Fuzzy Systems as a Fusion Framework for Describing Nonlinear Flow in Porous Media

A.-R. NAZEMI
Department of Civil Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

Email: ali_r_nazemi@yahoo.com

M.-R. AKBARZADEH-T
(IEEE Senior Member) Department of Electrical Engineering Ferdowsi University of Mashhad, Mashhad, Iran

Email: akbazar@eece.unm.edu

S. M. HOSSEINI
Department of Civil Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

Email: shossein@ferdowsi.um.ac.ir

Abstract

By increasing the velocity of flow in coarse grain materials, local turbulences are often imposed to the flow. As a result, the flow regime through rockfill structures deviates from linear Darcy law; and nonlinear or non-Darcy flow equations will be applicable. Even though the structures of these nonlinear equations have some physical justifications, they still need empirical studies to estimate parameters of these equations. Hence there is a great deal of uncertainty as an inherent part of the estimation process. In this paper we investigate fuzzy systems paradigm to combine three of the most commonly validated and utilized empirical solutions in the current literature. In this way, the results of the three empirical equations serve as inputs, and the combination framework serve as fusion algorithm. The results show that when learning injected to fuzzy logic based models, the system provides a powerful solution with a strong ability to track reality. Specifically, this paper concludes that ANFIS provide accurate combination framework with greatest performance among the considered conventional alternatives as well as Mamdani structures.

1. Introduction

In recent years, there has been considerable increase in application of rockfill and coarse grain materials for construction of hydraulic structures such as rockfill dams, breakwaters, and gabions [4, 5]. The availability of construction materials, increased knowledge about behavior of rockfill, and short construction duration are the main reasons for this development. The main feature of rockfill structures is that, because of increased local turbulence, flow through them is not accurately described by Darcy linear flow equation, and non-Darcy or nonlinear flow law is required to describe the flow. Furthermore, hydraulic behavior of rockfill depends on many parameters such as: size and size distribution, porosity, orientation, shape and roughness of the grains [4]. Hydraulic behavior resulting from interaction of these variables is hard to quantify and therefore uncertainty will be an inherent element in estimation of hydraulic parameters. A commonly used method to obtain hydraulic parameters is the use of empirical relations, based on the physical features of the media. Although the research in this area has been extensive, there is no general agreement on one specific equation. In other words, the empirical equations are biased and therefore produce underestimated and/or overestimated results [5]. Additionally, empirical studies based on laboratory conditions do not encompass all situations and therefore do not always reflect reality.

Considering the ever-increasing computational abilities and introduction of new methods of deduction, combination and modeling, it is now reasonable to use different equations simultaneously and then combine them together [16]. Averaging is the simplest method for combination. An improved but more complex method is weighted averaging or linear regression. However, these methods may produce weak results when interrelations among simultaneous equations are nonlinear. Hence, some nonlinear methods may be deemed superior. But, traditional nonlinear regression often leads to a time consuming and complex mathematical programming. These methods are also very sensitive, and a little change in parameters may change the regression line violently [7]. As an alternative to above conventional combination approaches, soft computing and specifically fuzzy logic-based methods are proposed to handle uncertainty and ambiguity in natural systems through the past decades. The core of soft computing is neural computation and fuzzy logic [3, 7]. On one hand neural networks are *low-level* (numeric) distributed processing units that have capability to learn and generalize nonlinear relations. On the other hand, fuzzy logic provides a *high-level* (linguistic) robust and accurate framework to model conceptual knowledge. Neuro-fuzzy systems are a combined framework of these two paradigms taking advantage of their strength for a more powerful approach. There is an extensive literature in soft computing, and everyday it finds new applications in solving real engineering problems.

In this study, a classic fuzzy system i.e. Mamdani and more recently developed neuro fuzzy system ANFIS are examined. Inputs to the above systems are the result of three most accurate empirical equations, and we hope to build a decision fusion algorithm that will track the observed output. In the next section, previous studies in the area of non-Darcy flow equations are summarized and three most accurate equations (as proposed by McCorquodale, Stephenson, and Adel) are selected using previous study of Hosseini [5]. In section 3, the data set of our case study will be introduced and the performance of selected equations will be evaluated. In section 4, the motivations and applications of combination methods are discussed and the combination model for this problem is presented. Then in section 5, Mamdani fuzzy system and ANFIS as a combination framework are introduced and different architecture are trained, checked and tested with the laboratory data. In section 6, the results are presented and finally, in section 7, conclusions are derived.

2. Non-Darcy flow studies

In general, two distinct approaches for describing nonlinear flow through coarse porous media can be found in the literature. The first approach was proposed by Forchheimer who introduced the following one-dimensional quadratic equation as the constitutive relationship for nonlinear flow [4, 5, 8].

$$i = aV + bV^2 \tag{1}$$

Where i is hydraulic gradient, V is the bulk velocity, and a and b are media and fluid constants. Another commonly used approach is the Missbach equation, which assumed the following exponential relation between hydraulic gradient and bulk velocity [4, 5, 10].

$$i = lV^{\lambda} \tag{2}$$

Where l and λ are constants which depend on media and fluid properties. λ is a variable between 1 and 2, which changes from case to case. Although above equations have some theoretical justification, constants (a and b in the Forchheimer equation, l and λ in the Missbach equation) relate to media characteristics and are generally estimated by empirical equations resulting from experimental studies. Many regulations of empirical equations can be found in the literature, based on the work of researchers such as in McCorquodale et al. [11], Stephenson [14], and Martins [10]. Hansen et al. [4] present a good review of different nonlinear empirical equations, which are widely used for estimating hydraulic properties in non-Darcy flow conditions. Also, there are several other studies that aim to evaluate the introduced nonlinear flow equations such as Joy [8], Hansen et al. [4] and Hosseini [5]. In general, it can be concluded from these studies that Forchheimer-based equations are superior and more accurate than Missbach type equations. Hosseini [5] found that the most accurate models are reported by Adel, McCorquodale, and Stephenson equations. The formulation of these equations can be found in many references such as [4, 5]. In short, these empirical equations have different mathematical structures, and include different media characteristics such as porosity, characteristic size (d_{10} or d_m), and parameters related to shape and roughness of the grains.

3. Data set used in this study

The data set used for this study was produced by Joy [8, 9]. He constructed a simple experimental device to collect a consistent set of i. vs. V data for 23 different materials. The device consisted of a vertical cylinder, 750mm long and 152mm in diameter, containing the media. Head losses were determined using 5 piezometer taps on the cylinder and discharge were determined using either an orifice for large flows or by timed weighting for smaller flows. 23 samples of different coarse media were tested. Mean (d_{50}) material sizes ranged from 3 to 31 mm while hydrodynamic conditions were all outside the laminar range with Reynolds numbers in the range of 50 to 600. The media was homogenous; the flow was steady and hydraulic gradient was from 0.014 to 1.5. Totally 483 bulk velocity-hydraulic gradient data were observed for all 23 coarse materials. Physical properties related to three selected empirical equations were extracted from references [8, 9], by following the recommendations made by the developer of the equations to calculate or estimate their physical properties. Therefore this data set can be used to compare simulated hydraulic gradients (resulting from any empirical equation or combination method) with the observed values.

These data are first used to investigate the performance of the empirical equations used in this study. To conduct this, physical properties associated with each empirical equation were applied to the equations to find the hydraulic gradients for all velocity values corresponding to permeameter tests. In Figure (1), plots (a) to (c) show scatter plot of i(simulated) vs. i(observed) curve for McCorquodale, Stephenson, and Adel equations, respectively. It can be seen that these plots scatter widely especially when the velocity increases. Hence there is a great deal of uncertainty in the mechanism of equations in representing actual hydraulic gradient.

4. Combinations methods: motivations and applications

From previous section, it is realized that no single equation provides an exact result. On one hand these equations may have some subjective features that should be determined from engineering judgment and on the other hand, the process has different sources of uncertainty that are not fully reflected in empirical equations. It should be considered that all empirical equations have been developed based on laboratory conditions so the real process in nature may have an obvious deviation from these equations as well. Assuming that each single empirical equation can best describe only one or more particular situations, conditions or relations, it can be expected that the hydraulic gradient estimates, which are obtained by combining the results from a number of different equations together through some appropriate weighting procedures are more comprehensive and accurate in representing the relationship between hydraulic gradient and bulk velocity than any single equation or relation [16]. Mathematically, if we have p different empirical equations for estimating the hydraulic gradient for a certain velocity, this combination process is generally expressed as [16],

$$\hat{i}_c = F(\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_p) \tag{3}$$

Where $\hat{i}_1, \hat{i}_2, \ldots, \hat{i}_p$ are the hydraulic gradients of p different equations, respectively, for a specified bulk velocity, $\hat{i}_c$ is fusion of their results, and $F(\cdot)$ is the decision fusion algorithm. The most obvious method for combining the results of different empirical equations is simple averaging. Another intuitive method, but more complex, is finding a linear relationship (Equation (4)) among the variables taking the results of empirical equations as independent variables and observed hydraulic gradients as dependent ones.

$$i_{observed} = a i_{Adel} + b i_{Stephenson} + c i_{McCorquodale} \tag{4}$$

In Equation (4), the best estimates of a, b, and c are obtained by applying multiple linear regression procedure. Figure (1), plots (d) and (e) show the results of simple averaging method (SAM) and multiple linear regression equation (MLR). As it can be realized, less scatter is observed in the results of multiple linear regression equation. However, the funnel-shaped trend is still dominant. By examining the results of different empirical equations and applied combination methods, the failure of these conventional methods in capturing the nonlinearity, which are inherent in the system, is understood. Therefore, the application of a decision fusion algorithm, based on fuzzy system that can handle this nonlinearity, is plausible.

5. Fuzzy systems as a fusion framework

The fuzzy systems can be easily used as a fusion framework [16]. In this configuration, output of each empirical equation is an input variable to the fuzzy fusion system. A fusion Mamdani fuzzy rule base consists of a set of rules in the following form:

If i Adel is A_k *and i Stephenson is* B_k *and i McCorquodale is* C_k *then i is* D_l (5)

Where A_k, B_k, and C_k are kth membership functions for Adel, Stephenson, and McCorquodale equations respectively. D_l is lth consequent membership function. The degree of membership functions is a positive real number in the interval [0,1]. A membership function assigns a degree of membership to an element and can be any convex shape [15]. In summary, the functionality of Mamdani fuzzy inference systems can be shown as follows:

$$\mu_{B'}(y) = \underset{l=1}{\overset{M}{Snorm}}[\sup_{X \in U}(\mu_{A'}(X) \times \underset{i=1}{\overset{n}{Tnorm}}(\mu_{A_i^l}(x_i)\mu_{B^l}(y)))] \tag{6}$$

For applying a fuzzy system as fusion algorithm, four triangular membership functions are assumed for each input. For rule generation, weighted counting algorithm [1, 13] is applied. If we choose Mamdani fuzzy system as the type of fuzzy inference model, algebraic product as AND (Tnorm) operator, algebraic sum as OR (Snorm) operator, Min as implication operator, Max as aggregation operator and center of average as the defuzzification method, then the results for training, checking and testing phases are as in Figure (2). For this purpose data is first randomly divided into three distinct sets. Training data set contains 350 data pairs, checking data set contains 50 data pairs, and testing data set consists of 82 data pairs. Each data pair contains results of Adel, Stephenson and McCorquodale equations to a specified velocity as inputs and real hydraulic gradient as output. Then data quantities are divided by their maximum value in the training data set. Hence a normalized training data set is created; however, checking and testing data sets may not be normal. Different simulations reveal that several other combinations of Snorms, Tnorms, and implication operators provided inferior performance. MATLAB® fuzzy logic toolbox was used for simulation [6].

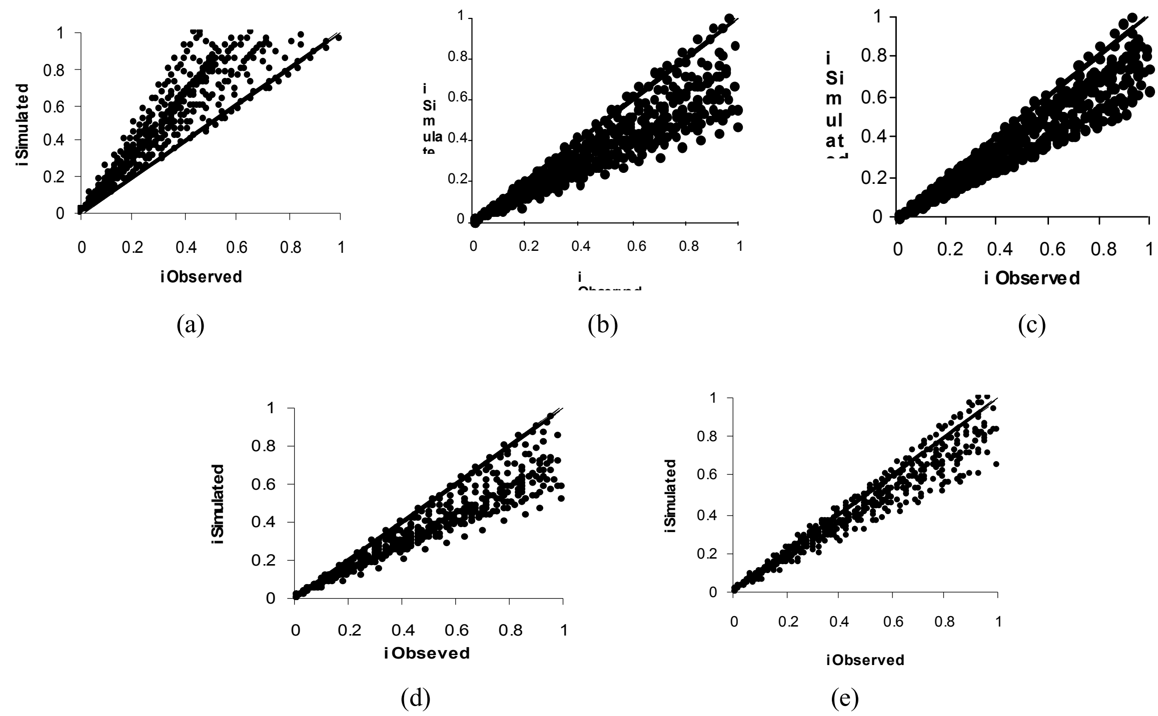


Figure (1): i(simulated) vs. i(observed) curve for selected relations; (a) McCorquodale, (b) Stephenson, (c) Adel equations, (d) simple averaging, (e) multiple linear regression

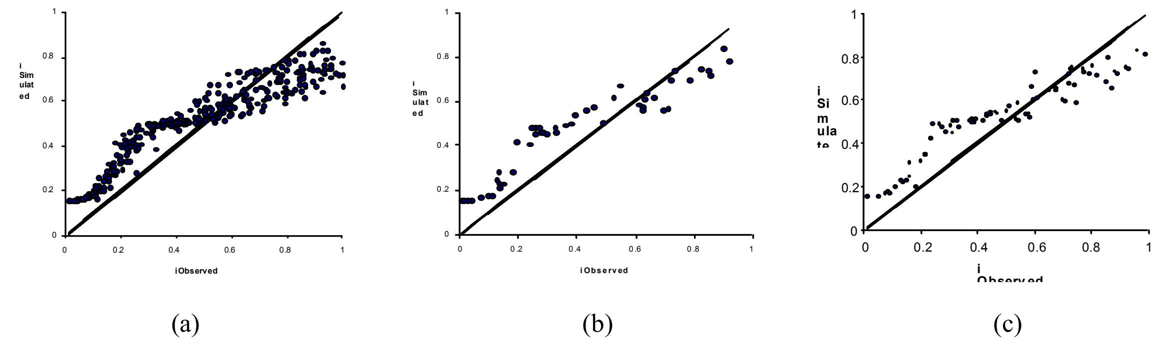


Figure (2): Combined hydraulic gradient obtained by Mamdani fuzzy system vs. real hydraulic gradient; (a) training phase, (b) checking phase, (c) testing phase

Hybrid combination of fuzzy logic and neural systems paradigm may provide a unique framework where the strength of each paradigm compensates for the weakness of the other. Fuzzy logic is known for its ability to model human knowledge qualitatively, whereas neural networks can be considered as physical and quantitative model of human brain. Combining these two differing views of human mind in a unifying framework allows for a stronger approach to modeling [7]. One of the most popular and powerful architectures is Adaptive Neuro-Fuzzy Inference System (ANFIS) [7] in which a Takagi-Sugeno-Kang (TSK) [3, 7, 15] fuzzy system is represented in a special five-layer feedforward network architecture. For this fusion process, rules are implemented in the following form:

If i Adel is $A_{1,r}$ and i Stephenson is $A_{2,r}$ and i McCorquodale is $A_{3,r}$ then $i_{fusion_r} = \alpha_0 + \sum_{k=1}^{3} \alpha_k i_{sim,k}$

(7)

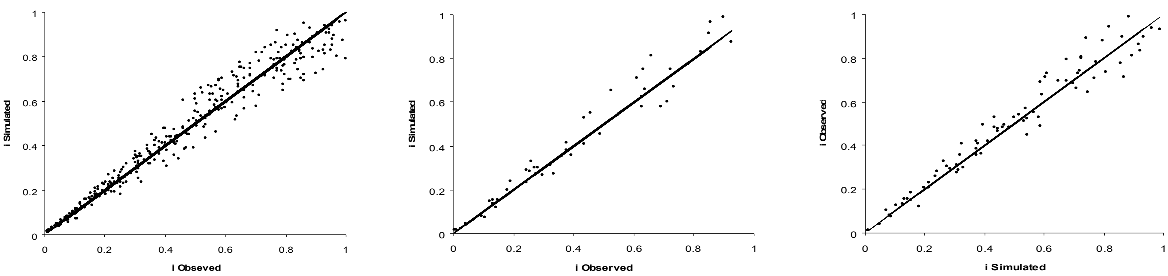
In Equation (7), (r=1,2,…R) is the rth rule, k is index of each empirical equation (Adel, Stephenson and McCorquodale, respectively), $A_{k,r}$ is the membership function of kth empirical equation representing linguistic descriptions of inputs in rth rule, $i_{sim,k}$ is the numerical value of kth empirical equation, and α_0 and $\alpha_k\ (k = 1,2,3)$ are equation constants. The rule-base must be known in advance and ANFIS adjusts the membership functions of the antecedents and the consequent parameters. A hybrid of Least Square Estimator (LSE) and backpropagation is used for the learning of ANFIS. Backpropagation is used to learn the antecedent parameters and LSE is used to determine the coefficients of rules consequents [6, 7]. However, the learned outcome of ANFIS may be difficult to interpret. ANFIS is therefore more suitable for applications where interpretation is not as important as performance such as our problem. MATLAB® fuzzy logic toolbox was used for simulation. In ANFIS system the type of membership function must be identified. Also two different methods for rule generation, i.e., sub-clustering and grid partitioning [6, 7] can be used. It was seen that ANFIS system with 3 Inputs-1 Output, 4 Gaussian membership function for each variable and 4 linear consequence functions obtained by sub clustering with 100 epochs hybrid training would be the best neuro-fuzzy model. Figure (9) shows the structure of ANFIS system with 3I/1O, 4 membership functions for each input variable obtained by grid partitioning and sub-clustering respectively. Performance of rule bases obtained by grid partitioning and sub-clustering are shown in following figures.

6. Results

In Table (1) the best fuzzy fusion algorithms in Mamdani and ANFIS structures are compared with two well-known conventional combination models, i.e. simple averaging, multiple linear regression, as well as three selected empirical non-Darcy equations. Four performance indices are selected for evaluation of these models. These are sum of square of errors, mean of errors, variance of errors and correlation between simulated and observed values. These parameters are compared in training, checking and testing phases. Generally, training indices represent the learning capability of different models, checking indices estimate generalization capability and testing ones show prediction ability. It can be seen that ANFIS system obtained by sub-clustering is the best fusion algorithm, among other illustrated models, for describing the nonlinear flow in porous media.

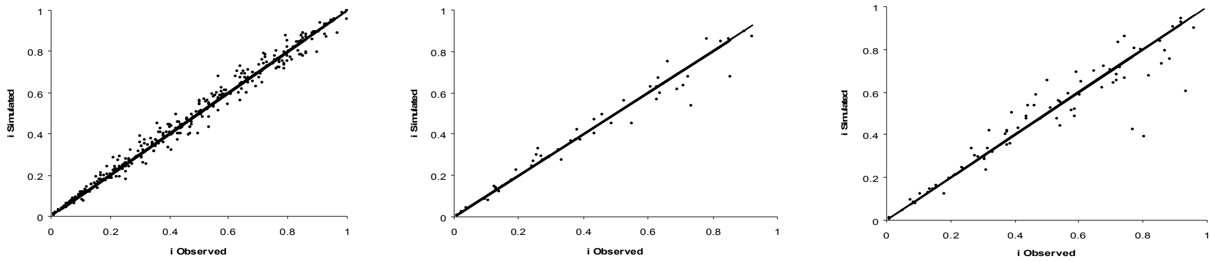
7. Conclusion

The most commonly used method for estimating hydraulic parameters for flow through coarse porous media is empirical-based equations. However, these equations do not fully reflect the flow behavior in coarse porous media. In this investigation, the results of three selected empirical equations, i.e., McCorquodale, Stephenson, and Adel, are fused by fuzzy system in order to find a better estimation. The results show significant improvement of proposed fusion algorithm based on ANFIS as compared with any other illustrated combination method or use of standard equation individually. Future directions of this research includes application of other soft computing-based methodologies for fusion and/or creating a direct mapping from actual physical properties and measurement of the system to a hydraulic gradient.



(a) (b) (c)

Figure (3): Combined hydraulic gradient resulting from ANFIS system obtained by sub clustering vs. real hydraulic gradient; (a) training phase, (b) checking phase, (c) testing phase



(a) (b) (c)

Figure (4): Combined hydraulic gradient resulting from ANFIS system obtained by grid partitioning vs. real hydraulic gradient; (a) training phase, (b) checking phase, (c) testing phase

Table (1): Comparison between different combination procedures and empirical equations

		Training phase (number of data: 350)				Checking phase (number of data: 50)				Testing phase (number of data: 82)			
No.	Description	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
1	Mamdani	5.5794	0.0384	0.0145	0.94	0.82	-0.0710	0.0120	0.93	1.16	-0.0400	0.0130	0.95
2	ANFIS	1.1108	0.0000	0.0032	0.98	0.16	-0.1390	0.0030	0.98	0.28	-0.0090	0.0030	0.98
3	SAM	9.1897	0.1243	0.0100	0.94	0.85	0.1011	0.0070	0.97	1.90	0.1220	0.0080	0.95
4	MLR	3.1980	0.0618	0.0050	0.97	0.25	0.0440	0.0030	0.98	0.57	0.0540	0.0040	0.97
5	Adel	5.1916	0.1061	0.0086	0.95	0.72	0.0820	0.0080	0.94	1.39	0.0956	0.0080	0.95
6	Stephenson	6.9427	0.1259	0.0135	0.93	0.93	0.1042	0.0080	0.96	2.21	0.1282	0.0110	0.93
7	McCorquad	10.232	0.1407	0.0164	0.91	1.23	0.1170	0.0110	0.93	2.73	0.1423	0.0130	0.91

(1) Sum of Square of Errors (2) Mean of Errors (3) Variance of Errors
(4) Correlation Coefficient

8. References

1. Bardossy, A., Duckstein, L.,: *Fuzzy rule-based modeling with applications to geophysical, biological & engineering systems*. CRC Press, Boca Raton, 1995.
2. Bardossy, A., *The use of fuzzy rules for the description of elements of the hydrological cycle*, Ecological modeling, VOL. 85, 1996, pp. 59-65.
3. Gorzalczany, M. B., *Computational Intelligence Systems and Applications,* Physica-Verlag Company, Heidelberg, Germany, 2002
4. Hansen, D., Garga, V. K., Townsend, D. R., *Selection and application of a one-dimensional non-Darcy flow equation for two-dimensional flow through rockfill embankments*, Can. Geotechnical Journal, VOL. 32, 1995, pp. 223-232.
5. Hosseini, S.M., *Statistical evaluation of the empirical equations that estimate hydraulic parameters for flow through rockfill*, in Stochastic Hydraulics 2000, Wang & Hu (eds.), Balkema, Rotterdam, The Netherlands, 2000, pp. 547-552.
6. Jang, J., S., R., Gulley, N., *Fuzzy logic toolbox for use with MATLAB®,* MathWorks, Inc., 1995.
7. Jang, J.-S. R., Sun, C., T., Mizutani, E., *Neuro-fuzzy and soft computing: a computational approach to learning and machine intelligence*, Prentice Hall, Upper Saddle River, NJ, 1997.
8. Joy, D. M., *Nonlinear flow in a coarse porous media*, Annu. Conf. of the CSCE, Vancouver, British Columbia, Canada, 1991 a, pp. 106-115.
9. Joy, D. M., *Nonlinear porous media flow: determination of parameters for a coarse granular media*, 13th Can. Cong. Appl. Mech., Winnipeg, Canada, 1991 b.
10. Martins, R., *Turbulant seepage flow through rockfill structures*, Journal of Water Power & Dam Construction, March 1990, pp. 41-45.
11. McCorquodale, J. A., Hannoura, A., Nasser, M. S., *Hydraulic conductivity of rockfill*, Journal of Hydraulic Research, VOL. 16, NO. 2, 1978, pp. 123-137.
12. Shamseldin, A. Y., O' Connor, K. M., Liang, G. C., *Methods for combining the outputs of different rainfall-runoff models*, Journal of Hydrology, VOL. 197, 1997, pp. 203-229.
13. Sherestha, B. P., Duckstein, L., *Fuzzy rule-based modeling of reservoir operation*, Journal of water resources planning and management, VOL. 122, NO. 4, 1996, pp. 262-269.
14. Stephenson, D., *Rockfill in Hydraulic Engineering, Developments in Geotechnical Engineering 27*, Elsevier Scientific Publishing Co., Amsterdam, The Netherlands, 1979.
15. Wang X. J., *A course in fuzzy systems and control*, Prentice Hall Inc., Upper Saddle River, NJ, 1997.
16. Xiong, L., Shamseldin A. Y., O' Connor, K. M., *A non linear combination of the forecasts of rainfall-runoff models by the first order Takagi-Sugeno fuzzy system*, Journal of Hydrology, VOL. 254, 2001, pp. 196-217.