

## Polarized structure of nucleon in the valon representation

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## Polarized structure of nucleon in the valon representation

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**Firooz Arash**

*Physics Department, Tafresh University,  
Tehran Road, Tafresh, Iran  
E-mail: farash@cic.aut.ac.ir*

**Fatemeh Taghavi-Shahri**

*Physics Department, Iran University of Science and Technology,  
University Street, Narmak, Tehran 16345, Iran  
E-mail: f\_taghavi@iust.ac.ir*

ABSTRACT: We have utilized the concept of *valon* model to calculate the spin structure functions of proton, neutron, and deuteron. The valon structure itself is universal and arises from the perturbative dressing of the valence quark in QCD. Our results agree rather well with all of the relevant experimental data on  $g_1^{p,n,d}$  and  $\frac{g_A}{g_V}$ , and suggests that the sea quark contribution to the spin of proton is consistent with zero. It also reveals that while the total quark contribution to the spin of a valon,  $\Delta\Sigma_{\text{valon}}$ , is almost constant at  $Q^2 \geq 1$  the gluon contribution grows with the increase of  $Q^2$  and hence requiring a sizable negative orbital angular momentum component  $L_z$ . This component along with the singlet and non-singlet parts are calculated in the Next-to-Leading order in QCD. We speculate that gluon contribution to the spin content of the proton is about 60% for all  $Q^2$  values. Finally, we show that the size of gluon polarization and hence,  $L_z$ , is sensitive to the initial scale  $Q_0^2$ .

KEYWORDS: Deep Inelastic Scattering, Spin and Polarization Effects.

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## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. Polarized valon structure</b>	<b>3</b>
<b>3. Polarized nucleon structure function</b>	<b>8</b>
<b>4. First moments and the spin of proton</b>	<b>9</b>
4.1 Role of gluon, orbital angular momentum, and the spin of proton	15
<b>5. Remarks and conclusions</b>	<b>15</b>

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## 1. Introduction

A central goal in the study of QCD is to understand the structure of hadrons in terms of their quark and gluon degrees of freedom. The most direct tool and sensitive test for probing the quark and gluon substructure of hadrons is the polarized Deep Inelastic Scattering (DIS) processes. In such experiments detailed information can be extracted on the shape and magnitude of the spin dependent parton distributions,  $\delta q_f(x, Q^2)$ . Deep inelastic scattering reveals that the nucleon is a rather complicated object consisting of an infinite number of quarks, anti-quarks, and gluon. It is a common belief that other strongly interacting particles also exhibit similar internal structure. However, under certain conditions, hadrons behave as if they were composed of three (or two) constituents. Examples are the magnetic moments of the baryons, meson and baryon spectroscopy, the meson-baryon couplings and the ratio of total cross sections such as  $\frac{\sigma(\pi N)}{\sigma(NN)}$  and so on. Thus, it seems to make sense to decompose a nucleon into three constituent quarks called U and D. They would carry the internal quantum numbers of the nucleon. On the other hand, in DIS one observes that a nucleon has a composition of essentially an infinite number of quark-antiquark pairs and gluons, in addition to its valence quarks. One might identify the valence quark with a constituent quark, but this would imply that the three quark picture is a very rough approximation and both  $q\bar{q}$  pair and gluon degrees of freedom need to be added to the picture. In doing so, it would be very difficult to understand why the three quark picture of a baryon works so well in many circumstances. One way of reconciling this apparent contradiction is to consider a constituent quark as quasi-particle with a non-trivial internal structure of its own; consisting of a valence quark and a sea of  $q\bar{q}$  pairs and gluons. Such an interpretation of constituent quark is not new, more that 30 years ago it was advocated by Altarelli and Cabibo [1]. R.C. Hwa developed a more elaborated version by introducing the so called valon model [2] (the term which we will use hereafter) and

applied it to a variety of phenomena with great success. More recent indication for the existence of the valon can be inferred from the measurements of the Nachtmann moments of the proton structure functions at Jefferson Laboratory. They point to the existence of a new scaling that can be interpreted as a constituent form factor consistent with the elastic nucleon data [3]. This finding suggests that the proton structure originates from elastic coupling with extended objects inside the proton. In References [4–6], the valon concept is utilized to calculate the unpolarized structure function of a number of hadrons. The results are in excellent agreement with the experimental data. Altarelli has also calculated the pion structure function in the constituent quark representation [7], using a deconvolution procedure. On a more theoretical front, M. Lavelle and D. McMullan [8, 9] proved that one can dress a QCD Lagrangian field to all orders in perturbation theory and construct a constituent quark in conformity with the color confinement. From this point of view, a valon is defined as a structureful object emerging from the dressing of a valence quark with gluons and  $q\bar{q}$  pairs in QCD. Chiral Models in the realm of non-perturbative QCD also require the dressing of a valence quark and thus producing a structureful object. These results and the success of the valon model in describing the unpolarized structure of hadrons and a number of other low  $P_T$  hadronic phenomena lend credit for the study of hadrons in the valon representation.

In this paper we calculate the polarized structure of a valon and extract the polarized structure function,  $g_1$ , of the nucleon and the deuteron and compare the results with the experimental data. A number of similar attempts are also made to derive the nucleon spin from the quark models [10–14]. Our model differs from those in that we calculate the polarized structure of a constituent quark (the valon) directly from QCD processes in the Next-to-leading and investigate its peculiarities and distinctive features. In order to be clear, we define a valon as a valence quark plus its associated sea partons, emerging from dressing processes. In a bound state problem those processes are virtual and a good approximation for the problem is to consider a valon as one integral unit whose internal structure cannot be resolved. Therefore, it is assumed that the spin of the nucleon is provided by the combination of the spins of the valons. In a scattering situation, on the other hand, the virtual partons inside a valon can be excited and be put on mass shell. It is therefore more appropriate to think of a valon as a cluster of partons with some momentum and helicity distribution. If the valon has a non-trivial internal structure then the question arises whether its spin structure is also complex as it seems to be the case for the nucleon. This issue will also be addressed. In Reference [15] this model is used to calculate polarized structure functions. While obtaining results that are in agreement with the experimental data, however, it contains misleading and at points even counter intuitive ingredients. We will address them throughout this paper.

The organization of the paper is as follows: First we will outline the formalism for calculating the spin structure of the valon, then the polarized structure function of the nucleon and deuteron will be evaluated and it will be shown that the orbital angular momentum of partons in a valon plays a central role in describing the spin of nucleon.

## 2. Polarized valon structure

In this section we will utilize the extended work done on the development of NLO calculation of the moments, to evaluate the polarized structure of a valon. We should stress that this is not a new next-to-leading order calculation, but it is an exploration of the existing calculations in the valon framework.

By definition, a valon is a universal building block for every hadron; that is, its structure is independent of the hosting hadron. The valons play a dual role in hadrons: (i) they interact with each other in a way that is characterized by the valon wave function and (ii) they respond independently in an inclusive hard collision with a  $Q^2$  dependence that can be calculated in QCD at high  $Q^2$ . In role (i) they are the constituents of bound state problem involving the confinement at large distances. In role (ii) they are quasi-particles whose internal structure are probed with high resolution and are related to the short distance problem of current operators. This picture suggests that the structure function of a hadron involves a convolution of two distributions: valon distribution in the hadron and the parton distribution in the valon. In an unpolarized situation we may write:

$$F_2^h(x, Q^2) = \sum_{valon} \int_x^1 dy G_{valon}^h(y) F_2^{valon}\left(\frac{x}{y}, Q^2\right) \quad (2.1)$$

where  $F_2^{valon}\left(\frac{x}{y}, Q^2\right)$  is the structure function of the probed valon and can be calculated in Perturbative QCD to a certain degree of approximation. If  $Q^2$  is small enough we may identify  $F^{valon}(x, Q^2)$  as  $\delta(z-1)$  at some point, for the reason that we cannot resolve its internal structure at that  $Q^2$  value. Similarly, for a polarized hadron we can write

$$g_1^h(x, Q^2) = \sum_{valon} \int_x^1 \frac{dy}{y} \delta G_{valon}^h(y) g_1^{valon}\left(\frac{x}{y}, Q^2\right) \quad (2.2)$$

where  $\delta G_{valon}^h(y)$  is the helicity distribution of the valon in the hosting hadron and  $g_1^{valon}\left(\frac{x}{y}, Q^2\right)$  is the polarized structure function of the valon. At high  $Q^2$  for a U-type valon one can write  $g^{valon}$  as follows:

$$2g_1^U(z, Q^2) = \frac{4}{9}(\delta G_{\frac{u}{U}} + \delta G_{\frac{\bar{u}}{U}}) + \frac{1}{9}(\delta G_{\frac{d}{U}} + \delta G_{\frac{\bar{d}}{U}} + \delta G_{\frac{s}{U}} + \delta G_{\frac{\bar{s}}{U}}) + \dots \quad (2.3)$$

where all the functions on the right-hand side are the helicity functions for finding polarized quarks with momentum fraction  $z$  in a U-type valon at that  $Q^2$ . These functions or certain combinations of them can be calculated in QCD and assumed to be known as we will meet them later. Similar expression can also be written for the D-type valon. To describe the polarized parton distribution inside a valon, we will work in the moment space, where the moment of the polarized parton distribution in a valon is defined as:

$$\Delta f(n, Q^2) = \int_0^1 z^{n-1} \delta f(z, Q^2) dz. \quad (2.4)$$

$\delta f(z, Q^2)$  corresponds to parton helicity densities in a valon. The moments of the valon structure function are expressed completely in terms of  $Q^2$  through the evolution

parameter  $t$ :

$$t = \ln \frac{\ln \frac{Q^2}{\Lambda^2}}{\ln \frac{Q_0^2}{\Lambda^2}}. \quad (2.5)$$

We work in  $\overline{MS}$  scheme where  $\Lambda_{QCD}^{\overline{MS}}$  is given by

$$\Lambda_{QCD}^{\overline{MS}} = \mu \text{Exp} \left\{ -\frac{1}{2} \left[ \frac{1}{\beta_0 \alpha_s(\mu^2)} - \frac{\beta_1}{\beta_0^2} \log \left( \frac{1}{\beta_0 \alpha_s(\mu^2)} + \frac{\beta_1}{\beta_0} \right) \right] \right\}, \quad (2.6)$$

where  $\mu$  is the factorization scale and the  $\beta$  functions are as follows

$$\beta_0 = \frac{11}{3} C_A - f \frac{4}{3} T_F, \quad \beta_1 = \frac{34}{3} C_A^2 - f \frac{20}{3} C_A T_F - 4f C_F T_F. \quad (2.7)$$

Here  $C_A = 3$ ,  $T_F = \frac{f}{2}$ ,  $C_F = \frac{4}{3}$ , and  $f$  is the number of active flavors. A NLO fit to the  $g_1/F_1$  with massless quarks is performed in [16] and favors a value  $\Lambda_{QCD} = 0.235 \pm 0.035$  GeV. This is very close to our choice of  $\Lambda_{QCD} = 0.22$  GeV for the unpolarized case. we will maintain this value along with  $Q_0^2 = 0.283$  GeV<sup>2</sup> as in [4].

Moments of the polarized valence and sea quarks in a polarized valon are:

$$\delta M_{\frac{\delta q_v}{\text{valon}}} = \delta M_{NS}(n, Q^2) \quad (2.8)$$

$$\delta M_{\frac{\delta q_{\text{sea}}}{\text{valon}}} = \frac{1}{2f} (\delta M_S - \delta M_{NS})(n, Q^2) \quad (2.9)$$

where  $f$  is the number of flavors and  $\delta M_{S,NS}$  are polarized singlet and non-singlet moments defined as:

$$\delta M_{NS\pm}(n, Q^2) = \left\{ 1 + \frac{\alpha_s(Q^2) - \alpha_s(Q_0^2)}{2\pi} \left( \frac{-2}{\beta_0} \right) \left( \delta \mathbf{P}_{NS\pm}^{(1)n} - \frac{\beta_1}{2\beta_0} \delta \mathbf{P}_{qq}^{(0)n} \right) \right\} \mathbf{L}^{-\left(\frac{2}{\beta_0}\right) \delta P_{qq}^{(0)n}} \quad (2.10)$$

$$\begin{pmatrix} \delta M_S(n, Q^2) \\ \delta M_G(n, Q^2) \end{pmatrix} = \left\{ \mathbf{L}^{-\left(\frac{2}{\beta_0}\right) \delta \hat{P}^{(0)n}} + \frac{\alpha_s(Q^2)}{2\pi} \hat{\mathbf{U}} \mathbf{L}^{-\left(\frac{2}{\beta_0}\right) \delta \hat{P}^{(0)n}} - \frac{\alpha_s(Q_0^2)}{2\pi} \mathbf{L}^{-\left(\frac{2}{\beta_0}\right) \delta \hat{P}^{(0)n}} \hat{\mathbf{U}} \right\} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.11)$$

The column matrix on the right hand side describes our initial input densities and they constitute an essential part of this work. We have inferred them from the properties of the model: The valon structure function has the property that it becomes  $\delta(z-1)$  as  $Q^2$  is extrapolated to  $Q_0^2$  (beyond the region of validity). This mathematical boundary condition means that the internal structure of the valon cannot be resolved at  $Q_0^2$  in the NLO approximation. Consequently, when this property is applied to eq. (2), the structure function of the nucleon becomes directly related to  $x \delta G_{\text{valon}}^h(x)$  at that value of  $Q_0^2$ ; that is,  $Q_0^2$  is the leading-order effective value at which the hadron can be regarded as consisting of only three (two) valons for baryons(mesons). In the moment space it is the Mellin transform of the  $\delta$ -function, being equal to one, that enters. Naturally, then  $\delta M_g(n, Q_0^2) = 0$  and it is reasonable to set  $\delta f(z, Q_0^2) = f(z, Q_0^2)$ , for the quark sector. As  $Q^2$  increases beyond a small enough value, say  $Q_v^2$ , where we may identify valon structure as  $\delta(z-1)$ , one expects the valon structure to develop a tail in the  $0 < z < 1$  region due to gluon radiation.

Reliable calculations are only possible for higher  $Q^2$  values; let it be for  $Q^2 > Q_1^2$ . Between  $Q_v^2$  and  $Q_1^2$  higher twist terms are involved and the picture is more complicated, but that is also the region where the most important part of  $Q^2$  evolution takes place. In reference [15] the initial moments of both gluon and singlet sectors are set equal to one. Such a choice has no justification in the valon model. Moreover, it is known that a fully saturated initial input density for the gluon is disfavored [16]. Basically, initial input densities are determined from the experimental data, but for the valon there is no experimental data, therefore our choice of initial input densities, based on the mathematical conditions of the model does not invalidate it, nor does it violate the positivity constraint.

In eqs.(10,11),  $\mathbf{L} \equiv \alpha_s(Q^2)/\alpha_s(Q_0^2)$ , and  $\delta\hat{P}^{(0)n}$  is  $2 \times 2$  singlet matrix of splitting functions, given by

$$\delta\hat{P}^{(0)n} = \begin{pmatrix} \delta P_{qq}^{(0)n} & 2f\delta P_{qg}^{(0)n} \\ \delta P_{gq}^{(0)n} & \delta P_{gg}^{(0)n} \end{pmatrix}, \quad (2.12)$$

where  $\delta P_{lm}^{(0)n}$  are the  $n^{th}$  moments of the polarized splitting functions and  $\mathbf{U}$  accounts for the 2-loop contributions as an extension to the leading order. The explicit forms of these functions are given in [17] in the next-to-leading order. Now it is straightforward to calculate the moments of polarized partons inside a valon at any  $Q^2$  or  $t$  value. These moments are shown in figure 1. The  $z$ -dependence of the polarized parton distributions is obtained by utilizing the usual inverse Mellin transformation.

$$\delta q_{NS,S,G}^{\text{valon}}(z, Q^2) = \frac{1}{\pi} \int_0^\infty \text{Im}[e^{i\phi} z^{-c-we^{i\phi}} \delta M^{NS,S,G}(n = c + we^{i\phi}, Q^2)] dw, \quad (2.13)$$

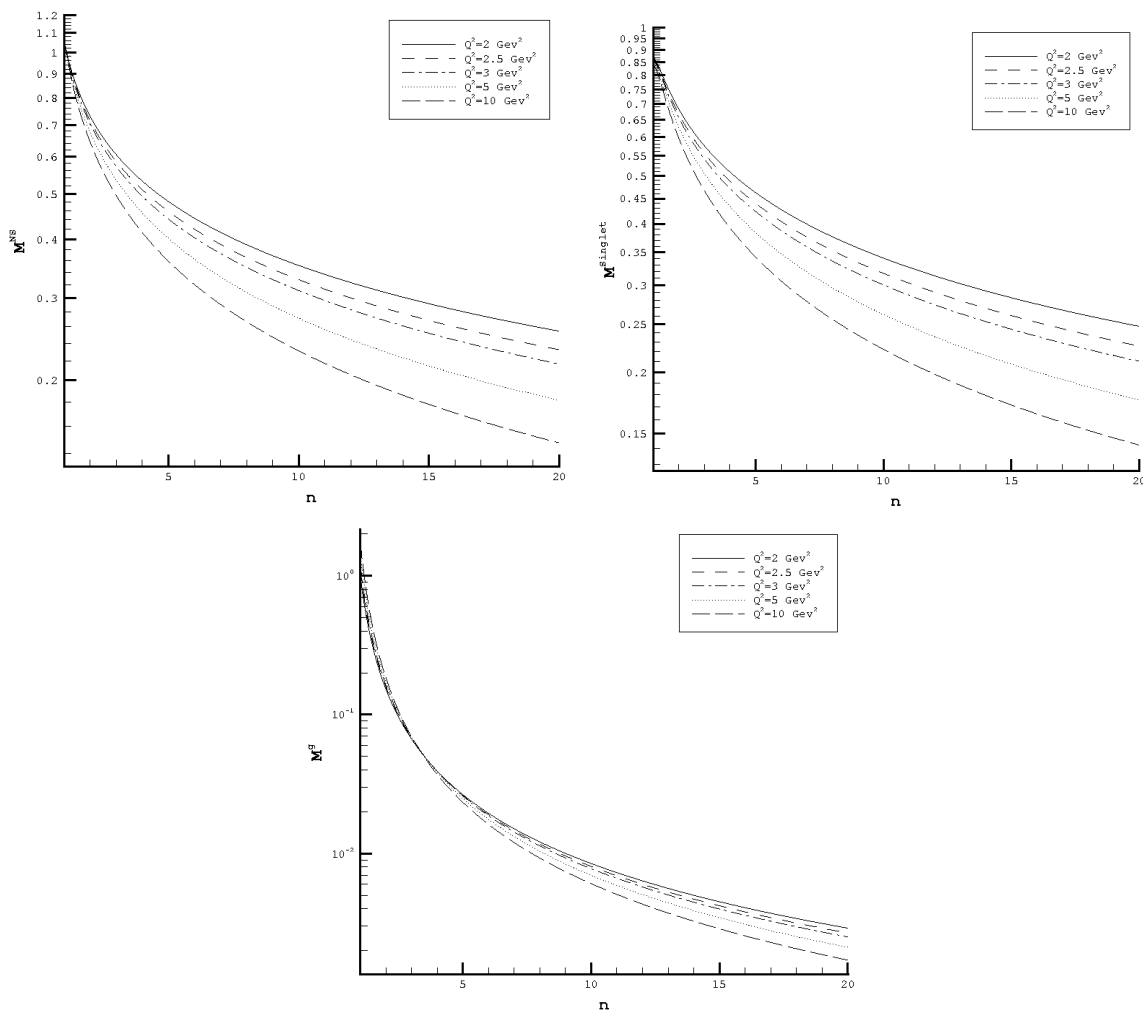
where NS, S, and G stand for non-singlet, singlet and gluon, respectively. In what follows we shall only be interested in quantities averaged over  $z$  and thus in the  $n = 1$  moments. The first moments are defined by

$$\Delta f(Q^2) = \int dz \delta f(z, Q^2) \quad (2.14)$$

There is a simple physical interpretation for the quantities like  $\Delta\Sigma$ ,  $\Delta q$  and  $\Delta g$ : they are related to the total  $z$  component of quark and gluon spins, thus

$$\langle S_z \rangle_q = \frac{1}{2} \Delta q, \quad \langle S_z \rangle_g = \Delta g \quad (2.15)$$

These quantities for the valon are shown in figure 2. The results imply that the total quark contribution to the spin of a valon, decreases from 1 to  $\Delta\Sigma = 0.88$ , in the range of  $Q^2 = [0.283, 1] \text{ GeV}^2$  and remains almost independent of  $Q^2$  thereafter. In other words, if the valon consisted only of quarks (valence plus sea) it would have been enough to account for  $\simeq 90\%$  of the valon spin at  $Q^2 \geq 1$ . Evidently, however, there is a sizable gluon component,  $\Delta g$ , which increases with  $Q^2$  as shown in figure 2. In figure 3 we have shown the variation of  $\Delta q_{\text{sea}}$  and  $\Delta q_{\text{valence}}$  as a function of  $Q^2$ . The variation of  $\Delta q_{\text{sea}}$  and  $\Delta\Sigma$  with  $Q^2$  is very marginal. we have checked that  $\Delta q_{\text{valence}} = [1, 1.08]$  for the range of  $Q^2 = [0.283, 10^6] \text{ GeV}^2$ ; whereas  $\Delta q_{\text{sea}}$  varies from 0 to -0.043 for the same range of

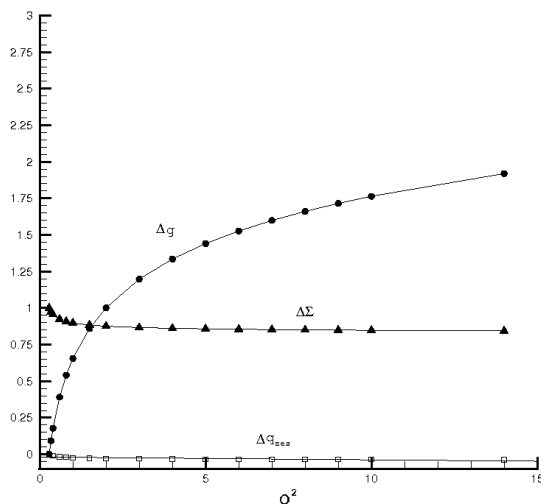


**Figure 1:** Moments of singlet, non-singlet quark, and gluon distributions in a valon at several  $Q^2$  values.

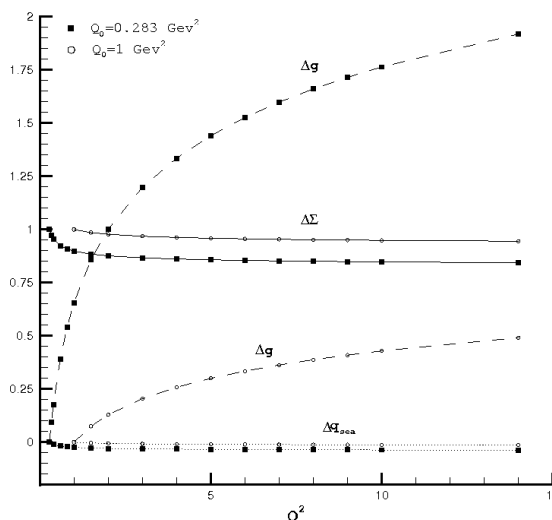
$Q^2$ . This weakly  $Q^2$  dependent behavior of  $\Delta q_{valence}$  and  $\Delta q_{sea}$ , are well understood: in the leading order one expects the total quark spin to be constant due to the vanishing of quark anomalous dimensions at  $n = 1$ . In the Next-to-Leading Order, however, they are marginally  $Q^2$  dependent due to  $\Delta P_{NS}^{(1)} \neq 0$ . The fact that sea quark polarization in the valon is consistent with zero can also be understood on theoretical grounds. The valon structure is generated by perturbative dressing in QCD. In such processes with massless quarks, helicity is conserved and therefore, the hard gluons cannot induce sea polarization perturbatively. It is also worth to note that  $\Delta q_{sea} \simeq 0$  is in good agreement with HERMES data [18, 19].

Had we chosen  $Q_0^2 = 1 \text{ GeV}^2$ , we would have obtained  $\Delta\Sigma = 1$  for all values of  $Q^2$  with little change in  $\Delta q_{sea}$ , and with a much reduced gluon polarization, as compared to our results with  $Q_0^2 = 0.238 \text{ GeV}^2$  as can be seen in figure 3. Such a choice with initial gluon helicity distribution equal to zero, however, is inconsistent with the mathematical





**Figure 2:** First Moments,  $\Delta g(n = 1, Q^2)$ ,  $\Delta \Sigma(n = 1, Q^2)$ , and  $\Delta q_{\text{sea}}(n = 1, Q^2)$  of various components in a valon as a function of  $Q^2$

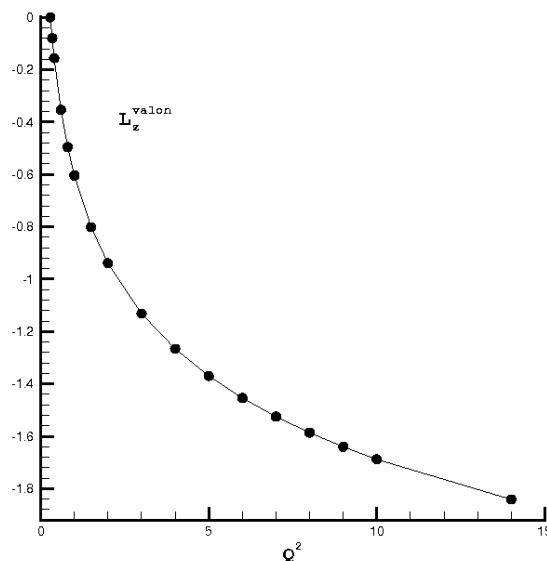


**Figure 3:** Variations of  $\Delta q_{\text{valence}}$  and  $\Delta q_{\text{sea}}$  in a valon as a function of  $Q^2$ . Full squares are the results calculated with  $Q_0^2 = 0.283 \text{ GeV}^2$  and open circles correspond to  $Q_0^2 = 1 \text{ GeV}^2$ .

condition of the model. The use of a different initial gluon helicity distribution, instead of zero, also would have been a pure guess and at best would require data fittings that we have avoided.

It is obvious that due to large gluon polarization in a valon, these results do not add up to give the spin  $\frac{1}{2}$  of the valon and do not satisfy the sum rule  $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g$ . The gluon contribution to the valon spin grows as  $Q^2$  increases, while  $\Delta\Sigma$  remains almost unchanged beyond  $Q^2 = 1 \text{ GeV}^2$ . It is the present wisdom that the above sum rule must be replaced with a more realistic one

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_z \quad (2.16)$$



**Figure 4:** Orbital angular momentum,  $L_z^{\text{valon}}(Q^2)$ , component of partons in a valon as a function of  $Q^2$ . The results corresponds to  $Q_0^2 = 0.283 \text{ GeV}^2$ .

where,  $L_z$  is the orbital angular momentum carried by the sea partons ( $q - \bar{q}$  pairs and gluons) within the valon. The size of this orbital angular momentum turns out to be large and negative, mainly competing with the gluon contribution. Ratcliffe [20] was the first to point out the necessity of including orbital angular momentum dependence of the evolution equation and predicted a negative value for  $\langle L \rangle_z$  of the sea partons in the proton. In figure 4 we present the Next-to-Leading Order result for the orbital angular momentum in a valon,  $L_z^{\text{valon}}(Q^2)$ . A leading order calculation is given in [21]. The existence of non-zero internal orbital momentum in the valon implies that there are substantial correlations among partons. It has been argued that the presence of quark pairs inside hadron resembles superconductivity [22]. An extension of the theory of superconductivity to the anisotropic case [23], shows that the presence of anisotropy leads to axial symmetry of pairing correlations around the anisotropy direction and to the particle currents induced by the pairing correlations. The particle number conservation, then requires that the cloud of correlated particles to rotate around the central particle in opposite direction, the so-called *Backflow* current. This pairing induced orbital angular momentum is proportional to the density of correlated particles. The analogy seems to match the valon picture, where the internal structure of the valon originates entirely from QCD processes.

### 3. Polarized nucleon structure function

In the previous section the polarized structure of a valon is completely specified. We are now in a position to carry forward and investigate the implications of the model at the hadronic level. Our starting point is eq. (2), where the only unknown element is the valon helicity distribution,  $\delta G_{valon}^h(y)$ , since the valon structure function  $g_1^{\text{valon}}$  is now completely given. In the analysis of [21], for the leading order, we assumed that the polarized valon

$valon(j)$	$N_j$	$\alpha_j$	$\beta_j$	$a_j$	$b_j$	$c_j$	$d_j$
$U$	3.44	0.33	3.58	-2.47	5.07	-1.859	2.780
$D$	-0.568	-0.374	4.142	-2.844	11.695	-10.096	14.47

**Table 1:** Numerical values of the parameters in eq. (18).

distribution is related to the, by now well determined, unpolarized valon distribution via:

$$\delta G_j(y) = \delta F_j(y) G_j(y) \quad (3.1)$$

where,  $G_j(y)$  is the unpolarized valon distribution of  $j = U, D$  kinds.  $G_j(y)$  are given in [4–6] for a variety of hadrons. They mimic the hadron wave functions.

In the absence of experimental knowledge on  $\delta G_{valon}^h(y)$ , the safe way to determine them is to fit the experimental values of  $g_1^p$  at some  $Q^2$  with the form given by

$$\delta F_j(y, Q_0^2) = N_j y^{\alpha_j} (1-y)^{\beta_j} (1 + a_j y^{0.5} + b_j y + c_j y^{1.5} + d_j y^2) \quad (3.2)$$

where,  $j$  stands for  $U$  and  $D$  type valon and parameters  $\alpha_j, \beta_j$ , etc. are given in table 1. Consequently, the polarized valon distributions in a proton are now completely specified and are given by:

$$\delta G_{\frac{U}{P}}(y) = \delta F_U G_{\frac{U}{P}} \quad \delta G_{\frac{D}{P}}(y) = \delta F_D G_{\frac{D}{P}} \quad (3.3)$$

Now we can calculate the polarized hadronic structure function,  $g_1^h$ . For this purpose all that we need is to substitute  $\delta G_{\frac{U}{P}}(y)$  and  $\delta G_{\frac{D}{P}}(y)$  from eq. (19) into eq. (2) and perform the convolution integral. In figure 5 we present the results for proton,  $g_1^p$ , and compare them with the experimental data [18, 19, 24, 25], and with the calculations of [16, 26], and [27].

It is evident that the model calculation is in good agreement with the experimental data. In figure 6 we compare the calculated results of  $x\delta u_v(x)$  and  $x\delta d_v(x)$  with the experimental values from HERMES at  $Q^2 = 2.5 \text{ GeV}^2$ . Results from other analysis are also shown.

As a further comparison, in figures 7 and 8 we present results for  $xg_1^p$  and  $xg_1^d$ . These results are also in agreement with the experimental data [24] and the analysis of References [16, 26], and [27].

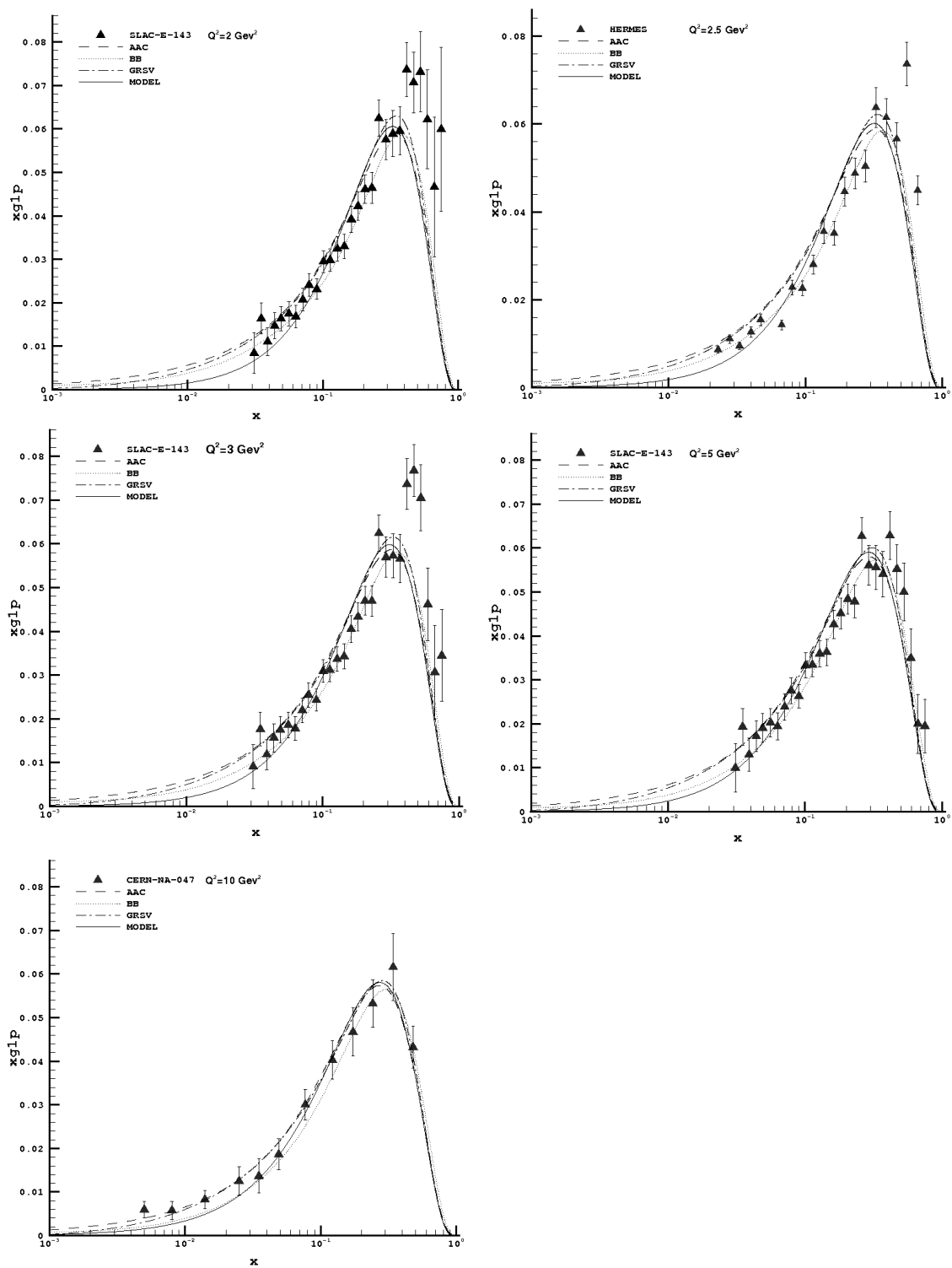
#### 4. First moments and the spin of proton

The first moment of polarized proton structure function, defined by

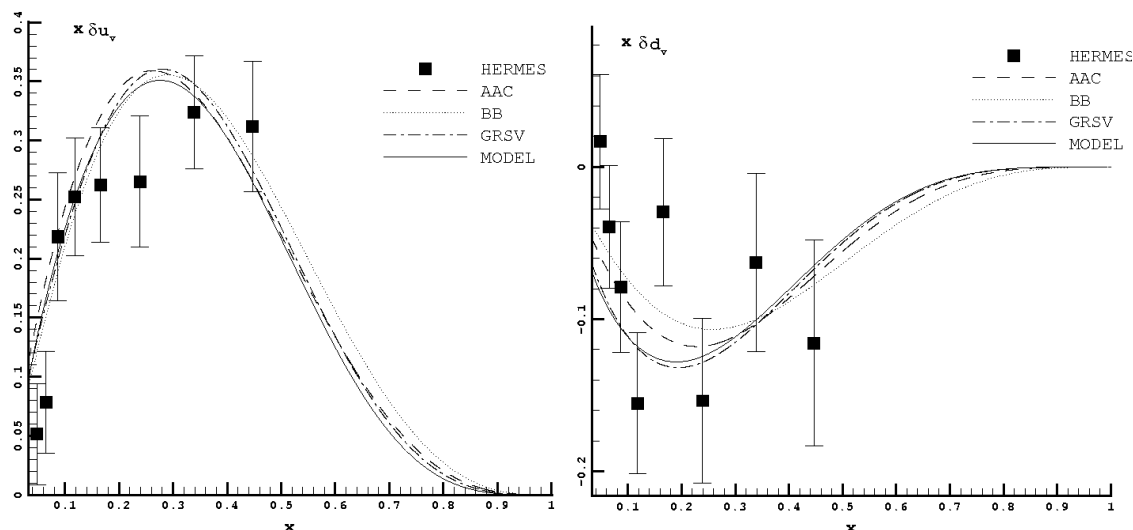
$$\Gamma_1^p = \int_0^1 g_1^p(x, Q^2) dx \quad (4.1)$$

can be related to the combinations of the quark spin components via

$$\Gamma_1^p = \frac{1}{2} \sum_q e_q^2 \Delta q(Q^2) = \frac{1}{2} \sum_q e_q^2 \langle p, s | \bar{q} \gamma_\mu \gamma_5 q | p, s \rangle s^\mu. \quad (4.2)$$



**Figure 5:** Polarized proton structure function,  $xg_1^p$ , as a function of  $x$  at various  $Q^2$  values. Data points are from [18, 19, 24, 25]



**Figure 6:**  $x\delta u_v(x)$  and  $x\delta d_v(x)$  at  $Q^2 = 2.5 \text{ GeV}^2$ . The curves are the model results and the data points are from ref. [18, 19].

$\Gamma_1^N$	$Q^2 = 2 \text{ GeV}^2$	$Q^2 = 2.5 \text{ GeV}^2$	$Q^2 = 3 \text{ GeV}^2$	$Q^2 = 5 \text{ GeV}^2$	$Q^2 = 10 \text{ GeV}^2$
$p$	0.1132	0.1153	0.1168	0.1200	0.1245
$n$	-0.0630	-0.0459	-0.0554	-0.0548	-0.0546
$d$	0.0305	0.0386	0.0341	0.0357	0.0377

**Table 2:** Numerical values for  $\Gamma_1^N$  at several  $Q^2$  values.

Our results for  $\Gamma_1^p$  are listed in table 2. The moments of the polarized quark contributions to the spin of proton at, say,  $Q^2 = 3 \text{ GeV}^2$  are

$$\Delta u_{\text{valence}} = 0.820, \quad \Delta d_{\text{valence}} = -0.422, \quad \Delta \bar{q}_{\text{sea}} \sim 0 \quad (4.3)$$

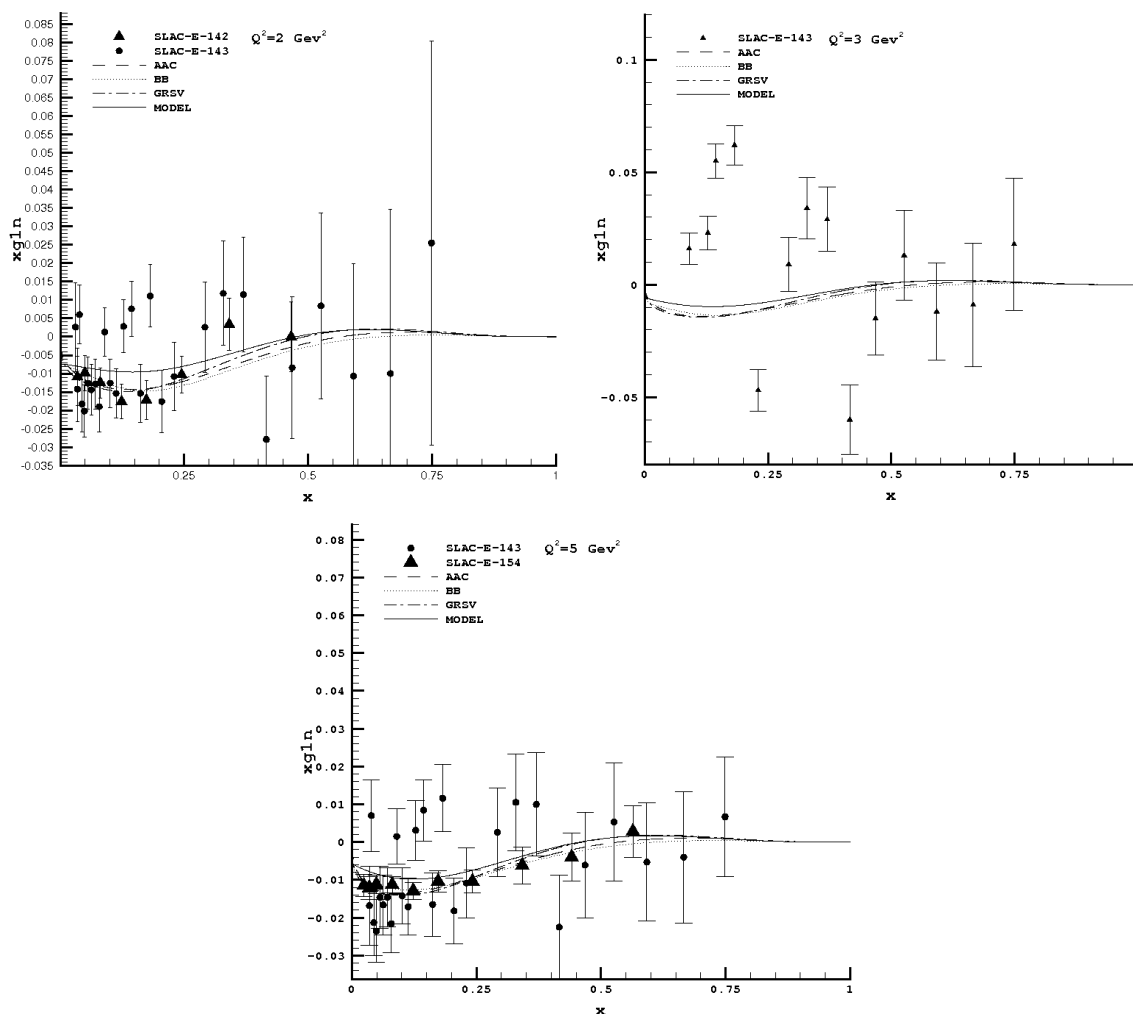
For other values of  $Q^2$  similar results are also obtained: for  $Q^2 = [2, 10]$ , we have  $\Delta u_v = [0.816, 0.827]$  and  $\Delta d_v = [-0.420, -0.426]$ . These results for the first moment of  $g_1^{p,n,d}$ , the quantity  $\Gamma_1^{p,n,d}$ , yield the values that are presented in table 2.

The available experimental values for  $\Gamma_1^{p,n}$  are obtained in a range of  $Q^2$ , rather than at a fixed  $Q^2$  and a fit to all data at  $Q^2 = 5$  yields  $\Gamma_1^p = 0.118 \pm 0.004 \pm 0.007$  and  $\Gamma_1^n = -0.048 \pm 0.005 \pm 0.005$  [28]. More recent data from HERMES [31] suggest that  $\Gamma_1^p = 0.1211 \pm 0.005 \pm 0.008$  and  $\Gamma_1^d = 0.0436 \pm 0.0012 \pm 0.0018$  at  $Q^2 = 5 \text{ GeV}^2$ . The values given in table 2 matches these experimental results.

Additional information on the quark polarization is also available from the low-energy nucleon axial coupling constants  $g_A^3$  and  $g_A^8$ :

$$\begin{aligned} g_A^3 &\equiv \langle p, s | \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d | p, s \rangle s^\mu = \Delta u(Q^2) - \Delta d(Q^2), \\ g_A^8 &\equiv \langle p, s | \bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d - 2\bar{s}\gamma_\mu\gamma_5 s | p, s \rangle s^\mu = \Delta u(Q^2) + \Delta d(Q^2) - 2\Delta s(Q^2). \end{aligned} \quad (4.4)$$

Since there is no anomalous dimension associated with the axial-vector currents,  $A_\mu^3$  and  $A_\mu^8$ , the non-singlet couplings,  $g_A^3$  and  $g_A^8$  do not evolve with  $Q^2$  and hence can be determined

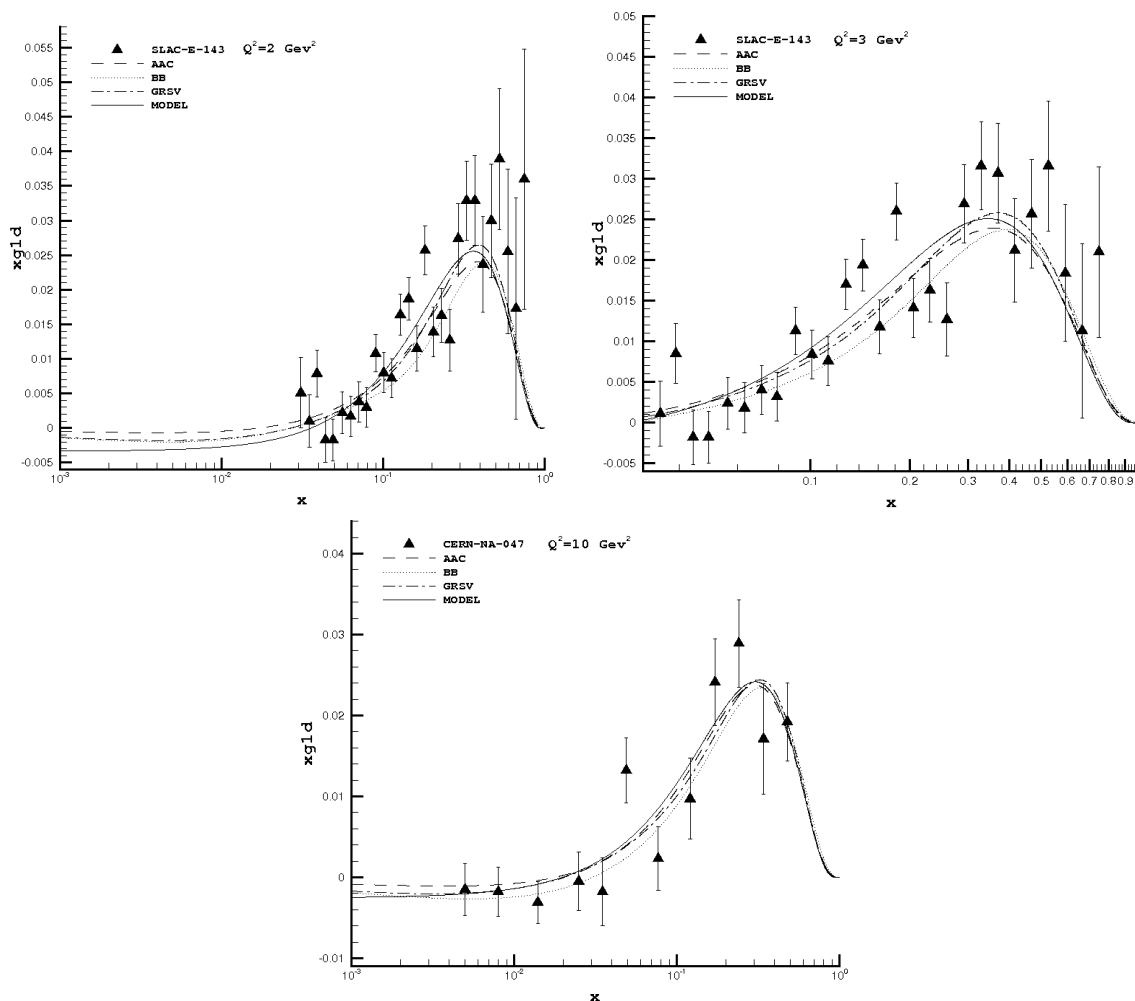


**Figure 7:** Polarized neutron structure function,  $xg_1^n$ , as a function of  $x$  at  $Q^2 = 2, 3$ , and  $5 \text{ GeV}^2$ . Data points are from [24].

from low-energy neutron and hyperon  $\beta$ -decays. The experimental values are  $g_A^3 = 1.2573 \pm 0.0028$  and  $g_A^8 = 0.579 \pm 0.025$ . The constraining values of  $g_A^{3,8}$  are the ones used in most analysis performed in order to fix sea quark contributions, in particular that of  $\Delta S$ . We have not considered this restriction a priori; instead we want to see if the results of the model can reproduce these numbers. From the stated values for  $\Delta u_v$  and  $\Delta d_v$  we obtain  $g_A^3 = 1.240 - 1.253$  which accommodates the experimental value with an accuracy of 2%. Our agreement with  $g_A^8$  is not as good as  $g_A^3$ . However, this does not invalidate the results of the model, for there are serious objections [29, 30] (mainly due to  $m_{u,d} \ll m_s$ ) to

$$g_A^8 = \Delta u + \Delta \bar{u} + \Delta d + \Delta \bar{d} - 2(\Delta s + \Delta \bar{s}) = 3F - D = 0.579 \pm 0.025, \quad (4.5)$$

in contrast to the unquestioned isospin  $SU(2)$  symmetry ( $m_u \simeq m_d$ ) that gives rise to the value of  $g_A^3$ . Experimental data from HERMES also puts the value of  $g_A^8$  at  $0.274 \pm 0.026 \pm 0.011$  in the range of  $0.02 < x < 0.6$  [31] which is substantially less than the value



**Figure 8:** Polarized deuteron structure function,  $xg_1^d$ , as a function of  $x$  at  $Q^2 = 2, 3$ , and  $10 \text{ GeV}^2$ . Data points are from [24].

inferred from hyperon decay. We have obtained  $g_A^8 = 0.39$ . Findings of [31] also suggests that  $\Delta s + \delta\bar{s}$  is consistent with zero. The HERMES data [31] in the measured region of  $x > 0.02$  gives  $\Delta s + \delta\bar{s} = 0.006 \pm 0.029 \pm 0.007$ . If the results reported in second reference of [31] is confirmed, it would also rule out any significant non-perturbative effects that predict strange quark contribution to the spin of proton from pion-nucleon sigma term  $\sigma_{\pi N}$  which in turn alters the value of  $g_A^8$ . Such a contribution can be calculated in the framework of chiral quark model. In fact, in this framework it is shown that [32] while light sea quark polarization is consistent with zero, the  $\Delta S = -0.051$ . If it is added to our result, it becomes in line with  $g^8$  value obtained from hyperon decay. Nevertheless, The crucial point in our model is that there is no room for the perturbative sea quark polarization, because the sea of the valon is generated entirely from gluon splitting and for the massless quark, the helicity is conserved; and yet the model nicely accommodates all of the experimental data with acceptable accuracy. Our finding that  $\Delta q_{\text{sea}}$  is consistent with

zero is in agreement with the HERMES [18, 19] and SMC collaboration [33] data. This can be seen yet in a different way as follows:

The moments of sea quarks in a proton can be written as

$$\Delta\bar{q}(n, Q^2) = \frac{1}{2f}[\Delta\Sigma(n, Q^2) - \Delta u_v(n, Q^2) - \Delta d_v(n, Q^2)] \quad (4.6)$$

where

$$\begin{aligned} \Delta u_v(n, Q^2) &= 2\Delta M^{NS}(n, Q^2) \otimes \Delta M_{\frac{U}{p}}(n) \\ \Delta d_v(n, Q^2) &= \Delta M^{NS}(n, Q^2) \otimes \Delta M_{\frac{D}{p}}(n) \\ \Delta\Sigma(n, Q^2) &= \Delta M^S(n, Q^2) \otimes \left[ 2\Delta M_{\frac{U}{p}}(n) + \Delta M_{\frac{D}{p}}(n) \right] \end{aligned} \quad (4.7)$$

where  $\Delta M_{\frac{U,D}{p}}(n)$  are moments of  $\delta G_{U,D}$ . For  $n = 1$  they are given by

$$\int_0^1 dy \delta G_{\frac{U}{p}}(y) = 0.403, \quad \int_0^1 dy \delta G_{\frac{D}{p}}(y) = -0.409 \quad (4.8)$$

Therefore, eq. (25) becomes

$$\Delta\bar{q}(n, Q^2) = (\Delta M^S(n, Q^2) - \Delta M^{NS}(n, Q^2))(2\Delta M_{\frac{U}{p}}(n, Q^2) + \Delta M_{\frac{D}{p}}(n, Q^2)) \quad (4.9)$$

and we see that for  $n = 1$  the range of variation for  $\Delta\bar{q}(1, Q^2)$  is  $0 - 0.016$ , since  $\Delta M^S(1, Q^2) = 1$  and  $\Delta M^{NS}(n = 1)$  varies between 1 and 0.80 for  $Q^2 = [0.283, 10^6] \text{ GeV}^2$ ; that is, the contribution of the sea quark to the spin of proton is consistent with zero. Reference [15] introduces two different polarized valon distribution in an attempt to avoid dealing with  $\Delta\bar{q}(1, Q^2) \approx 0$ . Such an scheme is quite counter intuitive, because it means that singlet and non-singlet quark distribution inside a valon alters the valon distribution in a hadron. A notion that is hard to understand within the context of the valon model.

Of course a 2% deviation of our results from the value of  $g_A^3$  can be attributed to the presence of sea polarization which then ought to be generated non-perturbatively. This point is not considered here nor have we attempted to fit the data in order to extract possible sea quark contribution, in particular that of strange sea. Another possible source for this deviation can be a poor determination of  $\delta G_{\frac{U,D}{p}}$ . We also note that a plausible alternative to the full  $SU(3)_f$  symmetry is a "valence" scenario where  $SU(3)_f$  symmetry is maximally broken which is based on the assumption that the flavor-changing hyperon  $\beta$ -decay data fix only the total helicities of valence quark at some appropriately chosen input scale  $Q^2 = Q_0^2$  [17].

Our prediction for  $\Delta\Sigma$ , the total quark contribution to the spin of proton, lies in the range of  $0.410 - 0.420$  for  $Q^2 = [2, 10]$ . The variation of  $\Delta\Sigma$  is due to (marginal)  $Q^2$  dependence of  $\Delta q_v$  in the Next-to-Leading Order; because  $\Delta P_{NS}^{(1)} \neq 0$ . This result is also compatible with the experimental data of refs. [17, 18], where for the measure range,  $0.023 < x < 0.6$ , they obtained  $\Delta\Sigma = 0.347 \pm 0.024 \pm 0.066$ . Further experimental support



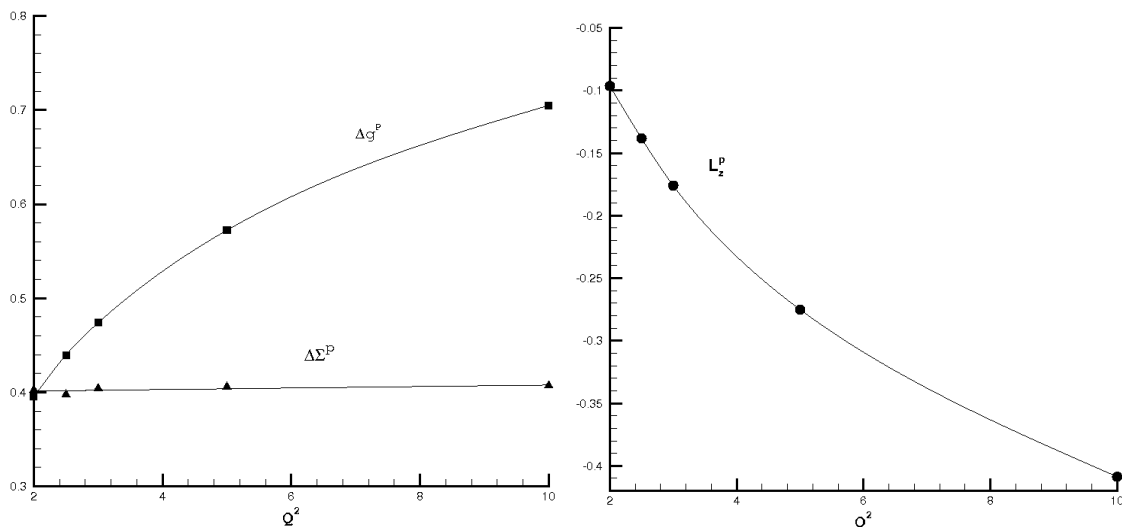
for our findings comes from recently published data from COMPASS collaboration [34] at  $Q^2 = 10 \text{ GeV}^2$  and  $0.006 < x < 0.7$ . It shows that  $\Delta u_v + \Delta d_v = 0.40 \pm 0.07 \pm 0.05$ ,  $\Delta \bar{u} + \Delta \bar{d} = 0.0 \pm 0.04 \pm 0.03$ , and  $\Delta s + \Delta \bar{s} = -0.08 \pm 0.01 \pm 0.03$ . This result is consistent with  $\Delta \bar{u} = \Delta \bar{d}$ . The conclusion is that: if we accept the validity of both HERMES [31] and COMPASS [34] data, then the role of sea quark polarization in calculation of polarized structure functions,  $g_1^{p,n,d}$  is marginal. We have arrived at the same conclusion directly by considering only the QCD processes up to the Next-to-Leading order in the valon representation.

#### 4.1 Role of gluon, orbital angular momentum, and the spin of proton

The phenomenological model that is described here is able to account for the experimental data with a good accuracy. However, it still remains to accommodate the spin of proton. The spin of valon in the absence of gluon is accounted for by the total spin contribution of quarks, as we saw in section 2. The large gluon polarization in valon, however requires a sizable negative orbital angular momentum to compensate for the gluon contribution beyond  $Q_0^2$ . The implication for proton is that  $\Delta g$  rises as  $Q^2$  increases; being around 0.4 at  $Q^2 = 2 \text{ GeV}^2$  and reaches to 0.7 at  $Q^2 = 14 \text{ GeV}^2$ . For the same range of  $Q^2$ , then  $L_z$  varies from  $-0.1$  to  $-0.4$ . These results are plotted in figure 9. The role of Orbital angular momentum,  $L_z$  in a valon is to cancel out the gluon polarization completely, but this cancelation in proton is partial. Therefore, it is reasonable to speculate that about 60% of the net spin of proton comes from gluon. This is comparable with  $\sim 50\%$  momentum contribution of gluon to the total momentum of proton. We do share the opinion expressed in [31] based on the experimental data from HERMES that the quark helicities contribute a substantial fraction to the nucleon helicity, but there is still need for a major contribution from gluon/orbital angular momentum. We have concluded that the orbital angular momentum is needed and arises from the structure of valon in order to compensate for the growing gluon helicity and produce a spin- $\frac{1}{2}$  valon. It also can be argued that the valons of nucleon themselves might have a relative orbital angular momentum other than zero. Such a situation would amount to the assumption that the nucleon is not in the  $S$ - state of orbital angular momentum, the case that we have not considered in this analysis. In the analysis of section 2 we stated that the magnitude of gluon helicity,  $\Delta g$ , depends on the initial scale,  $Q_0^2$ , chosen; and hence, so does the values of  $L_z$ . We believe that the mathematical boundary condition of the model provides a reasonable guideline for the choice of  $Q_0^2$ .

### 5. Remarks and conclusions

We have calculated the polarized structure of a valon form QCD processes in the next-to-leading order framework. While the valence quark completely accounts for the spin of a valon, the presence of large gluon polarization in the valon makes it more complicated, requiring a sizable and negative orbital angular momentum. Our finding indicates that the sea parton polarization in the valon remains very small, and hence its contribution to the spin structure of proton is consistent with zero. This finding is in agreement with



**Figure 9:** Gluon,  $\Delta g$ , and orbital angular momentum,  $L_z$ , components in proton as a function of  $Q^2$ .

the experimental results [18, 19]. The picture presented here is capable of reproducing all available data on  $g_1^{p,n,d}(Q^2)$  with good accuracy. We have further calculated the orbital angular momentum contribution, and its evolution, to the spin content of proton and valon. It appears that the size of gluon contribution to the spin content of proton is around 60%, somewhat similar to the momentum contribution of gluon to the momentum of proton in the unpolarized case. This value is also sensitive to the initial scale,  $Q_0^2$ . The model presented here does not have any free parameter, it is free of data fitting and solely relies on QCD processes, except the use of phenomenological concept of the valon model. Finally, we stress that there is the issue of initial input densities at scale,  $Q_0^2$ . While the most theoretical analysis and global fits begin with  $Q_0^2 \geq 1 \text{ GeV}^2$ , we have used the mathematical boundary conditions of the model and have shown that the results are compatible with the experimental data.

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