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A numerical damage identification approach by direct sensitivity of orthogonality conditions via stiffness modification index

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Abstract

This paper presents a numerical method for damage identification based on orthogonality conditions sensitivity methods based on stiffness modification index. Determination of damage severity is main objective of this article. Hence, the general eigenvalue problem and stiffness-orthogonality conditions sensitivity methods are developed to determine the damage severity index. In this study, the variation of stiffness matrices are introduced as damage index and based on its changes the eigenvalue problem and stiffness-orthogonality condition of damaged structure are expanded to determine the damage quantification. To achieve the damage detection algorithm, the initial physical properties of intact structures such as the mass and stiffness matrices as well as modal parameters consist of natural frequencies and mode shapes must be utilized. For damage assessment, two numerical structures as discrete and continuous dynamic systems are modelled. First, the extent of induced damage is estimated in a simple 6-story shear building. Then, a 15-bar planner truss is modelled and damage assessment methods are evaluated. Numerical results indicate that proposed damage quantification methods including the general eigenvalue problem and stiffness-orthogonality condition sensitivity methods can provide reliable tools to accurately determine the multiple structural damage quantification.

Keywords: Structural damage identification, General eigenvalue problem, Stiffness orthogonality condition, Modal parameters

1. Introduction

Structural damage detection using measured dynamic data has emerged as new research in civil, mechanical and aerospace engineering communities in recent years. The basic idea of this technique is that modal parameters are functions of the physical properties of the structures such as mass, stiffness or damping. Therefore, changes in the physical properties will cause changes in the modal parameters. Damage is generally defined as the adversely performance of dynamic behaviour in the structures that related to changes of physical properties.

Any damage identification method can be classified by its ability to characterize the damage. Generally three levels of damage characterization are derived as detection of the presence of damages, detection of the structural locations and estimation of the damage extents. For the last two stated above, most of the existing methods can be thought of as a two-stage algorithm in which damage locations are detected as first, and then damage extents are estimated. Generally the first step may be more important, but probably more difficult [1]. Recent surveys on the technical literature, such as on Doebling et al. [2] and Salawu [3] among others, show that extensive efforts have been developed to find reliable and efficient numerical and experimental models to identify damage in structures. Most prior work in damage identification is based on the modification of a structural finite element model (FEM). The goal of these methods is to use test data from the damage structure and the correlated FEM of the undamaged structure to determine changes to stiffness and/or mass matrices [4]. Assessment of structural damage based on sensitivity approach, Yang [5] proposed a mixed sensitivity method to identify structural damage by combining the eigenvalue sensitivity with the flexibility sensitivity. Then, the presented mixed perturbation approach was demonstrated and compared with the eigenparameter sensitivity method on a 31-bar truss structure. Also, Cao et al. [6] presented a sensitivity method as mode shape and static deflection changes for damage identification in cantilever beams. The efficiency of their approach was a rule in identifying damage using Euler–Bernoulli cantilever beams with a crack. The validity of the approach was supported by three-dimensional elastic finite element simulation. Lim [7] proposed a systematic method that provides precise identification of damage location and the extent when the exact measured modes at every finite element DOF are used. Also, a procedure was presented to perform damage detection with inaccurate, incomplete measured modes. Zimmerman and Kaouk [8] referred to a damage vector to locate the damage site first, and to assess the magnitude of damage using a minimum rank update theory. Changes in strain energy are also used as an indicator to represent damage by Shi and Law [9] and Fan et al. [10] introduced the curvature mode shapes. The objective of this study is damage quantification by orthogonality conditions sensitivity method. For this aim, a two stage of structures as intact and damaged states are required. On the other hand, the modal parameters such as natural frequencies and mode shapes are calculated of these two states. Typically, in the numerical investigation studies are preferred to utilize simulation method for identification of modal data. Therefore, with consideration of proportional damping in the modelling and assumption of existence of mass and stiffness matrices, the generalized eigenvalue problem is used. All part of these states is accomplished by finite element model and numerical methods. In this study, damage is also defined as stiffness reduction in stories of shear building and truss element. Based on this description, the damage extent based on sensitivities of eigenvectors (mode shapes) is formulated. For solving and calculating of damage severity, the SVD algorithm is used. The model of simple 6-story shear building is modelled and the proposed methods are validated by consideration of intact and damaged structures. Then, a 15-bar planner truss is used to determine the damage location and extent. All of these structures are constituted by finite element model. Eventually, numerical results will be accurately indicated the location and quantification of damage in the modelled structures.

2. Damage detection by orthogonality conditions sensitivity

2.1 General Eigenvalue problem of damaged structure

For the discretized damaged structure, the eigenvalue equation can be written as below:

$$K_d \{\phi_i\}_d = \{\omega_i^2\}_d M_h \{\phi_i\}_d, \; i = 1,...,N \quad (1)$$

where K_d is the stiffness matrix ω_{id} is the ith eigenvalue, ϕ_{id} is the ith eigenvector and N is degrees of freedom of mode shapes of the damaged structure. The mass matrix of the damaged structure is assumed to be equal to the mass matrix of the undamaged structure. Matrices K and M are assumed to be symmetric, positive definite, and therefore the eigenvalues ω_i are positive and the eigenvectors ϕ_i can be taken as K-orthogonal. Similar conditions apply to K_d, ω_{id} and ϕ_{id}.

For achieve to orthogonality condition sensitivity, the undamaged eigenvectors of structure ϕ_h pre-multiplying to Eq. (7) and can be written as:

$$\phi_h^T K_d \, \phi_d = \omega_d^2 \, \phi_h^T M \, \phi_d \tag{2}$$

Since the damage stiffness matrix is given by $K_d=K-\Delta K$ and on element e the corresponding error matrix is $\delta k_e=\beta k_e$, where β is the damage parameter, then Eq. (8) yields

$$\sum_{e=1}^{N} \left(\phi_d^T k_e \phi_d \beta \right)_e = \left(\omega_h^2 - \omega_d^2 \right) \phi_d^T M \phi_h \tag{3}$$

where ϕ_h is the element eth displacement vector of the ith mode shape of the damaged structure and N is the number of elements of the discretized structure. In the compact form, Eq. (9) is written as

$$S\beta = \delta\lambda \tag{4}$$

where

$$S = \left(\phi_d^T k_e \phi_d \right)_e \tag{5}$$

$$\delta\lambda = \left(\omega_h^2 - \omega_d^2 \right) \phi_d^T M \phi_h \tag{6}$$

S is the mixed sensitivity matrix of the undamaged/damaged structure and $\delta\lambda$ is the vector of modal parameters of the undamaged/damaged structure. Depending on the number of mode shapes and the number of elements, three cases can occur generally. The SVD solution is the one that minimizes the Euclidean norm $\|\beta\|$. The matrix S is written as the product of a column orthogonal matrix U, a diagonal matrix W, which contains the singular values, and the transpose of an orthogonal matrix V, as follow:

$$S = UWV^T \tag{7}$$

With substituting Eq. (13) into Eq. (10) and β computation is straightforward:

$$\beta = VW^{-1}U^T \left(\omega_h^2 - \omega_d^2 \right) \phi_d^T M \phi_h \tag{8}$$

2.2 Stiffness-orthogonality condition of damaged structure

In addition to Eq. (8) the damage assessment method can be evaluated by stiffness orthogonality condition without anticipation of mass matrix in the damage level. Considering the mass normalization of the mode shapes, the orthogonality conditions are defined by

$$\phi_d^T K_d \, \phi_d = \omega_d^2 \tag{9}$$

Substituting expression $K_d=K-\Delta K$ and provided error matrix of damaged element as $\delta k_e=\beta k_e$ into Eq. (15) the damage quantification relation is formulated as follow:

$$\sum_{e=1}^{N} \left(\phi_d^T k_e \phi_d \beta \right)_e = \phi_d^T K \phi_d - \omega_d^2 \tag{10}$$

where

$$S = \left(\phi_d^T k_e \phi_d \right)_e \tag{11}$$

$$\delta\lambda = \phi_d^T K \phi_d - \omega_d^2 \tag{12}$$

$$\beta = VW^{-1}U^T \left(\phi_d^T K \phi_d - \omega_d^2 \right) \tag{13}$$

Similar to previous section, S and $\delta\lambda$ are the sensitivity matrix and the vector of modal parameters of the undamaged/damaged structures. It can be indicated of above expressions, the damage quantification relations are also rewritten by the stiffness matrix of the undamaged structure as well as the modal parameters of the damaged structure. Similar to prior part, the Eq. (16) can be also solved via SVD method. The sensitivity matrix S is also rearranged similar to $S=UWV^T$. As a result, the damage parameter β can be also calculated by Eq. (19).

3. Numerical Evaluation

3.1 A 6-story shear building as discrete dynamic structure

The numerical model is considered a 6-story shear building shown in Fig. 1, which can be modelled as a 6-DOF system with following properties. The shear building can be considered as discrete dynamic system, hence the mass and stiffness matrices of shear building are determined by finite element method [11, 12]

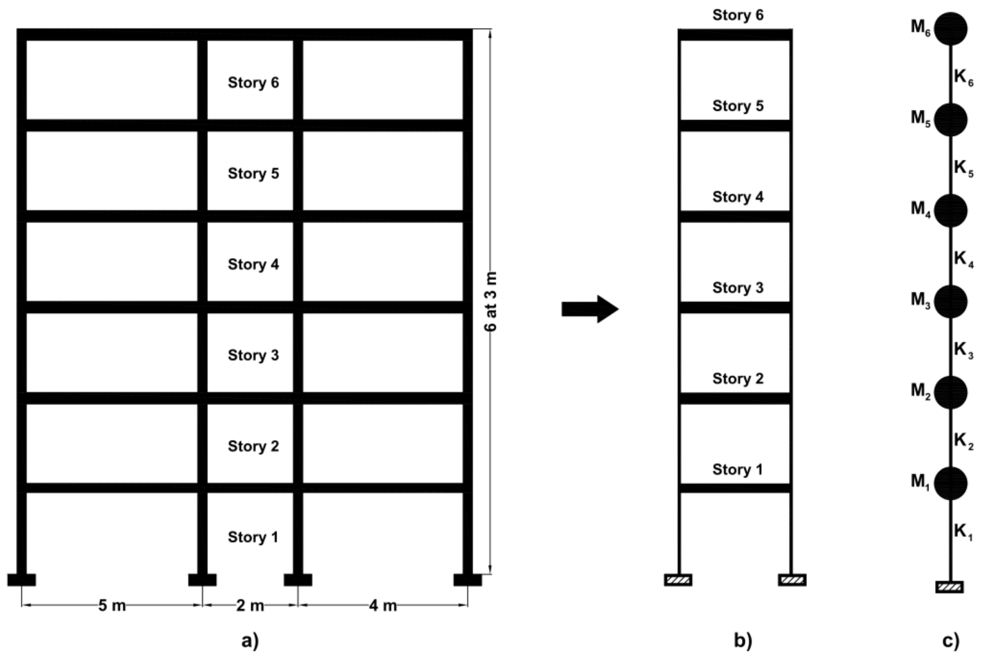

Figure 1. a) Full-scale shear building frame, b) Simulated shear building frame, c) Discrete dynamic system modelling of shear building frame

Table 1. Physical properties of 6 story shear building

Physical Properties	Story 1	Story 2	Story 3	Story 4	Story 5	Story 6
Mass (Ton)	10	10	10	8	8	6
Stiffness (Ton/m)	125	125	111	95	95	83

Four damage cases are considered to investigate the location and number of damaged stories on the results. In the first case, the stiffness of story 1 was decreased by 40%. In the second case, the stiffness of stories 2 and 5 were reduced by 30% and 20% respectively. In the damage case three, the stiffness of story 3 was decreased via 30%. Finally, in the fourth damage case, the stiffness of stories 3 and 4 were reduced by 10% and 20%, respectively.

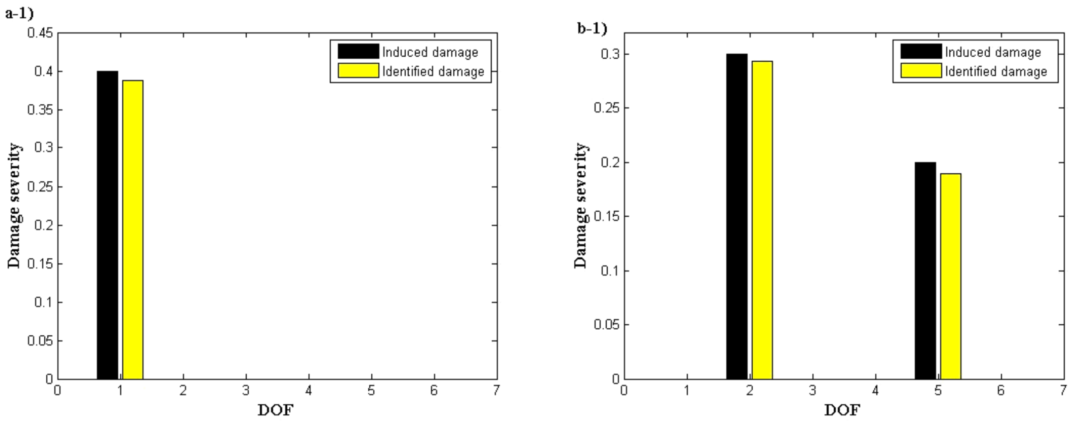

Figure 2. Damage severity based on general eigenvalue problem, a-1) Damage Case 1, b-1) Damage Case 2

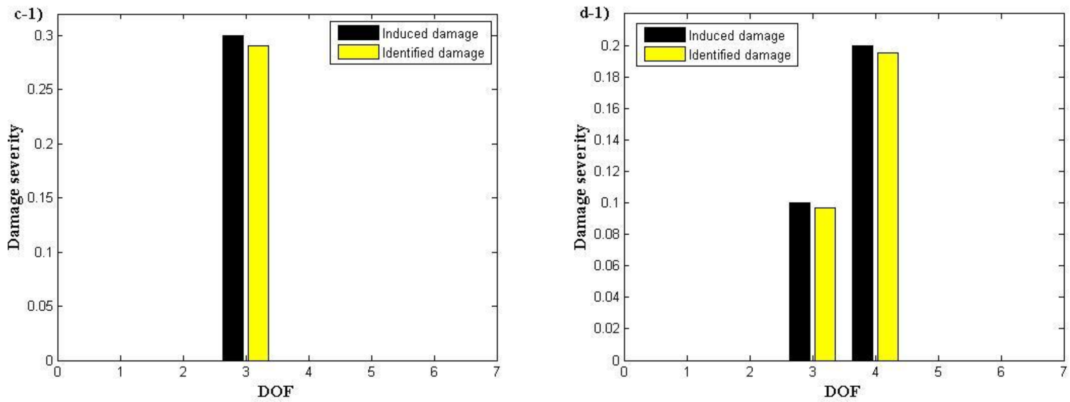

Figure 3. Damage severity based on General Eigenvalue problem, c-1) Damage Case 3, d-1) Damage Case 4

It can be indicated that the damage severity based on the general eigenvalue problem or orthogonality condition sensitivity method has been accurately estimated in damage cases and error in the identified damage is less than 2%.

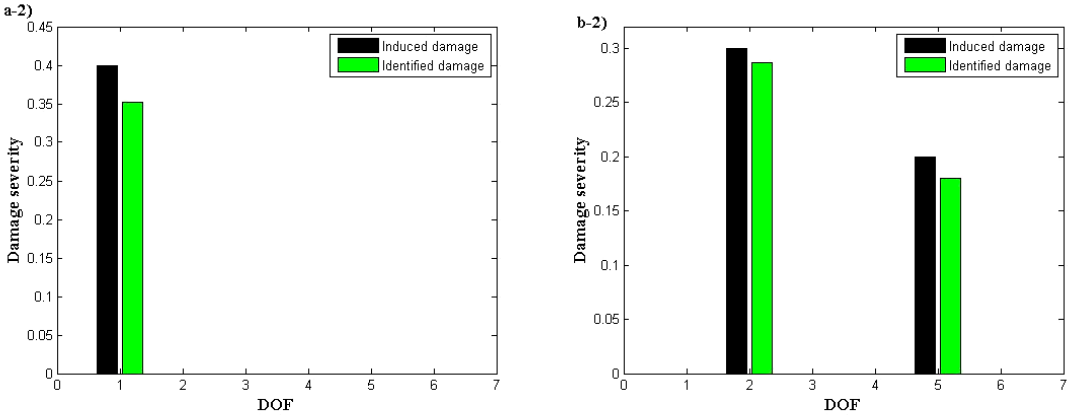

Figure 4. Damage severity based on K-orthogonality condition, a-2) Damage Case 1, b-2) Damage Case 2

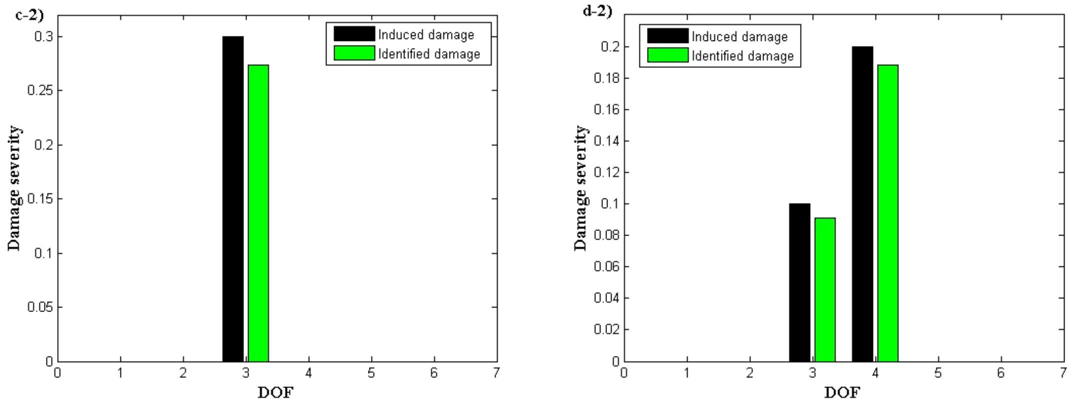

Figure 5. Damage severity based on K-orthogonality condition, c-2) Damage Case 3, d-2) Damage Case 4

These figures accurately show the damage severity of shear building. However, the Eq. (19) detected the damage quantification by stiffness-orthogonality condition; the error in the identified damage is less than 5% for all levels. As compare with Eqs. (14) and (19), it can be resulted that the Eq. (14) is the error less than the Eq. (19). Nonetheless, both of these equations correctly estimated the damage quantification.

3.2 A 15-bar planner truss

To illustrate characteristics of the proposed damage detection algorithm, a two-dimensional truss structure is presented as shown Fig. 6. The basic parameters of the structure are Young modules E=200 GPa, density ρ=7850 kg/m3. All element of truss are modelled with 100 mm × 100 mm equal double angels and 5 mm thickness. Each nodal of truss have two degrees freedom (DOF), hence the total degrees of freedom of truss is illustrated as shown Fig. 6.

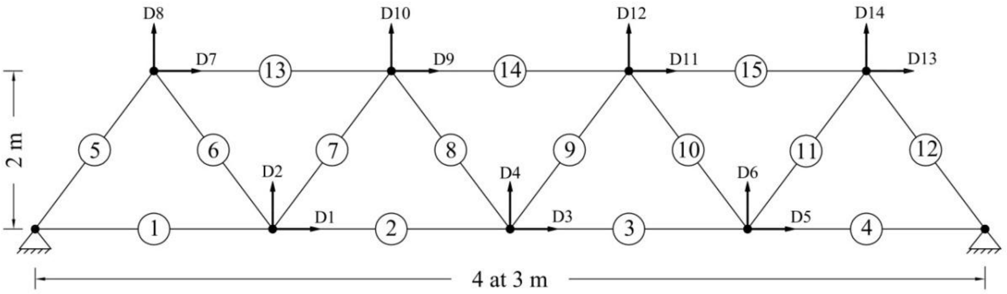

Figure 6. 15-Bar truss structure

The above structure is a continuous dynamic system and the mass and stiffness matrices can be determined by basic concept of finite element method [11]. After determination of physical parameters of intact truss structure, the generalized eigenvalue problem is used and the modal parameters including natural frequency and mode shapes are calculated. Assume that the proportional damping is dominated in the structure behaviour and consequently the modal parameters are extracted as real data. Four damage cases are considered to investigate the influence of the location, severity and number of the damaged elements on the results. In the first damage case, the stiffness of elements 2 and 15 were reduced by 40%. In damage case number two, the stiffness of elements 6, 9 and 12 were decreased by 20%, 25% and 30%, respectively. In damage case number three, the stiffness elements 1, 10 and 14 were reduced via 30%, 30% and 20%, respectively. Finally, in damage case number four, the stiffness of elements 7 and 8 were decreased by 20% and 30%, respectively. Figs 7-8 show extent of induced damage based on general eigenvalue problem in the truss structures.

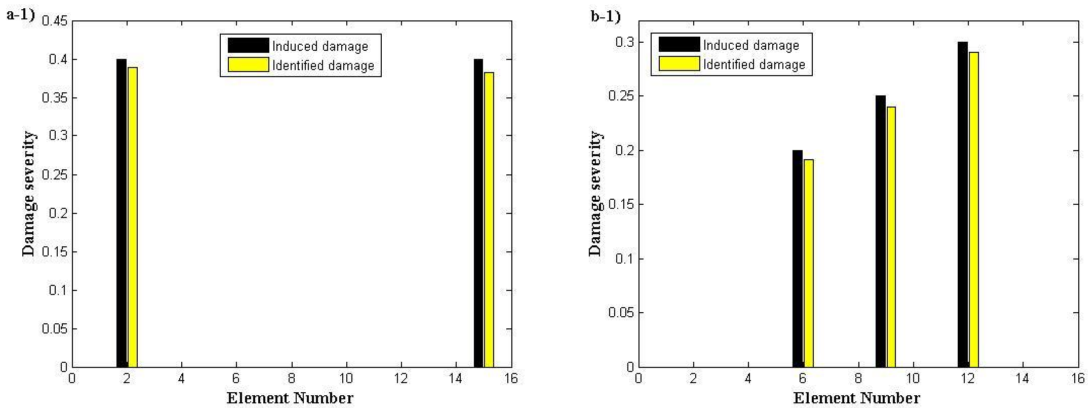

Figure 7. Damage severity based on general eigenvalue problem, a-1) Damage case 1, b-1) Damage case 2

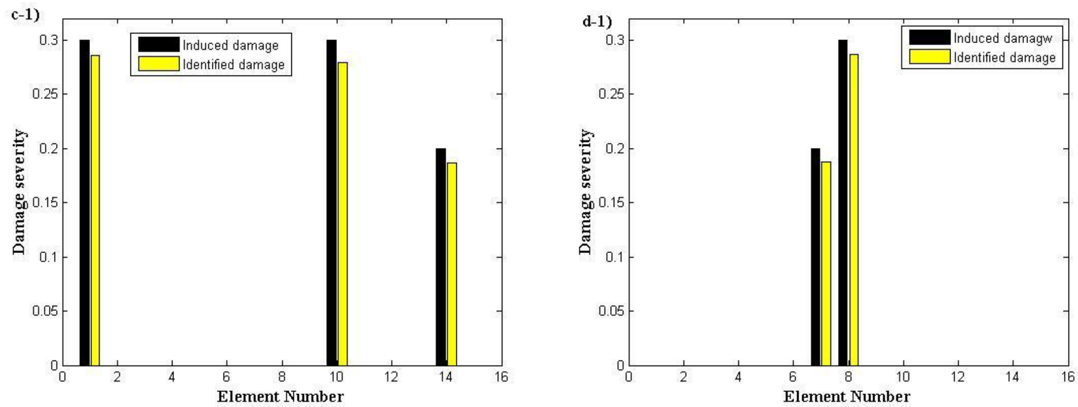

Figure 8. Damage severity based on general eigenvalue problem, c-1) Damage case 3, d-1) Damage case 4

It can be observed that the damage severity has been also identified by general eigenvalue problem sensitivity method. Consequently, the induced damage level was accurately estimated and identified error is less than 3%. Furthermore, the damage severity can be also determined by stiffness-orthogonality condition sensitivity method according to Eq. (19). On the other words, only using of the initial stiffness matrix and modal parameters of undamaged and damaged structures provide the easy calculations for damage detection. Indeed, the damage quantification by stiffness-orthogonality condition sensitivity method is simpler than general eigenvalue problem.

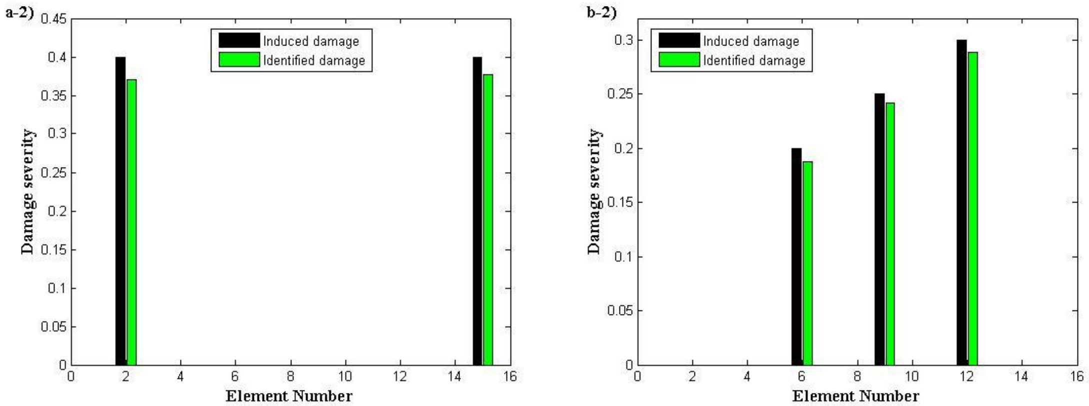

Figure 9. Damage severity based on K-orthogonality condition, a-2) Damage case 1, b-2) Damage case 2

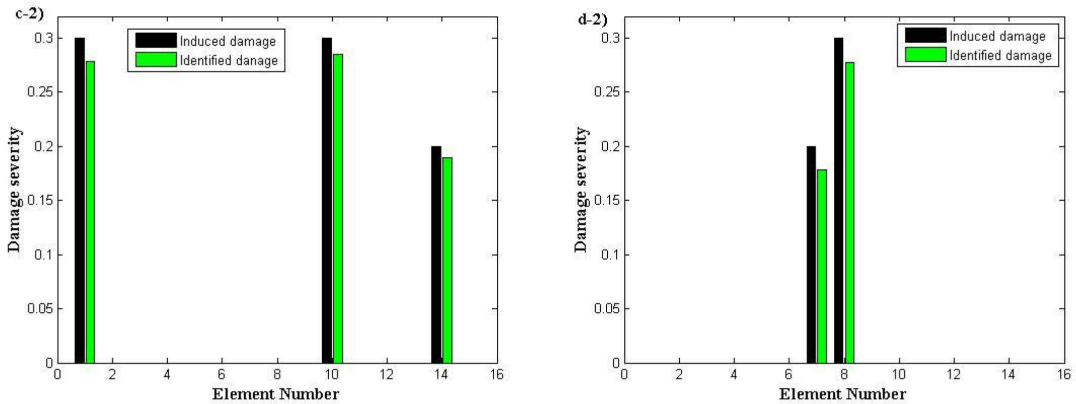

Figure 10. Damage severity based on K-orthogonality condition, c-1) Damage case 3, d-1) Damage case 4

Similar to prior relations, the stiffness-orthogonality condition sensitivity method has been also provided the well results in the truss structure. The estimated error in this stage is less than 5%; therefore, the damage severity has been accurately identified based on proposed approach. It should be noted that the two proposed damage quantification approaches approximately have similar results and indicate the accurate quantities of damage severity.

4. Conclusion

A numerical damage detection approach based on expansion and developments of orthogonality conditions sensitivity methods have been proposed. Based on objective of article, the damage severity is determined by modification of general eigenvalue problem and stiffness-orthogonality condition sensitivity methods. These relations do not require the previous knowledge of the damaged area or areas and allow the identification of multiple damage. In the first part of this stage, the mass matrix of undamaged structure and the stiffness error matrix have significant rules to damage detection. In addition to, the modal parameters of undamaged and damaged structures must be used. In the second part, the stiffness-orthogonality condition sensitivity method is utilized. At this part, the stiffness matrix of intact structure as well as modal parameters of undamaged and damaged structures must be defined. In order to assess the performance of proposed damage detection approaches two structures that related to discrete and continuous dynamic systems are modelled. These structures include a 6-story shear building and a 15-bar planner truss. The numerical results indicated that the damage severity of the 6-story shear building and the 15-bar planner truss are accurately identified by the two proposed methods. Eventually, the present methods permit the reliable location and quantification of damage even with the use of few damage structure mode shapes.

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