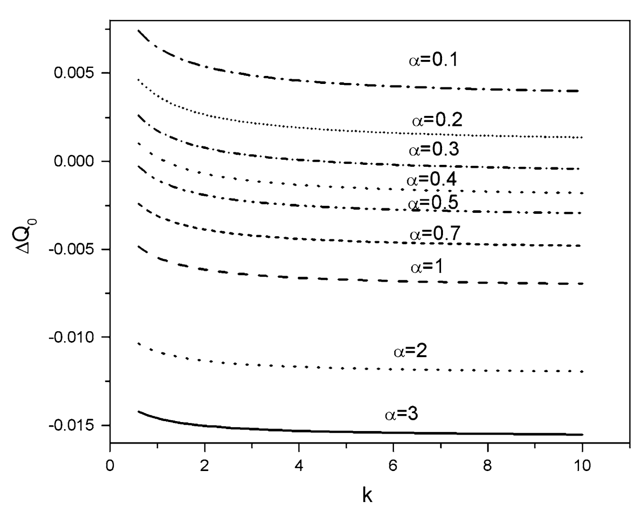

Fig. 1 Variation of phase shift ΔQ_0 with k for different values of α, where $\varepsilon = 0.1$ and $\varphi_A = \varphi_B = 1$

been also observed in study of the ion acoustic [44] and dust acoustic [26, 45, 46] solitary waves.

Figure 2 depicts variation in the phase shift with hot-to-cold electron density ratio, α, for several values of super-thermal parameter k in plasma. This figure shows that the phase shift ΔQ_0 decreases as k increases. The figure also shows that for a given value of the parameter k, when $\alpha \to 0$, the phase shift decreases quickly with α and phase shift decreases smoothly for larger values of α. In Fig. 3, we have plotted special solutions φ_1^1 and φ_2^1 for several values of τ for the time evolution of head-on collision, where $k = 1$, $\alpha = 0.5$. It is seen that the solitary wave solution φ_1^1 is shifted towards the right with onset of time, whereas solution φ_2^1 is shifted towards left with the onset of time.

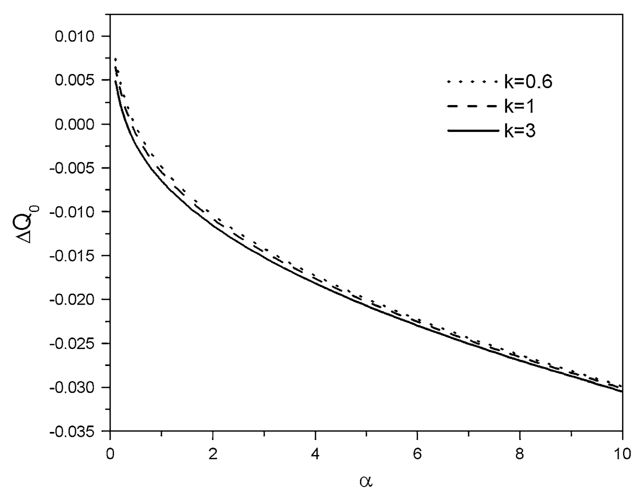

Fig. 2 Variation of phase shift ΔQ_0 with α for different values of k, $k = 0.6$, 1 and 3, where $\varepsilon = 0.1$ and $\varphi_A = \varphi_B = 1$

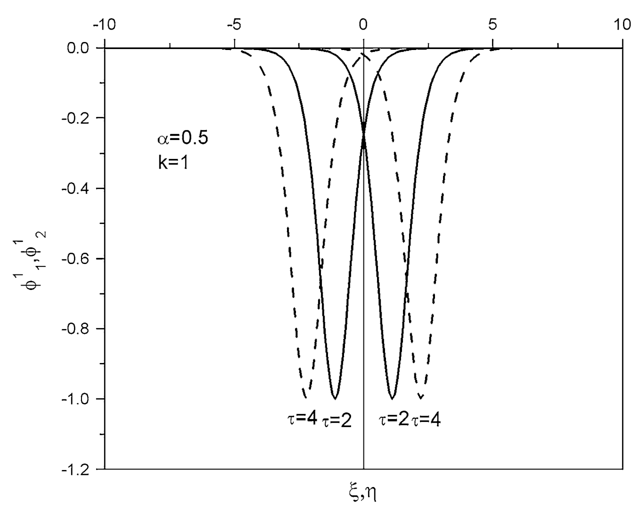

Fig. 3 Plot of φ_1^1 (*dashed line*) and φ_2^1(*solid line*) versus ξ and η, for several values of τ, where $k = 1$, $\alpha = 0.5$, $\varepsilon = 0.1$ and $\varphi_A = \varphi_B = 1$

4. Conclusions

By using extended *PLK* perturbation method, the leading-order analytical phase shifts of head-on collisions between two electron acoustic solitary waves in an unmagnetized plasma are derived. Effects of the plasma parameters k and α on the phase shift are studied. It is shown that hot-to-cold electron density ratio has a significant effect on the phase shift. For a given value of α, the phase shift ΔQ_0 decreases steeply with k when $k < 2$ and changes smoothly with k when $k > 2$. It is seen that for a given value of k, the phase shift ΔQ_0 decreases with an increase in the value of α. The results of this investigation are useful in understanding the collective phenomena related to EASWs collisions that are of vital importance in laboratory and space plasma.

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