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Assessment of structural damage by sensitivity of modal strain energy based on an efficient optimization function

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Abstract

Many methods for the assessment of structural damage based on vibration-based technique are surveyed for many decades. In this paper, an effective damage detection method by numerical evaluation of structural damage is presented based on using an improved sensitivity of modal strain energy. At first, damage localization approach in dynamic systems is formulated by fundamental concept of modal strain energy (MSE) to precisely locate the eventual damages. Subsequently, a new optimization function is presented that contain of sensitivity of modal strain energy and error matrix of mode shapes between healthy and damaged structures. Consequently, the optimization function is solved through Thikhonov regularization method to estimate the damage quantification. For verification of the proposed method, a numerical model of portal plane frame is used. Eventually, results show that the modal strain energy indicator can be exactly detected the location of damage. Then, the estimated damage extent indicates that the proposed method based on solution of optimization function by Thikhonov regularization method can provide the convenient results to predicting the severity of damages.

Keywords: Damage detection, Modal strain energy indictor, Sensitivity of modal strain energy, Optimization function, Thikhonov regularization

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1. Introduction

Vibration-based methods have been developed and applied to assessment of structural damage in many existing structures that related to civil, mechanical and aerospace engineering fields for many decades. Modal parameters depend on the physical properties of a structure and not the excitation applied. Therefore, parameter identification of structures has become very important as researchers attempt to correlate changes in test data to the changes in the structural element properties. Modification of physical parameters of structures such as mass, stiffness or damping properties are lead to occur changes in the vibrational response of structures or modal data. Hence, changes in the physical properties with adversely performance of dynamic behaviour of structures are described as structural damage. Typically, damage detection algorithms are categorized as three steps, namely detection of present of damages, detection of the structural damage locations and estimation of the damage extents [1]. Therefore, the knowledge of vibration-based methods can be used to determine the existence as well as the location and the extent of damage. For the damage detection problem, many researchers have worked in these fields for many decades. Doebling et el. [2], stubbs et al [3] and Yan et al. [4] have been provided literature reviews in damage detection process. Gudmundon [5] introduced a first order perturbation method to predict cracks, other geometrical changes and mathematically showed that the change in eigenvalue was related to the change in strain energy of the system. Lee and Chung [6] applied Gudmundson's theory to identify the location and severity of an edge crack in a cantilever beam. He et al, [7] presented a computationally attractive damage index method is proposed for structural damage detection of cylindrical shell. According this approach, the modal strain energy values computed for undamaged and damaged states were used in the corresponding damage index algorithms and two parameters as moment response power spectral density and curvature response power spectral density were divided. For all measured mode shapes, the damage index was defined by using the statistical parameter of relative root mean-square error of case before and after damage. Fan and Qiao [8] introduced a new strain-based damage detection for plate-type structures. They proposed the concepts of a damage location factor (DLF) matrix and a damage severity correction factor (DSCF) matrix, which can be derived from the elemental modal strain energy. Hence, the damage identification method using the DLF and DSCF was developed for damage localization and quantification in plate-type structures. Using finite element model updating for damage detection method, Jaishi and Ren [9] proposed a new multiobjective optimization technique for damage localization and quantification in the beam-like structures. In that methods, eigenfrequency residual and modal strain energy residual were used as two objective functions of the multiobjective optimisation. Also, Seyedpoor [10] provided a two-stage method to properly identify the site and extent of multiple damage cases in structural systems. In the first stage, a modal strain energy based index (MSEBI) was presented to precisely locate the eventual damage of a structure. The modal strain energy was calculated using the modal analysis information extracted from a finite element modelling. In the second stage, the extent of actual damage was determined via a particle swarm optimization (PSO) using the first stage results.

The objective of this article is identification of structural damage by sensitivity of modal strain energy with finite element model updating optimization function. For damage localization a modal strain energy indicator is expanded at first and then, changes of stiffness components as damage index case can be detected. Typically, the damage quantification pertains to location of damage. Therefore, the sensitivity of modal strain energy is determined to use in the model updating optimization function. After providing the optimization function, the extent of damage is estimated by minimization of proposed optimization function based on MSE sensitivity matrix and error vector containing the differences in mode shapes before and after damage. For verification of the proposed methods, a numerical model of a cantilever beam is used. Eventually, results show that the modal strain energy indicator can be exactly detected the location of damage. Then, the estimated damage extent indicates the proposed method based on sensitivity of modal strain energy and minimization of optimization function can provide the convenient results in the numerical and experimental evaluations.

2. Theory

2.1 Damage localization by modal strain energy method

In this study, an efficient index based on the modal strain energy is presented to accurately site the elements of a damaged structure. The modal analysis is a tool to determine the natural frequencies and mode shapes of a structure. Therefore, numerical simulation approach as generalized eigenvalue problem is used to identify of modal data.

$$K\varphi_i = \lambda_i M\varphi_i, \quad i = 1, 2, ..., N \tag{1}$$

Where K and M are the stiffness and mass matrices of the intact structures, respectively; ω_i and ϕ_i are the ith natural frequency and mode shape vector of the intact structure, respectively. Also, N is the total degrees of freedom of the structure. The mode shapes are usually normalized with respect to the mass matrix and moreover the relation ϕ_i^T M ϕ_i=1 and ϕ_i^T K $\phi_i=\omega_i^2$ can be established. Since the mode shape vectors are equivalent to nodal displacements of a vibrating structure, therefore in each element of the structure strain energy is stored. The strain energy of a structure due to mode shape vector are usually referred to as modal strain energy (MSE) and can be considered as a valuable parameter for damage identification. The modal strain energy of eth element in ith mode of the structure can be expressed as

$$MSE_i = \frac{1}{2}\varphi_i^T K\varphi_i \tag{2}$$

The eth element MSE can be then given by

$$MSE_i^e = \sum_{e=1} \frac{1}{2}\varphi_i^T K_e\varphi_i, \quad i = 1, 2, ..., N \tag{3}$$

where K_e is the stiffness matrix of eth element of the structure and NE is the number of element that the have been assigned to finite element modelling and damage detection. It is possible to use of Eq. (3) to detection of damage location. For computational propose, it is appropriate to normalized the MSE of elements with respect to the total MSE of the structure

$$NMSE_i^e = \left| \frac{\varphi_i^T K_e\varphi_i}{\sum \varphi_i^T K_e\varphi_i} \right| \tag{4}$$

where, NMSE is the normalized MSE of eth element in ith mode of the structure. Sometimes, achievement to complete modal data is impossible. Therefore, for only m identified modes the MSE can be rewritten as follow

$$MNMSE_i^e = \frac{\left|\sum_{i=1}^{m} NMSE_i^e\right|}{m} \tag{5}$$

The modal strain energy index of healthy and damaged structures as form $(NMSE)_h$ and $(NMSE)_d$ must be defined to detect the damage location, respectively. Generally, placing the mode shapes of healthy and damaged structures into Eq. (4), the corresponding MSE index for m modes are obtained. On the other hand, healthy modal strain energy index is similar to typical relationship for global modal strain energy method. The modal strain energy index for damaged structure is written as follow

$$MNSE_d = \sum_{e=1} \frac{1}{2}\varphi_{id}^T \alpha K_e\varphi_{id}, \quad i = 1, 2, ..., m \tag{6}$$

where, φ_d and α denote the damaged mode shape and stiffness modification factor for damaged states, respectively. The damage occurrence is led to increasing the MSE and consequently the efficient parameter NMSE for m modes. As a result, an indicator is termed as general modal strain energy index (λ_{MSE}), which can be determined as

$$\lambda_{MSE} = \frac{MNMSE_d - MNMSE_h}{(MNMSE)_h} \tag{7}$$

It should be noted that, as the damage locations are unknown for the damaged structure with respect to real data applications, therefore for this case the element stiffness matrix of the healthy structure is used for estimating the parameter $(MNMSE)_d$. According to the Eq. (7), for a healthy element the index will be equal to zero (λ_{MSE}=0) and for a damaged element the index will be greater than zero (λ_{MSE}>0).

2.2 Damage quantification by modal strain energy sensitivity analysis

Design sensitivity analysis is used to quantify the relationship between parameters used to define an optimum design and calculate outputs used to measure their performance. Design sensitivity analysis of structural and mechanical systems with respect to structural design parameters plays a critical role in inverse and identification problems in engineering applications [11]. Generally sensitivity analysis describes the rates of change of some of key properties of the dynamic model such as natural frequencies and mode shapes with small changes in some of the physical properties consist of individual mass and stiffness matrices [12]. Therefore, derivatives of dynamic response of structures toward to physical properties are usually described the sensitivity analysis in the dynamic structures.

$$\frac{\partial MSE_{ei}}{\partial p} = \varphi_i^T K_e \frac{\partial \varphi_i}{\partial p} + \frac{1}{2}\varphi_i^T \frac{\partial K_e}{\partial p}\varphi_i \tag{8}$$

The derivatives of eigenvalues with respect to the design variable p can be easily obtained by differentiation of the undamped eigenvalue, but the derivatives of mode shapes cannot be found directly due to it needs overcome the singular problem [11]. For dealing with these limitation to calculation of modal strain energy sensitivity, Yan and Ren [13] derived a compact analytical expression of the element MSE sensitivity based on the algebraic method. This method computes the sensitivity of element MSE using the following as

$$\frac{\partial MSE_{ei}}{\partial p} = \varphi_i^T K^* \varphi_i \tag{9}$$

where

$$K^* = \begin{bmatrix} K_e & 0 \end{bmatrix} . \begin{bmatrix} K-\lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0 \end{bmatrix}^{-1} . \begin{bmatrix} \lambda_i \frac{\partial M}{\partial p} - \frac{\partial K}{\partial p} \\ \frac{1}{2}\varphi_i^T \frac{\partial M}{\partial p} \end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p} \tag{10}$$

As it can be noted, this method is an accurate method, which only requires the eigenvector of interest. And it can find the design sensitivity of element MSE in a very simple and straightforward manner. In this study, damage is assumed to be directly related to a decrease in stiffness. Therefore, damage can be located using the sensitivity of the modal strain energy with respect to the stiffness parameters. With neglecting of the mass matrix modification, the Eq. (10) is rewritten to form

$$K^* = \begin{bmatrix} K_e & 0 \end{bmatrix} . \begin{bmatrix} K-\lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0 \end{bmatrix}^{-1} . \begin{bmatrix} -\frac{\partial K}{\partial p} \\ 0 \end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p} \tag{11}$$

Once the sensitivity of modal strain energy are computed, the sensitivity matrix is build and the change in the stiffness parameters is estimated by minimizing of the optimization function,

$$J = \left(S_{MSE}\Delta k - \Delta\varphi\right)^T W_{\varepsilon\varepsilon}\left(S_{MSE}\Delta k - \Delta\varphi\right) + \Delta k^T W_{kk}\Delta k \tag{12}$$

S_{MSE} is the modal strain energy sensitivity matrix, which can be described as follow

$$S_{MSE} = \varphi_i^T \left(\begin{bmatrix} K_e & 0 \end{bmatrix} . \begin{bmatrix} K-\lambda_i M & -M\varphi_i \\ -\varphi_i^T M & 0 \end{bmatrix}^{-1} . \begin{bmatrix} -\frac{\partial K}{\partial p} \\ 0 \end{bmatrix} + \frac{1}{2}\frac{\partial K_e}{\partial p} \right)\varphi_i \tag{13}$$

$\Delta\varphi$ is the error vector containing the differences in mode shapes before and after damage. $W_{\varepsilon\varepsilon}$, W_{kk} are positive definite weighting matrices. $W_{\varepsilon\varepsilon}$ is a diagonal matrix whose elements are given by the reciprocals of the variance of the corresponding measurements. $W_{kk}=\alpha I$ is a diagonal matrix whose elements are equal to the regularization parameter α (Tikonov regularization). A detailed explanation of the derivation of this equation is found in the book of Friswell and Mottershead [14]. The solution of Eq. (12) is obtained through least squares as follow,

$$\{\Delta k\} = \left[S_{MSE}^{T} W_{\varepsilon\varepsilon} S_{MSE} + W_{kk} \right]^{-1} S_{MSE}^{T} W_{\varepsilon\varepsilon} \Delta\varphi \tag{14}$$

where, Δk is the damage quantification based on modal strain energy sensitivity method. In the numerical evaluation, the damage detection process is usually carried out to induce the damage index and according to proposed method, predicated damage index can be estimated as stiffness reduction.

3. Numerical Results

In this section, the proposed damage detection algorithms are investigated to identify the damage site and subsequently determine the severity of damage on one-story portal plane frame as shown in Fig. 1. The corresponding model of frame was constructed and the FEM analysis was carried out to simulate the complete experimental data by using two-node beam elements. The number of nodes and elements are 21 and 22, respectively. Assume the portal frame has a rectangular cross sectional area with height h=0.2 m, width b=0.15 m and lengths B=1.2 m and H=1.6 m. The material has a Young Modulus of E=2.5×10^{10} N/m^2 and a material density of 2500 kg/m^3.

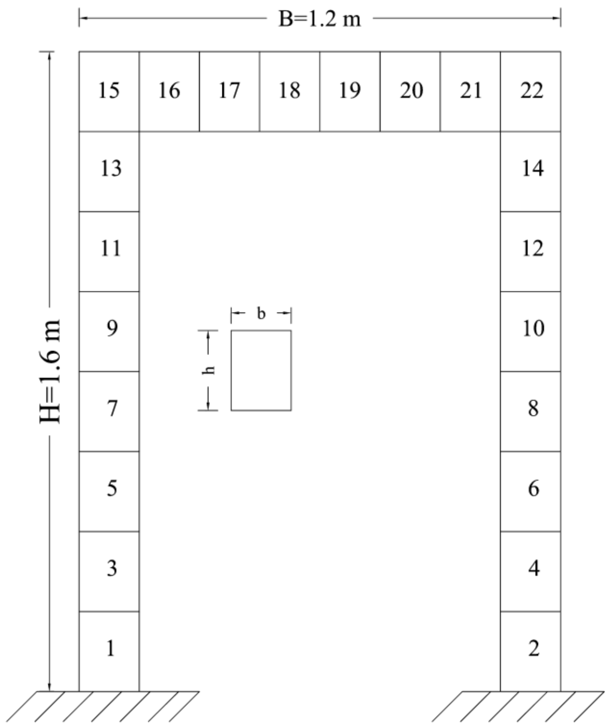

Figure 1. A portal plane frame

Here, four damage cases are assumed to investigate the capabilities of the proposed method to detecting of the occurred damages in portal frame. In the first damage case, the stiffness of elements 3 was decreased by 30%. In damage case number two, the stiffness of elements 4 and 10 reduced by 25% and 20%, respectively. In the third damage case, the stiffness of element 15 decreased by 40%. Finally, in the damage case number four, the stiffness of element 18 and 19 reduced by 10% and 15% respectively. Based on the proposed damage assessment algorithms, location of induced damages are detected by modal strain energy index from Eq. (7). Corresponding to damage quantification, sensitivity of MSE is firstly determined to specify the changes of dynamic behaviour. Subsequently, difference of eigenvectors (mode shapes) between healthy and damaged structures are computed. Eventually, using Eq. (14) the vectors of damage parameters will be estimated.

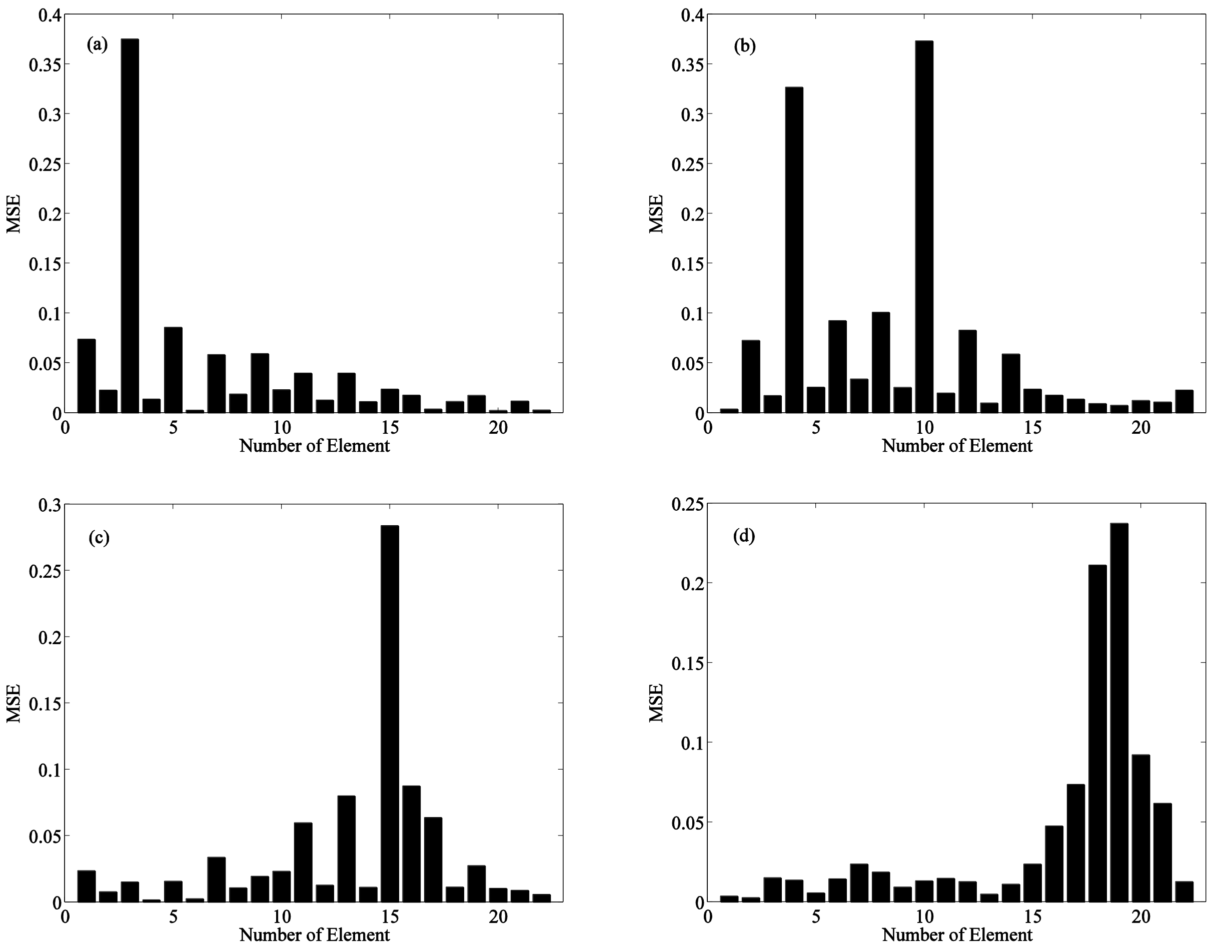

Figure 2. Damage localization in the portal frame by MSE, a) Damage scenario 1, b) Damage scenario 2, c) Damage scenario 3, d) Damage scenario 4

It can be observed that the modal strain energy index (MSE) achieves to precise location of damages, even for multiple damage cases. The computational error data for undamaged elements in the damage scenarios are inconsiderable values and based on Figs. 2a-d, peaks of damaged elements clearly demonstrate the site of damages. After identification of damage site, sensitivity matrix of modal strain energy by Eq. (13) is determined. The eigenvectors error matrix is also computed by comparison between mode shapes of healthy and damaged frame. As mentioned before, due to employing generalized eigenproblem similar to Eq. (1), the error matrix of eigenvectors is computed as complete data. Eventually, based on Eq. (14) the severity of identified damages is estimated. Accordingly, Figs. 3a-d, illustrate the extent of damages based on sensitivity modal strain energy and solution of optimization function by Tikonov regularization.

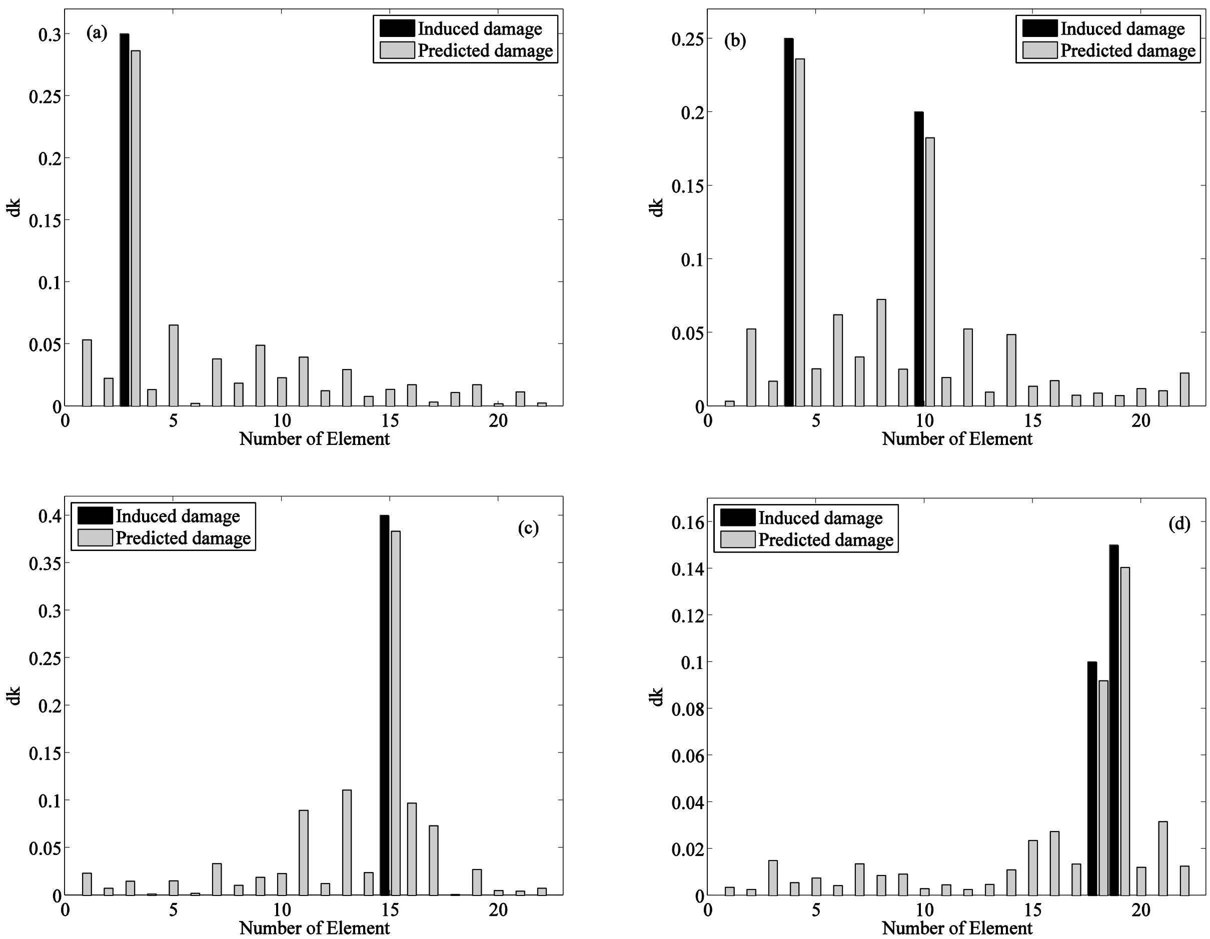

Figure 3. Damage quantification in the portal frame, a) Damage scenario 1, b) Damage scenario 2, c) Damage scenario 3, c) Damage scenario 4

As can be seen, Figs 3a-d show the severity of damages based on computational information of sensitivity of modal strain energy and mathematical solution of Tikhonov regularization. Due to continuity between elements of portal frame, computed errors in the undamaged elements which have joined to damaged members are greater than other elements. However, these errors are less than 10% for joined members.

4. Conclusion

This paper has presented a two-stage methodology to evaluate damage extent and location by basic concept of modal strain energy. In the first stage, modal strain energy has been formulated for healthy and damaged structures. Hence, modal strain energy indicator (MSE) has utilized to identify the location of damages. The proposed damage localization algorithm depends on little information of dynamic systems including stiffness matrix and eigenvectors of healthy and damaged structures. Subsequently, severity of damage has determined by a new optimization function. The basic components of this function were sensitivity of modal strain energy and error matrix of eigenvector (mode shape). Consequently, Tikhonov regularization method has used to solve the optimization function. The proposed method was numerically verified by a portal plane frame as continuous system. The initial model of the portal frame has constructed and based on generalized eigenproblem, complete modal parameters were identified. Accordingly, obtained information of system as well as proposed mathematical equations, the location and severity of damage cases were evaluated. Eventually, the numerical results can be shown that the proposed methodology has strong ability to identify the location of damages and then, estimate the damage quantification based on identified modal data in multiple damage cases.

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