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EFFECTS OF WEIGHING IN LM-INVERSE METHOD FOR OPTIMIZING 2D MODELING IN CASCADE TURBINE BLADES

Edris Yousefi Rad[1], Mohammad Reza Mahpeykar[2], Alireza Teymourtash[3]
[1]PhD Student in Mechanical Engineering, Ferdowsi University of Mashhad, Iran
edris.yoosefirad@gmail.com
[2]Professor in Mechanical Engineering, Ferdowsi University of Mashhad, Iran
mahpeymr@um.ac.ir
[3]Associate Professor in Mechanical Engineering, Ferdowsi University of Mashhad, Iran
teymourtash@um.ac.ir

Abstract. With the advancements of numerical upstream method in modeling flows in different paths including the flow inside turbine blades, employing the numerical CUSP technique in Jameson's finite volume method can simultaneously benefit from the positive features of both mentioned methods. The novelty of this paper is to improve Jameson's method in modeling a 2D supersonic flow between the blades of a steam turbine using the CUSP method, and defining the most optimum control function mode using Levenberg-Marquardt (LM) inverse method by effect of Weighing of experimental data and by accounting for the mass conservation equation. By considering the importance of the shock regions in the blade's surface suction side, the focus of the mentioned method is in this part which results in the significant improvement of the pressure ratio in Jameson's finite volume method. The results of the combined method (Jameson, CUSP and LM) at the shock region of the blade's suction surface desirably agree with the experimental data, and a decrease of numerical errors at this region is resulted. Furthermore, the results of the combined method shows that in comparison, by average, the conservation of mass condition is improved 16% at the shock region of the blade's suction surface.

Keywords: *Stationary turbine blade, Jameson's time marching, upstream method, CUSP method, Levenberg-Marquardt (LM) inverse method*

1. Introduction

Considering the importance of low pressure turbines in steam generators, the better design of these devices can result in higher efficiency [1]. For modeling these flows, finite volume numerical methods are commonly used. Reaching an exact numerical method that is able to capture the shock and flow discontinuities and has the least dissipation and oscillations is one of the most important challenges of Computational Fluid Dynamics modeling [2].

In numerical solutions, first, the differential equations are discreted using different methods and the resulting expansions are calculated by proper approximation of errors using numerical programs. In central difference schemes, a suitable approximation of the sentences eliminated during discretion should be added to the equations to prevent unwanted oscillations which are called artificial dissipation. When the shock phenomenon happens, it is appropriate to use second and higher degrees of accuracy for the solution region [3].

From the 1980s, vast efforts were made on flow upstream schemes that were categorized into flux vector splitting methods and flux vector difference methods which were developed for solving the Euler equations based on wave propagation. The common point among these methods lies within relationship between the direction of data propagation and direction of differential equation discretion or in other words, the discretion direction of the differential equations and receiving information concordant with the behavior of the inviscid flow [4,5].

In the presented work, the 2D CUSP (Convective Upstream Split Pressure) method which includes the upstream and finite difference methods is investigated which shows considerable improvement when employed in the Jameson's method for stator blades of a dry steam turbine. It is to be noted that modeling flow in this region (the middle section of the blade suction surface towards the end of the blade) is of great importance due to the existence of aerodynamic shock and condensation shock in two phase steam [6], but in this research the flow has been investigated as a single phase.

The novelty of this research is improvement of the Jameson's finite volume method by combining the CUSP method [7] and LM (Levenberg-Marquardt) method [8,9] for improving the solution and also the optimization of the control function mode by accounting for the mass conservation equation and effect of Weighing for two experimental data [10]. Using a simple calculation grid [8] for the modeling of single phase flows, although this proposed novel method can be used for complex two-phase flows. Considering the complexity of two-phase flows and the consequent high amount of calculations, employment of the standard grid is still suggested [11,12] which in this research, the steam flow is investigated at single phase, but the ultimate goal is developing the proposed model for two-phase steam flow and entropy theory in future studies.It is acknowledged that other finite volume methods could be used.

2. Jameson's Scheme

Jameson and his associates [2] presented a four-step method to enhance the efficiency of approaches based on finite volume and time marching. The proposed method was equivalent to central dismissal in space, and for integration with respect to time, they used to completely independent Rang-Kuta multi-step approach. To discrete location and time individually, the above mentioned method is very flexible, and the results obtained using this method are independent of the size of time steps. For the desirable capture the shocks, in this method, they added a mixture of category two and four waste terms to fluxsentences. Meanwhile, it is necessary to recall that in the early methods of Jameson, three techniques were introduced to accelerateconvergence, local time stepping, enthalpy damping, and implicit Residual averaging. The following are the mass and energy conservationequations for aninviscid and compressible steam in two-dimensional cartesian coordinates [4]:

$$\frac{\partial w}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0 \quad (1)$$

$$w = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho e_0 \end{bmatrix} F_x = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho u h_0 \end{bmatrix} F_y = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho v h_0 \end{bmatrix} \tag{2}$$

$$e_0 = e + \frac{V^2}{2} \tag{3}$$

In the upper equation, vector w includes survival variables;vectors $\mathbf{F}_x$, $\mathbf{F}_y$ indicate an inviscid flux and e_0 denotes the total energy.
The attributes for every finite volume are around the corners denoted using subscripts i and j. Given that the equations are two-dimensional, we integrate on the surface element of ω. This method leads to the following equations in the cartesian system:

$$\iint_{cell} \frac{\partial w}{\partial t} d\omega + \iint_{cell} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \right) = 0 \tag{4}$$

The above-mentioned vector equation shows the continuity and momentum equations in the direction of X and Y, the energy, and Ω denotes the constant area of each cell. The first term indicates the amount of change in the flow attributes with respect to time for every finite volume, and the second term represents the net rate of attributes that pass the flux from the faces of the finite volume.After integrating we can express the survival equation as follows:

$$\left(\frac{\partial w}{\partial t} \right)_{ij} = R_{ij}(w) \tag{5}$$

$R_{ij}(w)$indicates the residuals. It should be stipulated that the changes calculated for w are relative to the complete computational cell, while the flow variables need to be saved at control points. Hence, we assign the resulting changes to the corners of the cell. This is a fairly easy procedure and involves dividing the residuals equally between the cell`s points.
Hence, we obtain the following:

$$R_A(w) = 0.25\left[R_{ij} + R_{i-1,j} + R_{i,j-1} + R_{i-1,j-1}\right] \tag{6}$$

The resulting design is symmetric and logically reinforced, and the resulting separation equation for point A is as follows:

$$\left(\frac{\partial w}{\partial t} \right)_{A} = R_A(w) - D_A(w) \tag{7}$$

As can be seen, the above equation is a differential and can be solved using the existing methods. In this method for improving performance and computational speed, the waste term of $D_A(W)$ is calculated only in the first step of the Rang-Kuta multi-step methods; in the rest of the steps, we used the same value as that obtained in step one.

3. CUSP's Scheme

This method is based on the separation of the pressure terms in flow flux equations [13]. This method attempts to reduce the complexity and the time required for the computation as well as reaching an acceptable result. The flux term is divided into the flux displacement and the pressure terms, and a two-dimensional state flux vector is formed in the two directions of X and Y as follows[8]:

$$F = F_x.S_y + F_y.S_x \tag{8}$$

$$F_x = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho uv \\ \rho uH \end{bmatrix} F_y = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + P \\ \rho vH \end{bmatrix} \tag{9}$$

In the above equations, S_x and S_y are the surface vectors in directions X and Y, respectively. The first variable of flow (w) in the two-dimensional state is as follows:

$$w = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho \varepsilon \end{bmatrix} \tag{10}$$

The first step is to calculate the primary flow change vector on the two sides of (i,j) by using the switch function of $L(u,v)$.

$$L(u,v) = \frac{1}{2}\left(1 - \left|\frac{u-v}{|u|+|v|}\right|^z\right).(u+v) \tag{11}$$

The value of the power z ranges between values of 2 and 3[8].

$$w_R = w_{i+1,j} - \frac{1}{2}L(\Delta w_{i+3/2,j}, \Delta w_{i-1/2,j,}) \tag{12}$$

$\Delta w$is calculated as follows:

$$\Delta w_{i+1/2,j=} w_{i+1,j} - w_{i,j} \tag{13}$$

Upon dividing the flow flux terms, we obtain the following:

$$F_x = u.w + F_{px}$$
$$F_y - v.w + F_{py} \tag{14}$$

Having the displacement flux of q as given below, the terms of the flow flux pressure will be as follows:

$$q = u.S_y + v.S_x \tag{15}$$

$$F = F_x.S_y + F_y.S_x = qw + F_{px}.S_y + F_{py}.S_x \tag{16}$$

$$F_{px} = \begin{bmatrix} 0 \\ p \\ 0 \\ u.p \end{bmatrix} F_{px} = \begin{bmatrix} 0 \\ 0 \\ p \\ v.P \end{bmatrix} \tag{17}$$

The local Mach numbers are defined as $M = \frac{q}{c.S}$ and $\lambda^{\pm} = q \pm c.S$[8]. Therefore, factors $\alpha_x$and $\beta_x$can be expressed as follows:

$$\alpha_x = |M| \tag{18}$$

$$\beta_x = \begin{cases} +max\left(0, \frac{q+\lambda^-}{q-\lambda^-}\right) & 0 \le M \le 1 \\ -max\left(0, \frac{q+\lambda^-}{q-\lambda^-}\right) & -1 \le M \le 0 \\ \quad \text{sign}(M)|M \ge 1| & \end{cases} \tag{19}$$

In this state, we should correct the factorα_x, as in the one-dimensional state, by using the following:

$$\alpha_x = \frac{1}{2}\left(\alpha_0 + \frac{|M|^2}{\alpha_0}\right) \tag{20}$$

In this equation, the value of α_0 is very small ($0.0001 \approx$)[8]. To obtain more accurate results, we should apply the artificial dissipation terms more near the shock waves and less near the rest of the area. For achieving this, we insert a switch function of $L(u,v)$that has the capability of identifying the flow into the calculations.

In this research, for the first time for the purpose of improving the two-dimensional numerical method of Jameson for dry vapor, we use the above mentioned method for developing the code. The best value of the z is calculated using the inverse method given the mass conservation equation.

4. The LM's Method

In an inverse problem, the error is $\vec{e}$; the differences between the measured outputs of the process, $\vec{T}^m$, and the calculated output of the model on the spot, $\vec{T}^c$, which can be described as follows[14]:

$$\vec{e} = \vec{T}^m - \vec{T}^c \tag{21}$$

Vectors $\vec{T}^m$ and $\vec{T}^c$ are actually two elements because only the results obtained under a steady-state condition and at two experimental data calculated domain have been considered. These vectors have been reduced to scalar values (the experimentally measured pressure and the pressure calculated at a specified node). The goal is to compare these pressures to obtain a steady-state condition for the conservation of mass, which includes minor differences in the mass flow rate as compared to a simulated mass flow. However, the amount of pressure could be approximately determined by solving the analogous blades [9].

To minimize the above-mentioned error, the target function is defined in different ways. One of the common ways is to use the squared error method. The objective of solving an inverse problem is to minimize the sum of the squares:

$$S(\vec{P}) = \left(\vec{T}^m - \vec{T}^c\right)^T W\left(\vec{T}^m - \vec{T}^c\right) + \nu^k\left(\vec{P} - \vec{P}^k\right)^T \Omega^k\left(\vec{P} - \vec{P}^k\right) \text{ or}$$
$$X^T W\left(\vec{T}^m - \vec{T}^c\right) = \left(X^k.W.X + \nu^k \Omega^k\right).\Delta P \tag{22}$$

S is a function of $\vec{P}$. In this problem, the dimension of vertex $\vec{P}$ is $2 \times I_{max}$ and each of its elements represents the share of Jameson's method in a way that subsequently CUSP's share will be specified as well. In this study, the weight exchange has been conducted with two data in the aim position of calculation($0.7 < X/X\text{chord} < 0.95$)[9].The total of the squared errors could change every effect of the errors by a weight exchange W.One of the most important methods was presented by Levenberg [15], which is also called the least depreciated squares method. This method has a statistical basis. The calculation of the sensitivity matrix is one of the great difficulties in solving nonlinear inverse problems. The component of row "i" and column "j" for a comprehensive problem according to the explanation is defined using the following relation:

$$X_{ij} = \frac{\partial T_i^c}{\partial P_j} \tag{23}$$

In this problem, $I$is equal to 2 and $J = I_{max}$. Because of the measured and calculated parameters on a node that is considered and the number of undefined parameters, it observed that the Jameson's and subsequently CUSP's shares depend on the decrease in the mass flow rate.

Here, $\Omega^k$is a diagonal matrix that reduces the change along the desired path, thereby restricting the deviation, and when the related diagonal terms are bigger than the diagonal terms $\boldsymbol{X}^k.\boldsymbol{W}.\boldsymbol{X}$, causes a reduction in vibration and instability. Levenberg's method showed that if the adjustment coefficient is large at the beginning, S will decrease rapidly, and hence, this coefficient must be reduced because the answer of $\vec{P}^k$ could be incorrect otherwise. If this is so, the gradients can be used for finding the real answer as follows:

$$\Omega^k = diag\left[\left(\boldsymbol{X}^k\right)^T.\boldsymbol{X}^k\right] \tag{24}$$

Here, tonsorial summation has been used. This selection can cause the result of the method to be fixed under a linear transition. In this condition an appropriate coefficient is obtained by using the following equation:

$$\nu^k = \frac{\left(\vec{T}^m - \vec{T}^c\right)^T WX\Omega_m^k.W\left(\vec{T}^m - \vec{T}^c\right)}{S^k} \tag{25}$$

Marquardt's method is similar to Levenberg's. In this method, to change the adjustment coefficient, equation (25) is used along with the following relation:

$$\nu^k = \frac{\upsilon_0}{(a)^k} \tag{26}$$

Here υ_0 is the appropriate fixed number, and α is any number greater than value of 1[9].In general, this method is more useful in contexts related to inverse problems. Therefore, LM's method along with the revised functions as the follows:

$$P^{k+1} = P^k + \left[\left(\boldsymbol{X}^k\right)^T.\boldsymbol{X}^k + \nu^k\Omega^k\right]^{-1}\left(\boldsymbol{X}^k\right)\left[\vec{T}^m - \vec{T}^c\left(P^k\right)\right] \tag{27}$$

Where ν^k، is a scalar value and is called the depreciation parameter, and Ω^k is a diagonal matrix.

$\nu^k\Omega^k$ is introduced into a repetitive equation to depreciate the vibrations and instabilities caused by the problem criticality. The depreciative parameter at the beginning of the simulation when the unknown parameter with the initial guess is introduced is usually large; hence, because of the latter, we no longer need to study the singularity of the term$\left(\boldsymbol{X}^k\right)^T.\boldsymbol{X}^k$.The following relation is used for calculating this component [16]:

$$\boldsymbol{X}_{ij} = \frac{T_i^c\left(P_j(1+\varepsilon)\right) - T_i^c(P_j)}{\varepsilon P_j} \tag{28}$$

Thus, an equation for correcting $\vec{P}$and the criteria for ending the inverse solving obtained.

5. Combined Method

In this method, the domain that contains 12×115 standard meshes was used [8]. First, the flow is calculated with the finite volume using Jameson's method for the initial guess for the solution domain. After 20 iterations, the calculated results combine with CUSP's losses equations, and the solution continued. Each of them has a sensor for investigating the results for obtaining the convergence separately. To obtain better results, the calculated experimental data for pressure on one node of the solving domain as the $\vec{T}^m$ method is introduced to the inverse solving method, and the sensitivity matrix is calculated for obtaining the minor differences in the flow rates. As stated earlier in the case of a lack of experimental results for the selected blade, it is possible to estimate the required pressure using the results of the theoretical solution or the experimental results of the analogous blades [17].

6. Results and Discussion

The Domain of the cascade blades have 12×115 cells.The comparison results of the improved Jameson's finite volume method by combining with CUSP's method and LM-inverse technique is illustrated in Figures 2-5.

In Figures 1 the variations of the ratio of the static pressure to the initial stagnation pressure along the blade is shown on the suction surface respectively. As can be seen from these Figures, in the goal region ($0.7 < X/X_{Chord} < 0.95$) which is the sensitive and important region of the aerodynamic shock on the suction surface and where the combined method is also focused upon, the theoretical results are in agreeable accordance with the experimental results [10] when compared to the initial Jameson's method (without CUSP).

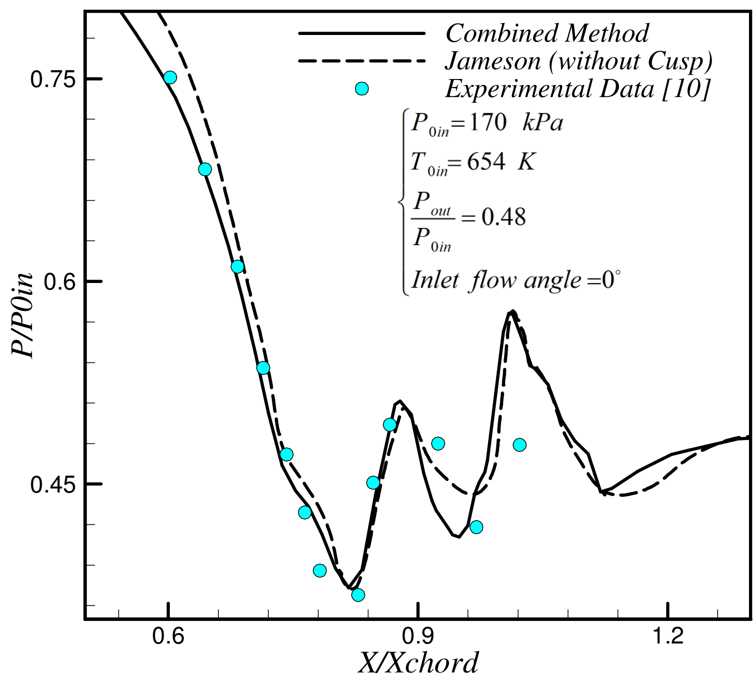

Figure 1. *Variations of the pressure ratio along the blade on the suction surface (SS)*

In this research the flow is adiabatic and inviscid therefore the stagnation pressure should remain constant except in the shock regions. In Figure 2 the ratio of the stagnation pressure differences to the initial stagnation pressure is illustrated.As can be seen the combined method has less errors.

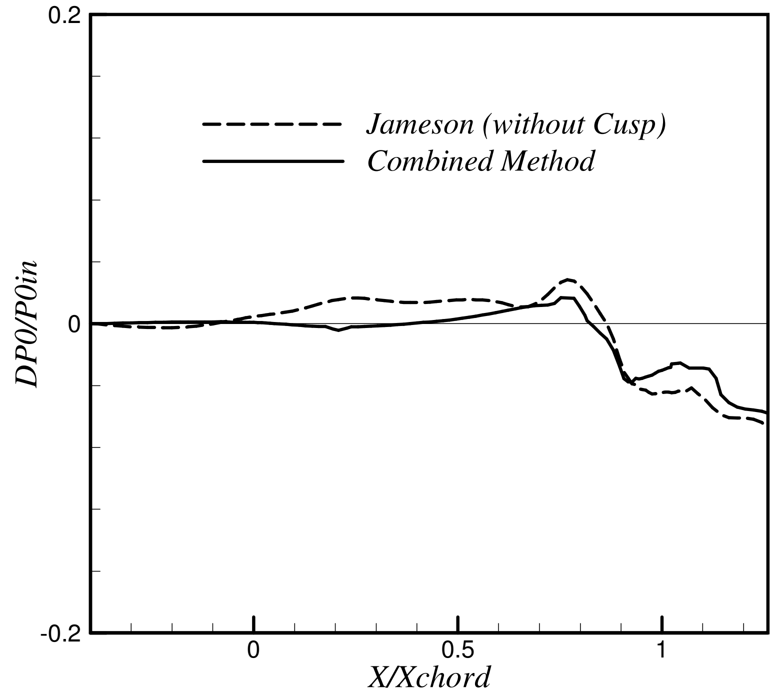

Figure 2. *. Percentage variation of the ratio of the stagnation pressure differences to the inlet stagnation pressure on the flow mid-passage*

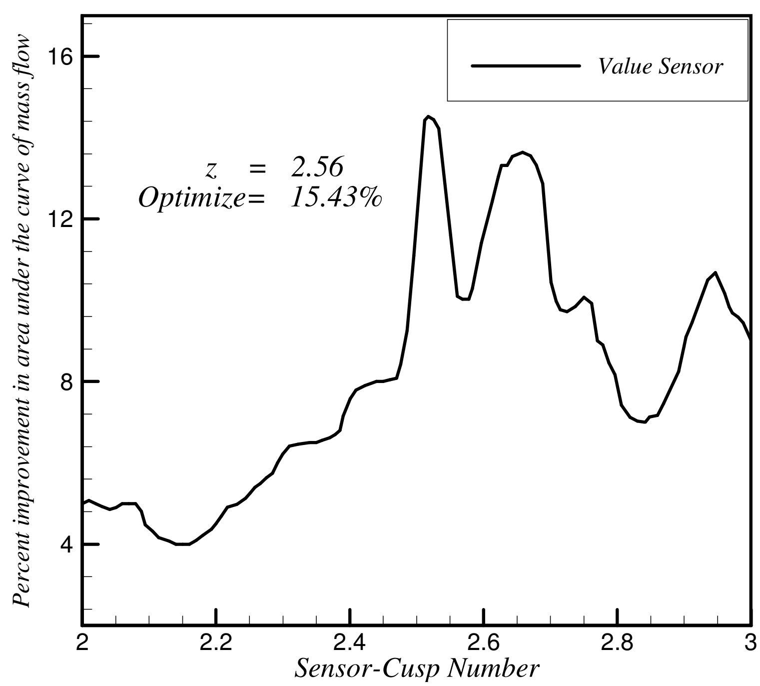

Figure 3. *Effects of z values (Eq. (11)) on the percentage variation of the mass flux*

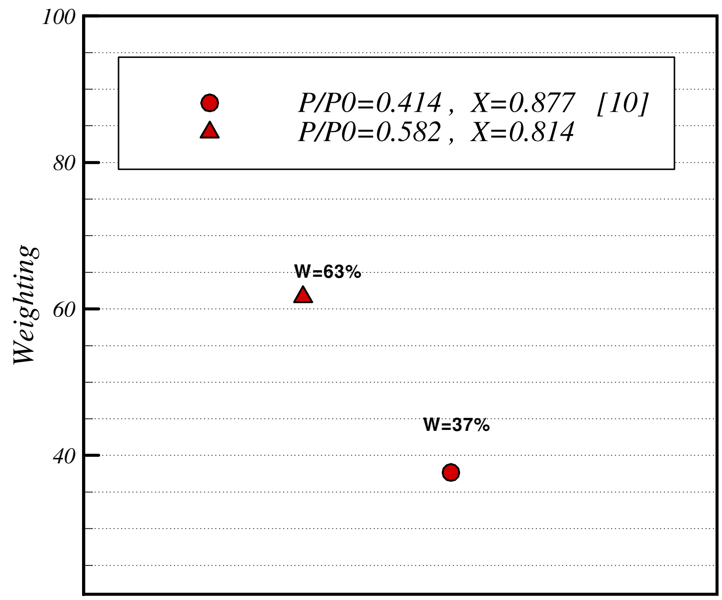

Figure 4. *Contribution of each experimental data with using Weighing effect [10]*

For the steady state conditions, the amount of inlet mass flux along the path must be constant. In Figure 3 by obtaining the best value for CUSP's convergence parameter ($z = 2.56$), it is observed that the combined method shows the least variations of the mass flux with respect to the inlet mass flux. Figure 4 shown that the percentage of two approximately data which are influenced the LM-inverse method for flow optimization.

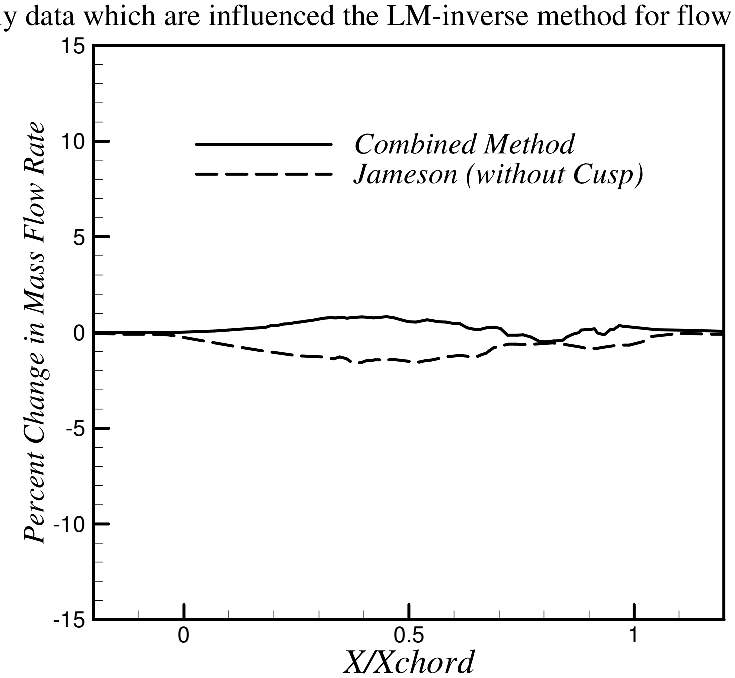

Figure 5. *Variations of the mass flux from the improved Jameson's method using LM*

Further, the use of the LM method results in a 16% improvement in the results when compared to Jameson's and the combined method (Figure 5). This new achievement assures better satisfaction of the law conservation of mass.

It is to be noted that the LM method can be simultaneously used for other regions, but the calculation burden will increase significantly. In this research, the goal was to combine Jameson's finite volume numerical method and the CUSP method and then to improve this combination by using the LM method. This new research shows desirable results in this respect.

7. Conclusions

As it has been explained, the idea of combining the two methods of finite volume and CUSP can significantly improve Jameson's finite volume method. In this research, for increasing the accuracy of the results, the LM method is used in the goal region. Also, using the LM method with contribution effect of two Weighing points of experimental data, the conservation of mass condition is significantly improved. Considering the attributes and vast implications of numerical finite volume methods which are employed for complex geometries such as flow inside turbines, conducting appropriate research is necessary for their improvement. It is acknowledged that any other finite volume method can be used in the proposed method. Defining the most optimum control function mode using LM-inverse method by effect of Weighing of experimental data and by accounting for the mass conservation equation.

In Figure 1 the comparison of the proposed models with experimental results and also Jameson's standard method, shows acceptable accordance of the proposed methods' result particularly in the goal region (shock region on the blade's suction surface). Also in Figure 5 the conservation of mass has achieved more appropriate conditions.

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