Astrophys Space Sci (2014) 352:185–191
DOI 10.1007/s10509-014-1892-0

ORIGINAL ARTICLE

Time evolution of nonplanar electron acoustic shock waves in a plasma with superthermal electrons

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Received: 12 November 2013 / Accepted: 17 March 2014 / Published online: 25 March 2014

Abstract The propagation of cylindrical and spherical electron acoustic (EA) shock waves in unmagnetized plasmas consisting of cold fluid electrons, hot electrons obeying a superthermal distribution and stationary ions, has been investigated. The standard reductive perturbation method (RPM) has been employed to derive the cylindrical/spherical Korteweg-de-Vries-Burger (KdVB) equation which governs the dynamics of the EA shock structures. The effects of nonplanar geometry, plasma kinematic viscosity and electron suprathermality on the temporal evolution of the cylindrical and spherical EA shock waves are numerically examined.

Keywords Electron acoustic waves · Nonplanar shock waves · Superthermal electrons · KdVB equation

1 Introduction

Electron acoustic waves (EAWs) are one of the basic wave processes in plasmas and they have been studied for several decades. EAWs can be created in a two-temperature (cold and hot) electron plasma. Multispecies models were originally used for laser-plasma interaction but there are several similar situations. The evidence of two populations of electrons in laboratory and space plasmas has already been reported. The observations (Parks et al. 1984; Onsager et al. 1993) in the plasma sheet boundary layer have shown that there exist two types of electrons, namely background plasma electrons and cold electron beams having energies of the order of few eV to few hundreds of eV. Intense broadband electrostatic noise is commonly observed in the plasma sheet boundary layer of the Earth's magnetosphere (Gurnett et al. 1976). Matsumoto et al. (1994) have shown that broadband electrostatic noise emissions in the plasma sheet boundary layer are not continuous noise but consist of electrostatic impulsive solitary waves. Polar cap boundary layer (Tsurutani et al. 1998), the magnetosheath (Pickett et al. 2003), the bow shock (Bale et al. 1998), and strong currents associated with the auroral acceleration region (Ergun et al. 1998) are other examples of plasmas consisting of two and three similar particle population. The EAWs are typically high frequency waves in comparison with the ion plasma frequency. On the EAW time scale, the ions are generally assumed stationary forming a neutralizing background. The phase speed of the EAW lies between the cold and hot electron thermal velocities, so that the Landau damping effects are ignored for the consistency of the fluid theory. To provide new insight into previously published papers, we propose here to examine the time evolution of nonplanar electron acoustic shock waves in a plasma with superthermal electrons. Watanabe and Taniuti (1977) used a linear electrostatic Vlasov dispersion equation to show that electron acoustic waves can be destabilized in such plasma. Later on, Yu and Shukla (1983) and Gary and Tokar (1985) obtained a dispersion relation for EAWs in a two (electron-

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ion) and three (two-temperature electrons and ions) component plasmas, respectively. The electron-acoustic solitary wave (EASW) is a localized nonlinear wave which arises due to a delicate balance between nonlinearity and dispersion. EASWs have been studied both theoretically (Schamel 2000) and numerically (Valentini et al. 2006). These waves have been observed in experiments with pure electron plasmas (Kabantsev et al. 2006) and in laser-produced plasmas (Sircombe et al. 2006) and have been studied in related subjects using numerical simulations (Ghizzo et al. 2006). The propagation of EASWs in a plasma system has been studied by several investigators in unmagnetized two electron plasmas (Mace et al. 1991; Dubouloz et al. 1991; Chatterjee and Roychoudhury 1995; Berthomier et al. 2000; Mamun and Shukla 2002; Clarmann et al. 2002; Berthomier et al. 2000) as well as in magnetized plasmas (Mace and Hellberg 2001; Berthomier et al. 2003; Shukla et al. 2004). Energetic electron distributions are observed in different regions of the magnetosphere. Gill et al. (2006) studied small amplitude EASWs in a plasma with nonthermal electrons. Recently, El-Shewy (2007) studied the higher order solution of EASWs with nonthermal electrons. However, numerous observations of space plasmas (Vasyliunas 1968; Leubner 1982; Armstrong et al. 1983) are often characterized by a particle distribution function with high energy tail and which deviates from the well-known Maxwellian. Vasyliunas (1968), was the first, who proposed an empirical functional form for describing the distribution of energy over the whole spectrum of the high-energy power-law tail, and is widely known as the kappa distribution. In order to provide the missing link between the Tsallis nonextensive q-statistics to the family of the phenomenologically introduced κ-distributions favored in space and astrophysical plasma modeling, we stress that fundamental and generalized physics is provided within the framework of an entropy modification consistent with q-nonextensive statistics, and we perform the transformation $\kappa = 1/(q-1)$ (Beck 2000) or $q = 1 + 1/k$ (Leubner 2002). The family of κ-distributions are obtained from the positive definite part $1/2 \le k \le \infty$ corresponding to $-1 \le q \le 1$ of the general statistical formalism where in analogy the spectral index κ is a measure of the degree of nonextensivity. In contrast to using Tsallis Statistical Mechanics, attempting to theoretically derive a κ-distribution from the standard BG Statistical Mechanics is highly problematic because the kinetically defined temperature T_k (Livadiotis and McComas 2009) in κ-distribution is not properly defined, and physical temperature T_q (Livadiotis and McComas 2009) defined in q-nonextensive distribution constitutes the appropriate definition of temperature over kinetic temperature T_k in κ-distribution. In fact, the Tsallis q-distribution provides a set of proven tools, including a grounded definition of temperature for systems in stationary states out of thermodynamic equilibrium. κ-distributed particle perceived a lack of theoretical justification, therefore, distribution very close to κ-distribution, particularly the nonextensive-q distribution, is a consequence of the generalized entropy favored by nonextensive statistics. It is proposed that this slightly modified functional form, qualitatively similar to the traditional κ-distribution, be used in fitting particle spectra in the future. Nonextensive q-distribution is successfully applied to demonstrate the solar neutrino problem (Kaniadakis et al. 1996), peculiar velocity distributions of galaxies (Lavagno et al. 1998), fractal like space-times, etc. On the other hand, kappa-distributions are favored in any kind of space plasma modeling (Mendis and Rosenberg 1994) among others, where a reasonable physical background was not apparent.

Superthermal particles may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave-particle interaction. Plasmas with an excess of superthermal electrons are generally characterized by a long tail in the high energy region. To model such space plasmas, generalized Lorentzian or kappa-distribution has been found to be appropriate rather than the Maxwellian distribution (Hasegawa et al. 1985; Thorne and Summers 1991; Summers and Thorne 1991, 1994; Mace and Hellberg 1995). Kappa-like distributions have been used by several authors (Hellberg and Mace 2002; Podesta 2005; Abbasi and Hakimi Pajouh 2007; Baluku and Hellberg 2008; Hellberg et al. 2009; Sultana et al. 2010; Baluku et al. 2010) in studying the effect of Landau damping on various plasma modes. "Superthermal" plasma behavior was observed in various experimental plasma contexts, such as laser matter interactions or plasma turbulence (Magni et al. 2005). At very large values of the spectral index k, the Maxwellian is recovered, whereas for low values of k, the distribution function exhibits a "hard" spectrum with a strong non-Maxwellian tail having a power-law form at high velocities. Direct measurement of *kappa*-like-distributions in association with electrostatic solitary structures is not available. However, studies of electron flux spectra in the auroral region where solitary waves are often observed have shown that *kappa* distributions rather than Maxwellian ones give a better fit to the observed distribution (Olsson and Janhunen 1998). Numerous observations of space plasmas (Feldman et al. 1973; Formisano et al. 1973; Scudder et al. 1981; Marsch et al. 1982) indicate clearly the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. The latter may arise due to the effect of external forces acting on the natural space environment plasmas or to the wave-particle interaction which ultimately leads to kappa-like distributions. Pakzad (2012) and Javidan and Pakzad (2012) investigated EA solitary waves in a plasmas by considering nonplanar geometry and noextensive distribution of electrons. Recently, Sahu and Tribeche (2012) addressed the problem of nonplanar EA shock waves in a nonextensive plasma. However and to the best of our knowledge, the propagation of nonplanar EA shock waves in a plasma consisting of cold fluid electrons, hot superthermal electrons and stationary ions have never been addressed in the plasma literature. In this paper, we try to show how the EA shock waves in cylindrical and spherical geometries differ qualitatively from previous analysis in planar geometry and how hot superthermal electrons affect on them.

The manuscript is organized in the following fashion. In next section, we present our theoretical model and carry out a weakly nonlinear analysis to derive a cylindrical/spherical KdV equation. Our results are presented and discussed in Sect. 3. Our findings are summarized in Sect. 4.

2 Basic equations and derivation of the KdV-Bergers equation

We consider a homogeneous, unmagnetized plasma consisting of a cold electron fluid, hot electrons obeying a superthermal distribution and stationary ions. In two temperature (cold and hot) electron plasmas, electron acoustic waves can be excited due to the conservation of equilibrium charge density $n_{e0h} + n_{e0c} = n_{i0}$. It is basically an acoustic (electrostatic) wave in which the inertia is provided by the cold electrons and the restoring force comes from the pressure of the hot electrons. The ions are stationary and provide only the a charge neutralizing background. This means that the ion dynamics does not influence the electron acoustic waves because the EA wave frequency is much larger than the ion plasma frequency. The nonlinear dynamics of the electron acoustic solitary waves is governed by the continuity and motion equations for cold electrons, along with the Poisson's equation

$$\begin{aligned}
&\frac{\partial n_c}{\partial t} + \frac{1}{r^m}\frac{\partial}{\partial r}(r^m n_c u_c) = 0 \\
&\frac{\partial u_c}{\partial t} + u_c\frac{\partial u_c}{\partial r} = \alpha\frac{\partial \varphi}{\partial r} + \eta\left(\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial u}{\partial r}\right) - \frac{mu}{r^2}\right) \\
&\frac{1}{r^m}\frac{\partial}{\partial r}\left(r^m\frac{\partial \phi}{\partial r}\right) = \frac{1}{\alpha}n_c + n_h - \left(1 + \frac{1}{\alpha}\right)
\end{aligned} \tag{1}$$

where $m = 0$, describes planar one-dimensional geometry and $m = 1, 2$ present cylindrical and spherical geometries, respectively. In the above equations, $n_c(n_h)$ is the cold (hot) electron number density normalized by its equilibrium value $n_{c0}(n_{h0})$, u_c is the cold electron fluid velocity normalized by $C_e = (k_B T_h/\alpha m_e)^{1/2}$, ϕ is the electrostatic wave potential normalized by $k_B T_h/e$, k_B is the Boltzmann's constant, e and m_e are the charge and mass of the electron, respectively, and $\alpha = n_{h0}/n_{c0}$. The time and space variables are in units of the cold electron plasma period ω_{pc}^{-1} and the hot electron Debye radius λ_{Dh}, respectively. n_h is the superthermal hot electron density and it is given by (Younsi and Tribeche 2010)

$$n_h = \left(1 - \frac{\phi}{\kappa - 1/2}\right)^{-\kappa - \frac{1}{2}} \tag{2}$$

The parameter κ shapes predominantly the superthermal tail of the distribution (Tribeche and Boubakour 2009) and the normalization is provided for any value of the spectral index $\kappa > 1/2$ (Boubakour et al. 2009). In the limit $\kappa \to \infty$, (2) reduces to the well known Maxwell-Boltzmann electron density. Low values of k represent distributions with a relatively large component of particles with speed greater than the thermal speed ("superthermal particles") and an associated reduction in "thermal" particles, as one observes in a "hard" spectrum. Such a very hard spectrum, with an extreme accelerated superthermal component, may be found near very strong shocks associated with Fermi acceleration (Mace and Hellberg 1995).

Let now study small but infinite amplitude EA waves in plasmas with superthermal electrons by using the reductive perturbation method. Firstly, we introduce the stretched coordinates as, $\tau = \varepsilon^{\frac{3}{2}}t$, $\xi = -\varepsilon^{\frac{1}{2}}(r + \lambda t)$, $\eta = \varepsilon^{\frac{1}{2}}\eta_0$ where ε is a small dimensionless expansion parameter and λ is the wave speed normalized by C_e. Secondly, dependent variables are expanded as follows,

$$\begin{bmatrix}
n_c = 1 + \varepsilon n_{1c} + \varepsilon^2 n_{2c} + \cdots \\
u_c = \varepsilon u_{1c} + \varepsilon^2 u_{2c} + \cdots \\
\phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots
\end{bmatrix} \tag{3}$$

Substituting (3) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lowest order of ε

$$n_{1c} = \frac{-\alpha\phi_1}{\lambda^2}, \qquad u_{1c} = \frac{-\alpha\phi_1}{\lambda}, \qquad \frac{1}{\lambda^2} = \frac{2\kappa + 1}{2\kappa - 1} \tag{4}$$

To the next higher order in ε, we obtain the following set of equations

$$\left|\begin{aligned}
&\frac{\partial n_{1c}}{\partial \tau} - \lambda\frac{\partial n_{2c}}{\partial \xi} + \frac{\partial}{\partial \xi}(u_{2c} + n_{1c}u_{1c}) + \frac{mu_{1c}}{\lambda\tau} = 0 \\
&\frac{\partial u_{1c}}{\partial \tau} - \lambda\frac{\partial u_{2c}}{\partial \xi} + u_{1c}\frac{\partial u_{1c}}{\partial \xi} = \alpha\frac{\partial \phi_2}{\partial \xi} + \eta_0\frac{\partial^2 u_{1c}}{\partial \xi^2} \\
&\quad\times\frac{\partial^2 \phi_1}{\partial \xi^2} - \frac{n_{2c}}{\alpha} \\
&\quad - \left[\frac{2\kappa + 1}{2\kappa - 1}\phi_2 + \frac{(2\kappa + 1)(2\kappa + 3)}{(2\kappa - 1)^2}\phi_1^2\right] = 0
\end{aligned}\right. \tag{5}$$

Finally, Eqs. (4) and (5) yield

$$\frac{\partial \phi_1}{\partial \tau} + \frac{m}{2\tau}\phi_1 + \mathrm{A}\phi_1\frac{\partial \phi_1}{\partial \xi} + \mathrm{B}\frac{\partial^3 \phi_1}{\partial \xi^3} - C\frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \tag{6}$$

where

$$A = -\left(\frac{3\alpha}{2\lambda} + \frac{2\kappa+3}{2\kappa-1}\lambda\right), \quad B = \frac{\lambda^3}{2}, \quad C = \frac{\eta_0}{2} \tag{7}$$

with $\frac{1}{\lambda^2} = \frac{2\kappa+1}{2\kappa-1}$.

Equation (6) is the cylindrical/spherical KdV-Burger equation describing the nonlinear propagation of the electron acoustic solitary waves in plasmas consisting of superthermal hot electrons and stationary ions.

3 Numerical results and discussion

There is not known exact analytical solution for the modified KdV-Burger equation (6) when the geometrical effect is taken into account ($m \neq 0$). Instead, we have to solve Eq. (6) numerically. The effects of superthermal electrons as well as other plasma parameters on the propagation of electron acoustic shock waves have been investigated using numerical simulations. During the numerical process, the equation is advanced in time using the standard fourth-order Runge-Kutta method (Press et al. 1992) with a time step of 10^{-4}. The spatial derivatives were expanded with centered finite difference approximations with a grid spacing of 0.1 (Maxon and Viecelli 1974a, 1974b). At large values of $|\tau|$ (e.g., $\tau = -10$), the spherical and cylindrical waves are similar to the one-dimensional waves in a flat geometry. In this situation, the term $\frac{m}{2\tau}\phi_1$ is no longer dominant and we obtain the usual planar KdV-Burger equation. This equation has the following shock wave

$$\begin{aligned}\phi_1(\xi,\tau) = {} & a_0 + a_1 \tanh\{\alpha(\xi - V\tau)\} \\ & + a_2 \tanh^2\{\alpha(\xi - V\tau)\}\end{aligned} \tag{8}$$

where

$$a_0 = \frac{1}{A}\left(V + 12B\alpha^2\right), \quad a_1 = -\frac{12C\alpha}{5A}, \quad a_2 = -\frac{12B\alpha^2}{A},$$
$$\alpha = \pm\frac{C}{10B} \tag{9}$$

Solution (8) can be reasonably used as an initial condition for the numerical integration of (6) starting from large values of $|\tau|$. However, as the value of $|\tau|$ decreases, the term $\frac{m}{2\tau}\phi_1$ becomes more and more significant until the spherical and cylindrical solutions become different from the one-dimensional solution of the KdV-Burger equation.

Figures 1 and 2 present the time evolution of the shock wave profiles in cylindrical and spherical geometries, respectively. These figures show that the effect of geometry becomes more pronounced for smaller values of $|\tau|$. This is attributed to the fact that the extra term $\frac{m}{2\tau}\phi_1$ becomes dominant for small values of $|\tau|$. Figure 1 shows that the shock amplitude increases as time elapses. A sort of oscillation appears in the knee of the shock profile during the time evolution. On the other hand, the steepness of the profile increases as the value of $|\tau|$ decreases. Figure 2 presents the time evolution of the shock wave profile in a spherical geometry. The situation is similar to what we have already seen in the case of the cylindrical geometry. But the increasing rate, oscillation amplitude and steepness of the shock wave profile are larger in spherical geometry in comparison with cylindrical one.

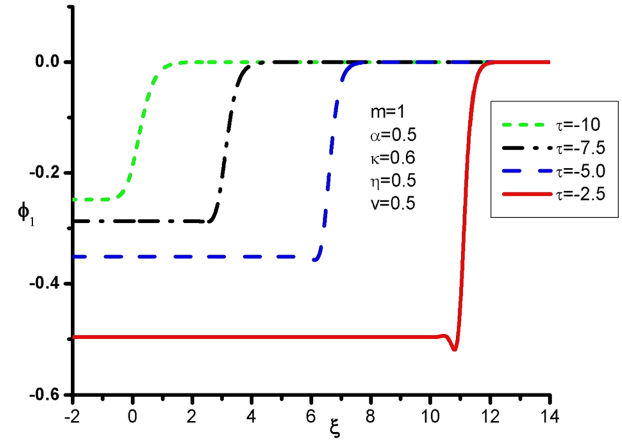

Fig. 1 Time evolution of Shock wave profile created in a plasma with parameters $\alpha = 0.5$, $\kappa = 0.6$, $\eta = 0.5$ and $V = 0.5$ in cylindrical geometry

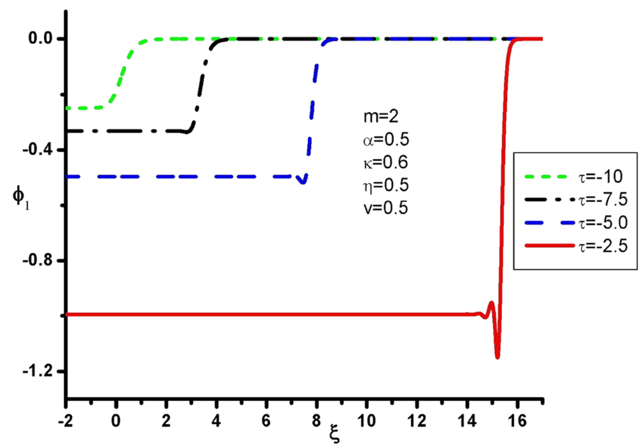

Fig. 2 Time evolution of Shock wave profile created in a plasma with parameters $\alpha = 0.5$, $\kappa = 0.6$, $\eta = 0.5$ and $V = 0.5$ in spherical geometry

Figure 3 depicts our numerical results for different values of the superthermal parameter ($\kappa = 0.6, 1, 2, 5, 10$) in cylindrical ($m = 1$) geometry at time $t = -2.5$. The remaining parameters are kept constant, viz., $\alpha = 0.1$, $\eta_0 = 0.5$ and $V = 0.5$. It is observed that the shape of developed shock wave appreciably changes as k decreases. The shock wave amplitude as well as the steepness of its profile increase with decreasing values of the superthermal parameter κ. More-

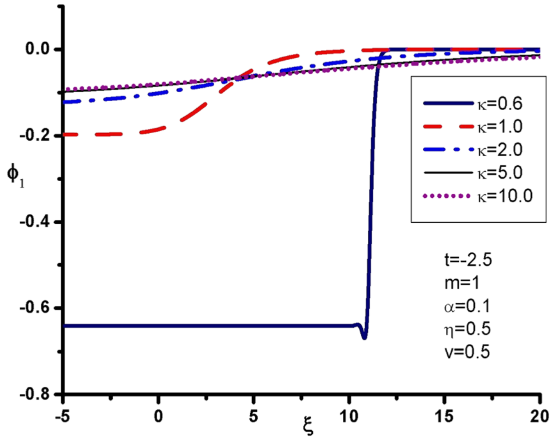

Fig. 3 Shock wave profiles with different values of superthermal parameter κ in cylindrical geometry at time $\tau = -2.5$. The other parameters are: $\alpha = 0.1$, $\eta = 0.5$ and $V = 0.5$

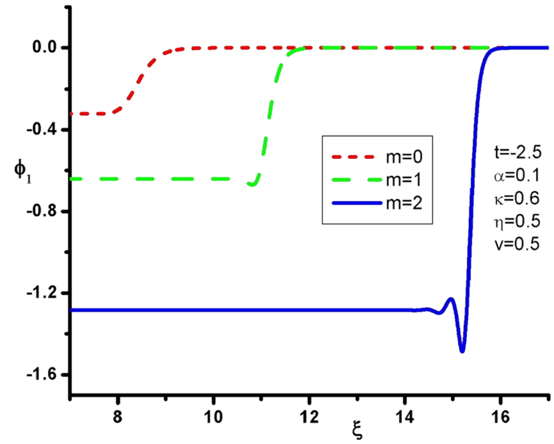

Fig. 5 Shock wave profiles as functions of ξ in different geometries at $\tau = -2.5$. Other parameters are: $\alpha = 0.1$, $\kappa = 0.6$, $\eta = 0.5$ and $V = 0.5$

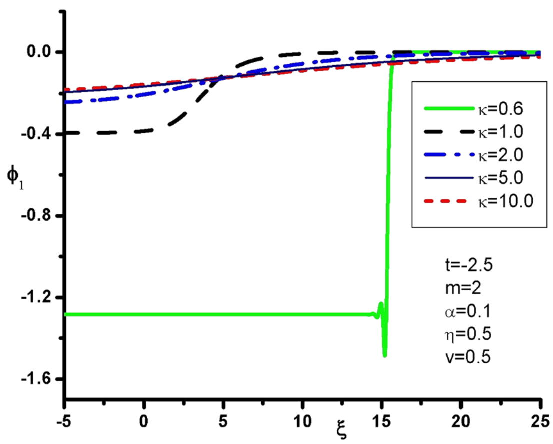

Fig. 4 Shock wave profiles with different values of superthermal parameter κ in spherical geometry at time $\tau = -2.5$. The other parameters are: $\alpha = 0.1$, $\eta = 0.5$ and $V = 0.5$

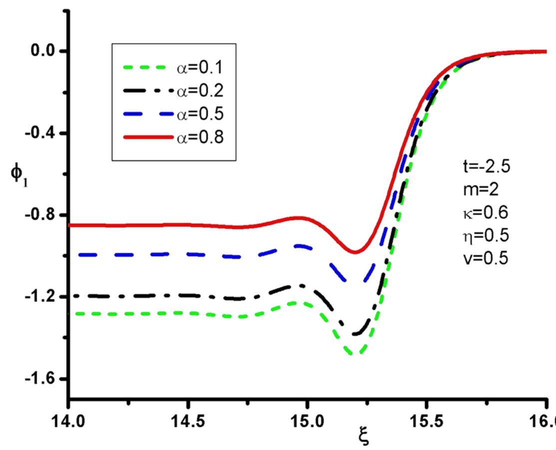

Fig. 6 Shock wave patterns as functions of ξ in spherical geometry with different values of α at time $\tau = -2.5$. Other parameters are: $\kappa = 0.6$, $\eta = 0.5$ and $V = 0.5$

over, a monotonic shock wave may change into an oscillatory one when the relative fast particles proportion is increased in the medium.

Similar behavior occurs in spherical geometry (see Fig. 4). Nevertheless, the effects of the electron superthermality seem to be more significant in the spherical geometry than in the cylindrical one. All the figures show that increasing the shock amplitude, steepness of the profile and small oscillations in the shape of the wave increase with decreasing values of superthermal parameter. These effects are also intensified in spherical geometry.

For the sake of comparison, we plot (simultaneously) in Fig. 5 the shock wave profile in different geometries ($m = 0, 1, 2$) at $\tau = -2.5$, with $\alpha = 0.1$, $\eta_0 = 0.5$, $V = 0.5$ and $\kappa = 0.6$. It is evident that the spherical shock wave exhibits the largest temporal growing rate. Figure 6 depicts the spatial variation of the spherical shock wave profile for different values of α at $\tau = -2.5$. It can be seen that the shock amplitude increases as α decreases. This means that the wave amplitude is proportional to the cold electrons population. Propagation of electron acoustic shock waves in plasmas with Tsallis distributed hot electrons has been investigated in Sahu and Tribeche (2012). The general results for the time evolution of the shock waves are the same in comparison with our results. Simulations in the paper Sahu and Tribeche (2012) have been done for plasmas in which hot electron equilibrium density is greater that the cold electron density i.e. $\alpha > 1$. Unfortunately, the effect of the parameter αon the behavior of the shock waves has not investigated in that paper. Nevertheless, the derived equations in Sahu and Tribeche (2012) reveal that the shock amplitude increases as α decreases, in accordance with the results shown in Fig. 6. Similar results have also been observed for ion acoustic shock waves in some other plasma systems. Formation of nonplanar ion acoustic shock waves in plasmas consisting of nonextensive electrons and thermal positrons has been investigated in Sahu (2012). It has been shown that the amplitude of the shock wave, its velocity, steepness and the small oscillations around the knee of the shock profile grow as time elapses.

4 Conclusions

Time evolution of nonplanar electron acoustic shock waves and its specifications have been investigated in plasmas containing superthermal distributed hot electrons and thermal cold electrons.

It is shown that the amplitude of shock wave, its steepness and its velocity increases as $|\tau|$ decreases. In this situation the term $\frac{m}{2\tau}\phi_1$ becomes dominant and nonplanar effects changes the wave specifications. Nonplanar effects are more significant in spherical geometry. Simulations also show that nonplanar effects are intensified by smaller values of superthermal parameter κ as well as smaller values of hot electron population which is characterized in parameter α. The results also compared with the results of other published related studies and it is shown that the presented results are in agreement with other investigations.

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