The 43^{rd} Annual Iranian Mathematics Conference, University of Tabriz
27 – 30 August 2012, Tabriz, Iran

Bifurcation Analysis of a Simplified BAM Neural Network Model with Time Delays

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Abstract

In this paper, a five-neuron bidirectional associative memory (BAM) neural network with two time delays is studied. Since the study of Hopf bifurcation is very important for the design and application of BAM neural networks, we investigate that Hopf bifurcation occurs and a family of periodic solutions appear when the sum of two delays passes through a critical value.

Keywords: Neural Network, Hopf bifurcation, Periodic solutions, Time delay.

Mathematics Subject Classification[2010]: 68T05, 37G15, 34D20, 37L10.

1 Introduction

The bidirectional associative memory (BAM) networks were first introduced by Kasko [3, 5]. The properties of periodic solutions are significant in many applications. It is well known that BAM NNs are able to store multiple patterns, but most of NNs have only one storage pattern or memory pattern. BAM NNs have practical applications in storing paired patterns or memories and have the ability of searching the desired patterns through both forward and backward directions.

The delayed BAM neural network is described by the following system:

$$\begin{cases} \dot{x_i}(t) = -\mu_i x_i(t) + \sum_{j=1}^{m} c_{ji} f_i(y_j(t-\tau_{ji})) + I_i & (i = 1, 2, \ldots, n) \\ \dot{y_j}(t) = -\upsilon_j y_j(t) + \sum_{i=1}^{n} d_{ij} g_j(x_i(t-\sigma_{ij})) + J_j & (j = 1, 2, \ldots, m) \end{cases} \tag{1}$$

where c_{ji} and d_{ij} are the connection weights through the neurons in two layers: the X-layer and the Y-layer. The stability of internal neuron processes on the X-layer and Y-layer are described by μ_i and υ_j, respectively. On the X-layer, the neurons whose states are denoted by $x_i(t)$ receive the input I_i and the inputs outputted by those neurons in the Y-layer via activation function f_i, while the similar process happens on the Y-layer. Also, τ_{ji} and σ_{ij} correspond to the finite time delays of neural processing and delivery of signals. For further details, see [3, 5].

Since a great number of periodic solutions indicate multiple memory patterns, the study of Hopf bifurcation is very important for the design and application of BAM NNs. In fact, various local periodic solutions can arise from the different equilibrium points of BAM NNs by applying Hopf bifurcation technique. But the exhaustive analysis of the dynamics of such a large system is complicated, so some authors have studied the dynamical behaviours of simplified systems [1, 2, 4, 6, 7, 8, 9, 10].

Motivated by the above, in this paper, we consider the following five-neuron BAM neural network:

$$\begin{cases} \dot{x_1}(t) = -\mu_1 x_1(t) + c_{11} f_1(y_1(t-\tau_2)) + c_{31} f_1(y_3(t-\tau_2)) \\ \dot{x_2}(t) = -\mu_2 x_2(t) + c_{22} f_2(y_2(t-\tau_2)) + c_{32} f_2(y_3(t-\tau_2)) \\ \dot{y_1}(t) = -\upsilon_1 y_1(t) + d_{11} g_1(x_1(t-\tau_1)) + d_{21} g_1(x_2(t-\tau_1)) \\ \dot{y_2}(t) = -\upsilon_2 y_2(t) + d_{12} g_2(x_1(t-\tau_1)) + d_{22} g_2(x_2(t-\tau_1)) \\ \dot{y_3}(t) = -\upsilon_3 y_3(t) + d_{13} g_3(x_1(t-\tau_1)) + d_{23} g_3(x_2(t-\tau_1)) \end{cases} \tag{2}$$

where $\mu_i > 0 (i = 1, 2)$ and $\upsilon_j > 0 (j = 1, 2, 3)$. The time delay from the X-layer to another Y-layer is τ_1, while the time delay from the Y-layer back to the X-layer is τ_2. In the next section, we state our main results on the Hopf bifurcation analysis of the system (2). We should mention that it is the first time to deal with (2).

2 Main Result

To establish the main results for system (2.1), it is necessary to make the following assumption:

$$\text{(H1)} \quad f_i,\, g_j \in C^n, \quad f_i(0) = g_i(0) = 0, \quad (i = 1, 2;\, j = 1, 2, 3)$$

It is easy to see that the origin is an equilibrium point of (2). Under the hypothesis (H1) and letting $u_1(t) = x_1(t-\tau_1)$, $u_2(t) = x_2(t-\tau_1)$, $u_3(t) = y_1(t)$, $u_4(t) = y_2(t)$, $u_5(t) = y_3(t)$ and $\tau = \tau_1 + \tau_2$, we can get the associated characteristic equation of (2):

$$\lambda^5 + a\lambda^4 + b\lambda^3 + c\lambda^2 + d\lambda + e + (a_1\lambda^3 + b_1\lambda^2 + c_1\lambda + d_1)e^{-\lambda\tau} + (a_2\lambda + b_2)e^{-2\lambda\tau} = 0 \tag{3}$$

Now, by assuming

$$\text{(H2)} \quad a_1 = b_1 = c_1 = d_1 = 0,$$

it can be proved that $\lambda = i\omega (\omega > 0)$ is a root of (3) if and only if $z = \omega^2$ satisfies

$$z^5 + pz^4 + qz^3 + rz^2 + sz + v = 0. \tag{4}$$

Then by assuming $h(z) = z^5 + pz^4 + qz^3 + rz^2 + sz + v$ and z_k^*, $k = 1, 2, 3, 4, 5$ as the positive roots of (4), we have

$$\tau_0 = min_{k\in\{1,\dots,5\}} \frac{1}{2\omega_k}[cos^{-1}(\frac{a_2\omega_k^6 + (ab_2 - a_2 b)\omega_k^4 + (da_2 - cb_2)\omega_k^2 + eb_2}{-b_2^2 - a_2^2\omega_k^2})],$$

where $\omega_k = \sqrt{z_k^*}$.

Letting $\lambda(\tau) = \alpha(\tau) + i\omega(\tau)$ and $\alpha(\tau_0) = 0$, $\omega(\tau_0) = \omega_0$, we can state the following theorem:

Theorem 2.1. *Suppose* $z_0 = \omega_0^2, h'(z_0) \neq 0$. *Then, at* $\tau = \tau_0$, $\pm i\omega_0$ *is a pair of simple purely imaginary roots of* (3) *and* $\frac{dRe(\lambda(\tau_0))}{d\tau} \neq 0$.

Proof. By differentiating equation (3) with respect to τ, we can easily prove this theorem. □

Theorem 2.2. *Assume that (H1) and (H2) hold. (a) if $v < 0$, then the zero solution of system (2) is asymptotically stable for all $\tau \in [0, \tau_0)$. (b) if $v < 0$ and $h'(z_0) \neq 0$, then system (2) undergoes a Hopf bifurcation at the zero solution when τ passes through τ_0.*

Proof. It should be noted that when $v < 0$, the equation (4) has at least one positive root. By using this fact and bifurcation theory, this theorem follows from Theorem 2.1. □

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