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PROPERTIES OF BIFURCATING PERIODIC SOLUTIONS IN A DELAYED FIVE-NEURON BAM NEURAL NETWORK MODEL

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Abstract. In this paper, some properties of the bifurcating periodic solutions in a bidirectional associative memory (BAM) neural network model will be studied. The model considered consists of two neurons in the X-layer and three neurons in the Y-layer with two time delays. In fact, the direction and the stability of bifurcating periodic solutions on the center manifold are discussed. Furthermore, the formula of period will be resulted.

1. Introduction

The bidirectional associative memory (BAM) networks were first introduced by Kasko (see [4]). BAM neural networks have practical applications in storing paired patterns or memories and possess the ability of searching the desired patterns through both forward and backward directions. Furthermore, since a great number of periodic solutions indicate multiple memory patterns, the study of Hopf bifurcation is very important for the design and application of BAM neural networks. For example, Hopf bifurcation has been discussed in

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three-neuron, four-neuron and five-neuron BAM neural networks with multiple delays [2, 5, 3].

The delayed BAM neural network is described by the following system:

$$\begin{cases} \dot{x_i}(t) = -\mu_i x_i(t) + \sum_{j=1}^{m} c_{ji} f_i(y_j(t-\tau_{ji})) + I_i \; (i = 1, 2, \ldots, n), \\ \dot{y_j}(t) = -\upsilon_j y_j(t) + \sum_{i=1}^{n} d_{ij} g_j(x_i(t-\sigma_{ij})) + J_j \; (j = 1, 2, \ldots, m) \end{cases} \tag{1.1}$$

where c_{ji} and d_{ij} are the connection weights through the neurons in two layers. The stability of internal neuron processes on the X-layer and Y-layer are described by μ_i and υ_j, respectively. Also, τ_{ji} and σ_{ij} correspond to the finite time delays. On the Y-layer, the neurons whose states are denoted by $y_j(t)$ receive the input J_i and the inputs outputted by those neurons in the X-layer via activation function g_j, while the similar process happens on the X-layer. For further details, see [4].

Since the exhaustive analysis of the dynamics of system (1.1) is complicated, it is useful to consider and study the dynamical behaviors of simplified forms. Motivated by the above, in this paper, we consider the following five-neuron BAM neural network model:

$$\begin{cases} \dot{x_1}(t) = -\mu_1 x_1(t) + c_{11} f_1(y_1(t-\tau_2)) \\ \qquad\qquad + c_{21} f_1(y_2(t-\tau_2)) + c_{31} f_1(y_3(t-\tau_2)) \\ \dot{x_2}(t) = -\mu_2 x_2(t) + c_{12} f_2(y_1(t-\tau_2)) \\ \qquad\qquad + c_{22} f_2(y_2(t-\tau_2)) + c_{32} f_2(y_3(t-\tau_2)) \\ \dot{y_1}(t) = -\upsilon_1 y_1(t) + d_{11} g_1(x_1(t-\tau_1)) + d_{21} g_1(x_2(t-\tau_1)) \\ \dot{y_2}(t) = -\upsilon_2 y_2(t) + d_{12} g_2(x_1(t-\tau_1)) + d_{22} g_2(x_2(t-\tau_1)) \\ \dot{y_3}(t) = -\upsilon_3 y_3(t) + d_{13} g_3(x_1(t-\tau_1)) + d_{23} g_3(x_2(t-\tau_1)) \end{cases} \tag{1.2}$$

where $\mu_i > 0 (i = 1, 2)$, $\upsilon_j > 0 (j = 1, 2, 3)$, $c_{j1}, c_{j2} (j = 1, 2, 3)$ and $d_{i1}, d_{i2}, d_{i3} (i = 1, 2)$ are real constants. The time delay from the X-layer to another Y-layer is τ_1, while the time delay from the Y-layer back to the X-layer is τ_2. In [3], it has been discussed that the zero solution of system (1.2) loses its stability and Hopf bifurcation occurs when $\tau = \tau_1 + \tau_2$ passes through a critical value τ_0. In the next section, we will determine the direction, stability and the period of the bifurcating periodic solutions.

2. Main results

In this section, we will work out an algorithm for determining the direction, stability and the period of the bifurcating periodic solutions through the normal form theory and center manifold reduction due to [1]. In what follows, it is essential to assume that $f_i, g_j \in$

$C^3\,(i = 1,2;\ j = 1,2,3)$. Without loss of generality, we assume that $\tau = \tau_0 + \mu = (\tau_1^* + \mu) + \tau_2^*$, where τ_0 has been defined in [3], so $\mu = 0$ is the Hopf bifurcation value for system (1.2). Letting $X(t) = (x_1(t), x_2(t), y_1(t), y_2(t), y_3(t))^T$ and $X_t(\theta) = X(t+\theta)$, system (1.2) is transformed into an operator equation of the form:

$$\dot{X}(t) = L_\mu(X_t) + G(\mu, X_t) \tag{2.1}$$

with

$$L_\mu(\phi) = -B\phi(0) + B_1\phi(-\tau_1^* - \mu) + B_2\phi(-\tau_2^*)$$

and

$$G(\mu,\phi) = \begin{bmatrix} \frac{f_1''(0)}{2}(c_{11}\phi_3^2(-\tau_2^*) + c_{21}\phi_4^2(-\tau_2^*) + c_{31}\phi_5^2(-\tau_2^*)) \\ \frac{f_2''(0)}{2}(c_{12}\phi_3^2(-\tau_2^*) + c_{22}\phi_4^2(-\tau_2^*) + c_{32}\phi_5^2(-\tau_2^*)) \\ \frac{g_1''(0)}{2}(d_{11}\phi_1^2(-\tau_1^* - \mu) + d_{21}\phi_2^2(-\tau_1^* - \mu)) \\ \frac{g_2''(0)}{2}(d_{12}\phi_1^2(-\tau_1^* - \mu) + d_{22}\phi_2^2(-\tau_1^* - \mu)) \\ \frac{g_3''(0)}{2}(d_{13}\phi_1^2(-\tau_1^* - \mu) + d_{23}\phi_2^2(-\tau_1^* - \mu)) \end{bmatrix} + \begin{bmatrix} \frac{f_1'''(0)}{3!}(c_{11}\phi_3^3(-\tau_2^*) + c_{21}\phi_4^3(-\tau_2^*) + c_{31}\phi_5^3(-\tau_2^*)) \\ \frac{f_2'''(0)}{3!}(c_{12}\phi_3^3(-\tau_2^*) + c_{22}\phi_4^3(-\tau_2^*) + c_{32}\phi_5^3(-\tau_2^*)) \\ \frac{g_1'''(0)}{3!}(d_{11}\phi_1^3(-\tau_1^* - \mu) + d_{21}\phi_2^3(-\tau_1^* - \mu)) \\ \frac{g_2'''(0)}{3!}(d_{12}\phi_1^3(-\tau_1^* - \mu) + d_{22}\phi_2^3(-\tau_1^* - \mu)) \\ \frac{g_3'''(0)}{3!}(d_{13}\phi_1^3(-\tau_1^* - \mu) + d_{23}\phi_2^3(-\tau_1^* - \mu)) \end{bmatrix} + O(\|\phi\|^4), \tag{2.2}$$

where $\phi(\theta) = (\phi_1(\theta), \phi_2(\theta), \phi_3(\theta), \phi_4(\theta), \phi_5(\theta))^T \in C$.

Now, we can state the following main theorem:

Theorem 2.1. *Suppose that f_i, $g_j \in C^3\,(i = 1,2; j = 1,2,3)$. Then,*

(i) *the direction of the Hopf bifurcation is determined by the sign of ρ_2: if $\rho_2 > 0 (\rho_2 < 0)$, then the Hopf bifurcation is supercritical (subcritical) and the bifurcating periodic solutions exist for $\tau > \tau_0 (\tau < \tau_0)$;*

(ii) *ϑ_2 determines the stability of the bifurcating periodic solutions: the bifurcating periodic solutions are stable (unstable) if $\vartheta_2 < 0 (\vartheta_2 > 0)$;*

(iii) *T_2 determines the period of the bifurcating periodic solutions: the period increases (decreases) if $T_2 > 0 (T_2 < 0)$. Furthermore, the period of the bifurcating periodic solutions of (1.2) can be estimated by $T(\mu) = \frac{2\pi}{\omega_0}(1 + T_2\mu^2 + \dots)$;*

where

$$\rho_2 = -\frac{Re\{c_1(0)\}}{Re\{\lambda'(\tau_0)\}},\ \vartheta_2 = 2Re\{c_1(0)\},$$
$$T_2 = -\frac{1}{\omega_0}[Im\{c_1(0)\} + \rho_2 Im\{\lambda'(\tau_0)\}]$$

and

$$c_1(0) = \frac{i}{2\omega_0}(h_{20}h_{11} - 2\left|h_{11}\right|^2 - \frac{1}{3}\left|h_{02}\right|^2) + \frac{1}{2}h_{21}.$$

Proof. Since the associated characteristic equation of system (2.1) has a pair of simple imaginary roots $\pm i\omega_0$ when $\mu = 0$, there is a neighbourhood of $\mu = 0$ such that for any μ in it there exists a two-dimensional local center manifold C_0 of (2.1) in C, which contains the zero element of C and the orbits of Hopf periodic solutions are also located in C_0. By constructing the local coordinates z and $\bar{z}$ for center manifold C_0, we can describe the center manifold near $\mu = 0$. Hence, we have

$$\dot{z}(t) = i\omega_0 z(t) + h(z, \bar{z}),$$

where

$$h(z, \bar{z}) = h_{20}\frac{z^2}{2} + h_{11}z\bar{z} + h_{02}\frac{\bar{z}^2}{2} + h_{21}\frac{z^2\bar{z}}{2} + \dots. \tag{2.3}$$

By the above results and (2.2), we compute the coefficients in (2.3), and so, the quantities ρ_2, ϑ_2 and T_2 can be computed. By using the center manifold method, the proof is complete. □

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