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Optimum Parameters for Tuned Mass Damper Using Shuffled Complex Evolution (SCE) Algorithm

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ABSTRACT: This study is investigated the optimum parameters for a tuned mass damper (TMD) under the seismic excitation. Shuffled complex evolution (SCE) is a meta-heuristic optimization method which is used to find the optimum damping and tuning frequency ratio for a TMD. The efficiency of the TMD is evaluated by decreasing the structural displacement dynamic magnification factor (DDMF) and acceleration dynamic magnification factor (ADMF) for a specific vibration mode of the structure. The optimum TMD parameters and the corresponding optimized DDMF and ADMF are achieved for two control levels (displacement control and acceleration control), different structural damping ratio and mass ratio of the TMD system. The optimum TMD parameters are checked for a 10-storey building under earthquake excitations. The maximum storey displacement and acceleration obtained by SCE method are compared with the results of other existing approaches. The results show that the peak building response decreased with decreases of about 20% for displacement and 30% for acceleration of the top floor. To show the efficiency of the adopted algorithm (SCE), a comparison is also made between SCE and other meta-heuristic optimization methods such as genetic algorithm (GA), particle swarm optimization (PSO) method and harmony search (HS) algorithm in terms of success rate and computational processing time. The results show that the proposed algorithm outperforms other meta-heuristic optimization methods.

Keywords: Dynamic Magnification Factors, Earthquake Excitation, Response Reduction, Shuffled Complex Evolution (SCE), Tuned Mass Damper (TMD)

INTRODUCTION

Tuned mass damper (TMD) has attracted the attention of many researchers in the field of passive control devices in recent years. A TMD consists of a lumped mass with a spring and viscous damper that is usually attached to the top of a building to attenuate undesirable vibration in the structure. The TMD concept was firstly suggested by Frahm (1909), who invented an undamped dynamic vibration absorber to decrease the rolling motion of a ship and ship hull vibration. Ormondoyd and Den Hartog (1928) used a damped TMD to mitigate the vibration of an undamped single degree of freedom (SDOF) structure subjected to sinusoidal force excitation. After that, Den Hartog (1956) investigated the optimum TMD parameters in an undamped SDOF structure for external and support harmonic excitation. He also developed closed-form expressions for optimum tuning frequency and damping ratio of TMD in the term of TMD mass.

Later on, many researchers such as (Snowdon 1959; Falcon et al. 1967; Gupta

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and Chandrasekaran 1969; Wirsching and Yao 1973; Wirsching and Campbell 1973; Dong 1976; McNamara, 1977; Jagadish et al., 1979; Luft, 1979; Warburton and Ayorinde, 1980; Kaynia et al., 1981 and Thompson, 1981) investigated optimum TMD parameters where the damping was included in the main structure. They found optimum parameters for different types of excitations including: stationary and non-stationary base acceleration modeled as Gaussian white-noise random processes. Sladek and Klingner (1983) used the Den Hartog formula to find the optimum frequency and damping ratio for a TMD. The results of analysis of a 25-storey building using 2 time history records revealed that the TMD was not effective in decreasing the structural response. Villaverde (1985), Villaverde and Koyama (1993), Villaverde (1994), Villaverde and Martin (1995) analytically determined that a TMD performed best in seismic applications when the first 2 complex modes of vibration of the combined structure and the TMD have the same modal damping ratio as the average damping ratio of the structure and TMD. Tsai and Lin (1993) proposed a numerical searching procedure to find the optimum tuning frequency and damping ratio for a TMD by minimizing the steady-state response of a structure for two different harmonic excitations including: displacement and acceleration base excitation. They also derived explicit formulae using a curve-fitting sequence.

There was no general agreement on the efficiency of TMD in decreasing the structural response for seismic applications until Sadek et al. (1997). They showed that the Villavede’s formulation does not result in an equal modal damping ratio in the first 2 modes of vibration for mass ratios larger than 0.005. They investigated the optimum parameters for TMD and found that TMD effectively decreased the peak response of a multi-DOF structure for earthquake excitations. After that, numerical searching method is widely used for optimization of TMD and multi-TMDs parameters (Rana and Soong, 1998; Li, 2002; Lee et al., 2006; Bakre and Jangid, 2007).

Rana and Soong (1998) used the numerical approach and determined optimum tuning frequency and damping ratio for a TMD. They also studied the effect of detuning on TMD performance and suggested that the optimum TMD parameters differ depending on whether a TMD is designed for the fundamental frequency of base excitation or the dominant frequency of main mass vibration. Bakre and Jangid (2007) employed a numerical searching method by minimizing the mean square responses of displacement and velocity of the main system and forces transmitted to the support under external force and base acceleration modeled as a Gaussian white-noise random process. They found that the optimum tuning frequency of TMD was strongly affected by the damping level of the system, but the optimum damping ratio was not sensitive to the structural damping ratio.

Optimizations of TMD problems have been carried out using either conventional mathematical methods (Villaverde, 1985; Villaverde and Koyama, 1993) or numerical searching techniques (Tsai and Lin, 1993; Sadek et al., 1997; Rana and Soong, 1998; Li, 2002; Bakre and Jangid, 2007). The numerical searching approaches are usually time-consuming and conventional methods require exact mathematical calculations.

The difficulties associated with mathematical optimization methods and numerical searching procedures have contributed to the development of evolutionary algorithms. These types of algorithms are stochastic numerical search methods derived from biological evolution or the social behavior of a species. All the meta-heuristic methods have a common approach. They find the optimum solution of a problem like the numerical searching techniques, but with the better convergence rate and lower computational processing times. The oldest evolutionary-based method is the genetic algorithm (GA) developed by Holland (1992). GA is based on the Darwinian principle of the survival of the fittest and evolution through reproduction. In GA algorithm, the solution is saved in the form of a chromosome consisting of a set of genes. The GA process continues by producing the offspring chromosomes. If they provide a better solution, they will be used to evolve the population. Finally, the individual with the best fitness is chosen as the optimum solution. Particle swarm optimization (PSO) method is another meta-heuristic algorithm developed by Kennedy and Eberhart (1995). It is based on the social behavior of a bird flock migrating to an unknown destination. In this method, each solution is a bird in the flock called as a particle. In PSO process, the birds do not spawn, but evolve a social behavior to reach a destination. Harmony search (HS) is a random search method developed by Geem et al. (2001) based on a musical performance. Music increases in enjoyment for the listeners when the musician searches for better harmony in a musical instrument. Like this, optimization is developed to find the optimum solution (perfect state) of a given problem with minimal cost.

These meta-heuristic methods have been successfully applied to optimize TMD parameters. Hadi and Arfiadi (1998) used GA to find the optimum damping and tuning frequency ratio of a TMD for a 10-storey shear building by minimizing the maximum displacement of the building floors. Leung et al. (2008) found the optimum TMD parameters for a SDOF structure subjected to a non-stationary base excitation using PSO algorithm. Leung and Zhang (2009) also used PSO to find the optimums TMD parameters by minimizing the mean square displacement response and displacement amplitude of the main system for base ground acceleration and external dynamic forces. Explicit formulae have also been developed using the curve-fitting technique. Bekdaş and Nigdeli (2011, 2013) proposed HS for optimization of TMD parameters under harmonic base acceleration. Farshidianfar and Soheili (2013 a,b) used the artificial bee colony (ABC) and ant colony (AC) approaches to find the best TMD parameters for decreasing the seismic vibration of a 40-storey building considering soil-structure interaction (SSI) effects. The effectiveness of TMD in wind excitation considering SSI effects has been also demonstrated by Liu et al (2008). The optimum parameters of TMD in inelastic structures is investigated by Woong (2008), Woong and Johnson (2009), Sgobba and Marano (2010), Woo et al. (2011), Mohebbi, and Joghataie (2012).

Duan et al. (1994) presented an optimization method known as the shuffled complex evolution (SCE) for dealing with the exotic problems encountered in conceptual watershed model calibration. This method is based on a combination of the best features from several existing methods. SCE has been used to optimize many types of engineering problems such as for water distribution networks (Eusuff and Lansey 2003, Liong and Atiquzzaman 2004). SCE is a meta-heuristic algorithm designed to explore space for obtaining global optima using a population of potential solutions. This algorithm solves the optimization problems through the following steps: 1) combination of deterministic and probabilistic approaches, 2) systematic evolution of a complex of points, 3) competitive evolution and 4) complex shuffling. Compared with other meta-heuristic methods like GA, PSO and HS algorithms, SCE has attractive features that include ease of application, a fast convergence rate and low computational time.

To the best of our knowledge, there is no published paper on the optimization of TMD parameters using SCE algorithm. In this paper, SCE is utilized to get the optimum tuning frequency and damping ratio of a TMD system to control the first vibration mode of a real structure. For a fast and general optimization, a harmonic base acceleration is used. The minimization of displacement and acceleration dynamic magnification factors (DDMF and ADMF) is chosen as the optimization criteria. The final optimum TMD parameters are examined for a 10-storey building under earthquake excitation and the results are compared with the results of other approaches (Den Hartog 1956, Sadek et al. 1997, Hadi and Arfiadi 1998). To show the efficiency of the adopted algorithm (SCE), a comparison is also made between SCE and other meta-heuristic optimization methods (GA, PSO and HS) in terms of success rate and computational processing time.

STRUCTURAL MODEL

A TMD is assumed to be a point mass connected to a structure through a spring and viscous damper. Most studies show that only the first mode of a structure gives a good approximation of structural responses. As a result, the structure is modeled as a SDOF system. In this study, TMD is taken into account for the control of the first vibration mode of a structure and the analytical model shown in Figure 1. Since TMD is also modeled as a SDOF system, the total DOF of the combined structure and TMD is considered to be 2.

The equations of motion for the structure and TMD under base excitation are expressed with Eqs. (1) and (2), respectively.

$$m_s(\ddot{x}_s+\ddot{x}_g)+c_s\dot{x}_s+k_sx_s = c_d(\dot{x}_d-\dot{x}_s)+k_d(x_d-x_s) \quad (1)$$

$$m_d(\ddot{x}_d+\ddot{x}_g)+c_d(\dot{x}_d-\dot{x}_s)+k_d(x_d-x_s)=0 \quad (2)$$

The parameters of the system are described by first-mode modal characteristics; m_s: is the first-mode modal mass, c_s: is the first-mode modal damping and k_s: is the first-mode modal stiffness of a multi-DOF structure. Similar to the main system, m_d, c_d and k_d: are the mass, damping and stiffness of the TMD system, respectively. x_s: is the displacement of the main mass system and x_d: is the displacement of the TMD system -both modeled as a SDOF system- with respect to the ground. $\ddot{x}_g$: is the ground acceleration.

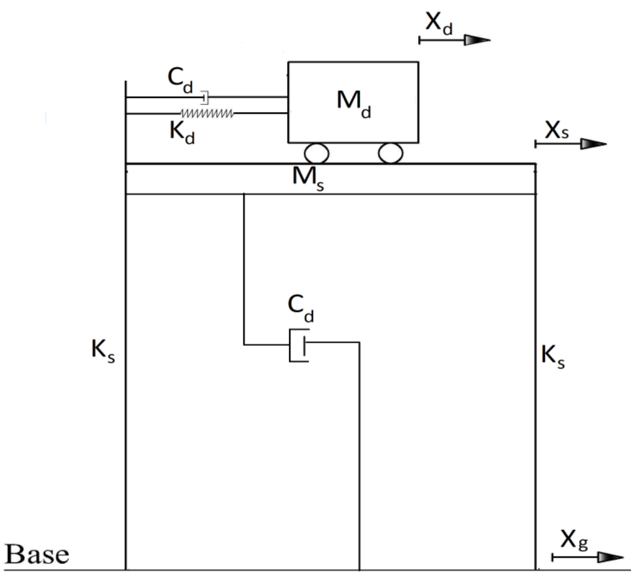

Fig. 1. SDOF structure with a TMD.

The natural frequency and viscous damping ratio of the main system are defined as the first mode frequency and the first modal damping ratio of a multi-DOF structure, respectively, shown by Eq. (3).

$$w_s = \sqrt{\frac{k_s}{m_s}}, \ \xi_s = \frac{c_s}{2m_s w_s} \quad (3)$$

Similary, $w_d = \sqrt{\frac{k_d}{m_d}}$ and $\xi_d = \frac{c_d}{2m_d w_d}$ denote the natural frequency and damping ratio of the TMD, respectively.

For a fast and general analysis, the external force is considered to be harmonic base acceleration, $F_{ext} = -m\ddot{x}_g$ where $\ddot{x}_g = e^{iwt} = Cos\, wt + i\, Sin\, wt$ and w is the natural frequency of harmonic loading. The steady-state response of the main and TMD systems can be estimated as $x_s = H_s e^{iwt}$ and $x_d = H_d e^{iwt}$, respectively. By substituting the response equations (x_s and x_d) into Eqs. (1) and (2), the transfer function of the combined structure with TMD can be expressed as in Eq. (4).

$$H_s = -\frac{m_d(-m_d w^2 + c_d iw + k_d) + .. \ m_d(c_d iw + k_d)}{(-m_s w^2 + c_s iw + k_s + c_d iw + k_d)^* \ (-m_d w^2 + c_d iw + k_d) - (c_d iw + k_d)^2} \quad (4)$$

After some manipulation and simplification, the DDMF is derived as given in Eq. (5), where λ is the ratio of the excitation frequency to the first mode frequency of the structure ($\lambda = \frac{w}{w_s}$). The mass ratio (μ) is the mass TMD to the main mass system ($\mu = \frac{m_d}{m_s}$) and tuning frequency ratio (f) is the TMD frequency to the first mode frequency of the structure ($f = \frac{w_d}{w_s}$).

$$DDMF = \left| w_s^2 H_s \right| = \frac{\sqrt{\left[f^2(1+\mu) - \lambda^2\right]^2 + \ldots \ 2\lambda f\, \xi_d (1+\mu)^2}}{\sqrt{\left[\mu f^2 \lambda^2 - (\lambda^2 - 1)(\lambda^2 - f^2) + 4\xi_s \xi_d f \lambda^2\right]^2 + \ldots \ \left[2\lambda \xi_d f (\lambda^2 + \mu\lambda^2 - 1) + 2\lambda \xi_s (\lambda^2 - f^2)\right]^2}} \quad (5)$$

The ADMF is calculated as following:

$x_s = H_s e^{-iwt}$, $\dot{x}_s = (-iw) H_s e^{-iwt}$, $\ddot{x}_s = (-w^2) H_s e^{-iwt}$

The absolute acceleration of the main system is obtained by adding ground acceleration $\ddot{x}_g$ to relative acceleration $\ddot{x}_s$,

$$\ddot{x}_s + \ddot{x}_g = -w^2 H_s e^{-iwt} + e^{-iwt}$$
$$= (1 - w^2 H_s) e^{-iwt} = H_a e^{-iwt}$$

The absolute value of H_a is the ADMF and is expressed as in Eq. (6).

$$ADMF = \left| H_a \right| = \frac{\sqrt{\left[\mu f^2 \lambda^2 - (\lambda^2 - 1)(\lambda^2 - f^2) + 4\xi_s \xi_d \lambda^2 + f^2 \lambda^2 (1+\mu) - \lambda^4\right]^2 + \ldots \ \left[2\lambda \xi_d f (\lambda^2 + \mu\lambda^2 - 1) + 2\lambda \xi_s (\lambda^2 - f^2) + 2\lambda^3 f\, \xi_d (1+\mu)\right]^2}}{\sqrt{\left[\mu f^2 \lambda^2 - (\lambda^2 - 1)(\lambda^2 - f^2) + 4\xi_s \xi_d f \lambda^2\right]^2 + \ldots \ \left[2\lambda \xi_d f (\lambda^2 + \mu\lambda^2 - 1) + 2\lambda \xi_s (\lambda^2 - f^2)\right]^2}} \quad (6)$$

Now, the cost function is defined by Eq. (7):

$$CF(x_{de}) = \alpha.DDMF + \beta.ADMF \qquad (7)$$

where x_{de}: is the decision variable - $x_{de} = \{\xi_d, f\}$ - α and β: are the constant coefficients changes between 0 and 1.

The SCE algorithm is implemented to find the optimum damping (ξ_d) and tuning frequency ratio of the TMD (f) to minimize the cost function for different values of μ and α, β. In most studies, the mass ratio (μ) is not regarded as an important factor in optimization process (Hadi and Arfiadi, 1998; Bakre and Jangid, 2007; Leung and Zhang, 2009); however, considering mass ratio as an optimization parameter results in selection of a large and impractical mass. In this study, the optimization is performed for different mass ratios where a fixed mass ratio is assigned for each step of optimization.

When the optimum parameters of TMD are obtained, they will be examined for a 10-storey building (Sadek et al., 1997) under the Imperial Valley Irrigation District (El-Centro) 1940 NS ground acceleration record. For the purpose of time history analysis, the equation of motion of the structure with TMD is given by Eq. (8):

$$M\ddot{x} + C\dot{x} + Kx = -M\{1\}\ddot{x}_g \qquad (8)$$

where M, C and K: are the mass, damping, and stiffness matrices of the combined structure, respectively.

For an easy and fast analysis, the equation of motion is converted into a state space equation (Eq. 9).

$$\dot{X} = AX + BU \qquad (9)$$

Where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -1 \end{bmatrix},\ X = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix},\ \dot{X} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix},\ U = \ddot{x}_g \qquad (10)$$

SHUFFLED COMPLEX EVOLUTION (SCE)

The SCE algorithm is designed to find the global minimum of a function and was introduced by Duan et al. (1994). The SCE algorithm is not based on mathematical calculations, as are traditional optimization methods. In this algorithm, *s* solution vectors are randomly generated to form a population. Each solution vector consists of a set of values (ξ_d, f). The SCE algorithm operates according to the following steps:

Step 1: Initialize parameters

$$CF(x_{de}) = \alpha.DDMF + \beta.ADMF \qquad (11)$$

where *CF*: is objective function, x_{de}: is decision variables, $x_{de} = \{\xi_d, f\}$, α, β: are coefficients of the objective function, *n*: is number of decision variables, *s*: is number of solution vectors, *p*: is number of complexes, *m*: is number of points in each complexes, *q*: is number of sub-complexes, *B*: is number of iterations in CCE and *NI*: is number of iterations in SCE.

Step 2: Generate sample

Randomly generate *s* solution vectors in the probable parameters space and evaluate the objective function.

Step 3: Rank solution vectors

Sort the solution vectors in order of increasing objective function.

Step 4: Partition into complexes

Partition s into p complexes, each containing m points. Duan et al. (1994) suggested that the best value for m is $2n+1$. The complexes are partitioned so that the first complex contains every $p(k-1)+1$ ranked point; the second complex contains every $p(k-1)+2$ ranked point and so on, $k=1,2,....,m.$

Step 5: Evolve each complex

Each complex is then divided into q sub-complexes and propagates each sub-complex to find a new point with a smaller cost according to the Competitive Complex Evolution (CCE) algorithm. The CCE algorithm uses simplex downhill search schemes to effectively explore optimum solution vectors and is repeated for B times (B usually equals m). Parallel searching on several populations and application of the gradient simplex method to find the optimum solution indicates the superiority of the SCE over other meta-heuristic algorithms.

Step 6: Shuffle complexes

Combine the m points in the evolved complexes into a single sample population and sort the sample population in order of increasing objective function. Repartition the sample population into p complexes according to the procedure defined in step 4.

Step 7: Check the termination criterion

Steps 4, 5 and 6 are repeated until the termination criterion is satisfied. Figure 2 shows the schematic diagram of SCE algorithm.

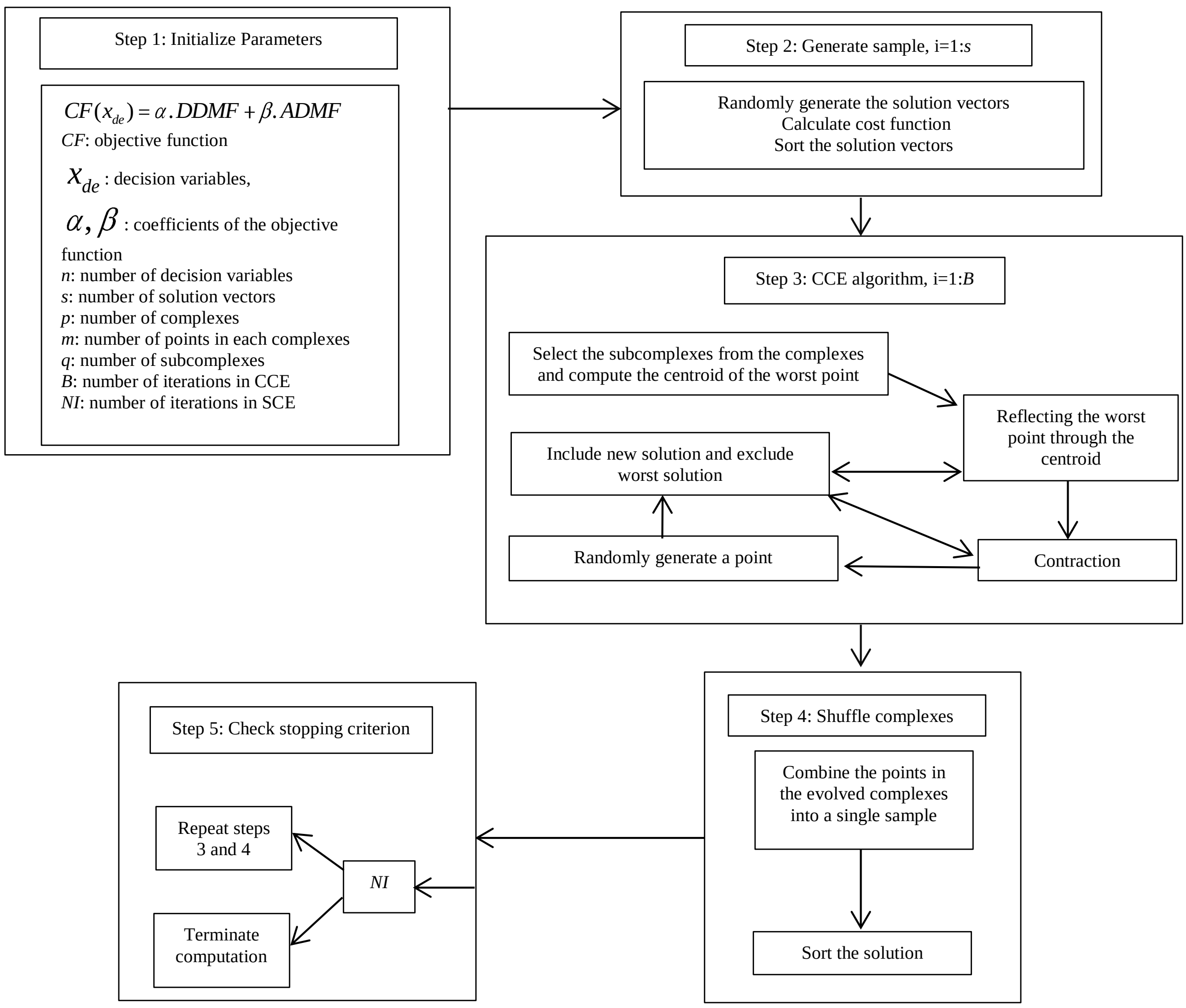

Fig. 2. Flowchart of SCE algorithm.

OPTIMUM TMD PARAMETERS

The SCE method is employed to find the optimum TMD parameters (i.e. the damping ratio and the tuning frequency ratio of the TMD). The upper bound and the lower bound of the tuning frequency ratio are assumed to be 0.8 and 1.2, while the upper and lower bounds for damping ratio are considered to be 0 and 0.5, respectively. The mass ratio is given as a fixed value for each step. The parameters of the SCE algorithm are chosen as: *n*=2, *s*=20, *p*=4, *m*=5, *q*=2, *B*=5 and *NI*=100. Optimum TMD parameters (ξ_d , f) and the corresponding response quantities of the main system (DDMF and ADMF) are obtained for different structural damping ratios (ξ_s), different mass ratios of the TMD system (μ) and two control levels.

Figures 3 and 4 show the variation of DDMF and ADMF versus the mass TMD ratio for different damping values of the main system and two cost functions ($\alpha=1, \beta=0$ and $\alpha=0, \beta=1$). Two bounds for each structural damping ratio demonstrate two distinct control levels (i.e. displacement control and acceleration control). In Figure 3, the lower bound indicates DDMF values when TMD is tuned for displacement control $(\alpha=1, \beta=0)$ and the upper bounds indicates DDMF values when TMD is tuned for acceleration control $(\alpha=0, \beta=1)$. In Figure 4, the lower bound indicates ADMF values when TMD is tuned for acceleration control $(\alpha=0, \beta=1)$ while the upper bounds indicates ADMF values when TMD is tuned for displacement control $(\alpha=1, \beta=0)$

Figures 3 and 4 show that the response of the main system (maximum displacement and acceleration) decreases as mass TMD ratio and structural damping ratio increase. It can also be understood that the maximum response of the main system differs according to the cost function. The differences in responses become clearer for low-level damping of the structure and the large mass ratio of the TMD system. The DDMF and ADMF can be obtained for any ideal cost function by interpolation method using Figures 3 and 4. All the results are presented in Table 1.

Table 1. Optimum TMD parameters for a single degree of freedom main system under harmonic base acceleration for different mass ratio and different cost function (ξs = 0.02, 0.05 and 0.1).

$\xi_{s=}$ 0.02	α=1, β=0 (Displacement Control)				α=0, β=1 (Acceleration Control)			
μ	$\boldsymbol{\xi_d}$	**f**	**DDMF**	**ADMF**	$\boldsymbol{\xi_d}$	**f**	**DDMF**	**ADMF**
0.005	0.0467	0.9901	11.7409	12.005	0.0467	0.9926	12.039	11.713
0.010	0.0656	0.9827	9.5283	9.8078	0.0646	0.9869	9.8819	9.4723
0.015	0.0797	0.9756	8.3392	8.6261	0.0794	0.9814	8.7154	8.2612
0.020	0.0923	0.9688	7.5567	7.8412	0.0913	0.9761	7.9541	7.4597
0.025	0.1036	0.9621	6.9885	7.2672	0.1019	0.9709	7.4034	6.8743
0.030	0.1138	0.9556	6.5504	6.8246	0.1112	0.9658	6.9817	6.4207
0.035	0.1230	0.9491	6.1989	6.4702	0.1205	0.9608	6.6396	6.0549
0.040	0.1318	0.9428	5.9086	6.1752	0.1293	0.9558	6.3574	5.7512
0.045	0.1406	0.9366	5.6636	5.9224	0.1374	0.9509	6.1210	5.4935
0.050	0.1489	0.9304	5.4531	5.7053	0.1444	0.9461	5.9229	5.2711
0.055	0.1566	0.9244	5.2700	5.5173	0.1522	0.9414	5.7450	5.0765
0.060	0.1647	0.9184	5.1087	5.3475	0.1592	0.9367	5.5913	4.9042
0.065	0.1719	0.9125	4.9653	5.1998	0.1662	0.9320	5.4541	4.7502
0.070	0.1795	0.9067	4.8367	5.0633	0.1725	0.9274	5.3339	4.6115
0.075	0.1868	0.9009	4.7207	4.9406	0.1793	0.9229	5.2228	4.4856
0.080	0.1937	0.8952	4.6154	4.8295	0.1857	0.9184	5.1229	4.3707
0.085	0.2007	0.8895	4.5192	4.7273	0.1917	0.9140	5.0332	4.2653
0.090	0.2071	0.8839	4.4310	4.6347	0.1974	0.9096	4.9523	4.1681
0.095	0.2142	0.8785	4.3498	4.5456	0.2033	0.9053	4.8761	4.0780
0.100	0.2209	0.8730	4.2746	4.4647	0.2091	0.9010	4.8063	3.9943
ξs= 0.05	α=1, β=0 (Displacement Control)				α=0, β=1 (Acceleration Control)			

μ	ξd	f	DDMF	ADMF	ξd	f	DDMF	ADMF
0.005	0.0511	0.9826	7.1242	7.3085	0.0502	0.9877	7.3075	7.1413
0.010	0.0699	0.9732	6.2862	6.4983	0.0693	0.9808	6.5194	6.2813
0.015	0.0856	0.9648	5.7702	5.9896	0.0834	0.9744	6.0382	5.7468
0.020	0.0982	0.9567	5.4018	5.6279	0.0958	0.9684	5.6938	5.3621
0.025	0.1096	0.949	5.1188	5.3471	0.1073	0.9626	5.428	5.0641
0.030	0.1205	0.9416	4.8911	5.1184	0.1167	0.9569	5.2195	4.8226
0.035	0.13	0.9343	4.7022	4.9299	0.1263	0.9514	5.0435	4.6206
0.040	0.1398	0.9273	4.5418	4.7659	0.1352	0.946	4.8954	4.448
0.045	0.1486	0.9204	4.4033	4.6254	0.143	0.9408	4.7704	4.2979
0.050	0.1574	0.9136	4.282	4.5002	0.1511	0.9356	4.6588	4.1655
0.055	0.166	0.907	4.1746	4.3886	0.1588	0.9305	4.5602	4.0475
0.060	0.1738	0.9005	4.0786	4.2901	0.1656	0.9255	4.4757	3.9412
0.065	0.182	0.8941	3.9922	4.1985	0.1729	0.9205	4.3967	3.8449
0.070	0.1895	0.8877	3.9138	4.117	0.1791	0.9157	4.3292	3.7569
0.075	0.1967	0.8814	3.8423	4.0427	0.1861	0.9109	4.2638	3.6761
0.080	0.2044	0.8753	3.7768	3.9724	0.1925	0.9062	4.2061	3.6015
0.085	0.2119	0.8693	3.7165	3.9071	0.1989	0.9015	4.1528	3.5324
0.090	0.2191	0.8633	3.6607	3.847	0.205	0.8969	4.1043	3.4681
0.095	0.2261	0.8574	3.609	3.7914	0.2109	0.8924	4.0598	3.4079
0.100	0.2327	0.8515	3.5609	3.7403	0.2166	0.8879	4.019	3.3516

ξs= 0.1	α=1, β=0 (Displacement Control)				α=0, β=1 Acceleration Control)			
μ	**ξd**	**f**	**DDMF**	**ADMF**	**ξd**	**f**	**DDMF**	**ADMF**
0.005	0.0573	0.9652	4.2706	4.441	0.0562	0.9758	4.3819	4.3458
0.010	0.0784	0.9526	3.9839	4.1726	0.0759	0.9672	4.1368	4.0427
0.015	0.0946	0.9417	3.7898	3.9882	0.0915	0.9595	3.9706	3.8342
0.020	0.1084	0.9317	3.6423	3.8467	0.1044	0.9524	3.846	3.6736
0.025	0.1211	0.9223	3.5238	3.731	0.1153	0.9457	3.7482	3.5428
0.030	0.1326	0.9134	3.4249	3.6337	0.1257	0.9392	3.6658	3.4325
0.035	0.1433	0.9048	3.3405	3.5503	0.1351	0.933	3.5969	3.3372
0.040	0.154	0.8966	3.2672	3.4756	0.1444	0.927	3.5363	3.2534
0.045	0.1636	0.8886	3.2025	3.4107	0.153	0.9211	3.4839	3.1787
0.050	0.1733	0.8808	3.1449	3.3512	0.1612	0.9154	3.4379	3.1115
0.055	0.1822	0.8731	3.0931	3.2985	0.169	0.9098	3.3968	3.0504
0.060	0.1914	0.8658	3.0462	3.249	0.177	0.9043	3.3588	2.9944
0.065	0.1998	0.8584	3.0035	3.2048	0.1842	0.899	3.3258	2.9429
0.070	0.2088	0.8514	2.9643	3.1622	0.1911	0.8937	3.2964	2.8952
0.075	0.2171	0.8444	2.9283	3.124	0.1976	0.8886	3.2701	2.8509
0.080	0.2255	0.8376	2.895	3.0875	0.2048	0.8835	3.2439	2.8094
0.085	0.2333	0.8308	2.8641	3.0547	0.2118	0.8785	3.2201	2.7706
0.090	0.2414	0.8242	2.8354	3.023	0.2178	0.8736	3.2	2.734
0.095	0.2497	0.8177	2.8085	2.9926	0.2237	0.8688	3.1819	2.6996
0.100	0.2578	0.8114	2.7834	2.9641	0.2307	0.864	3.1621	2.667

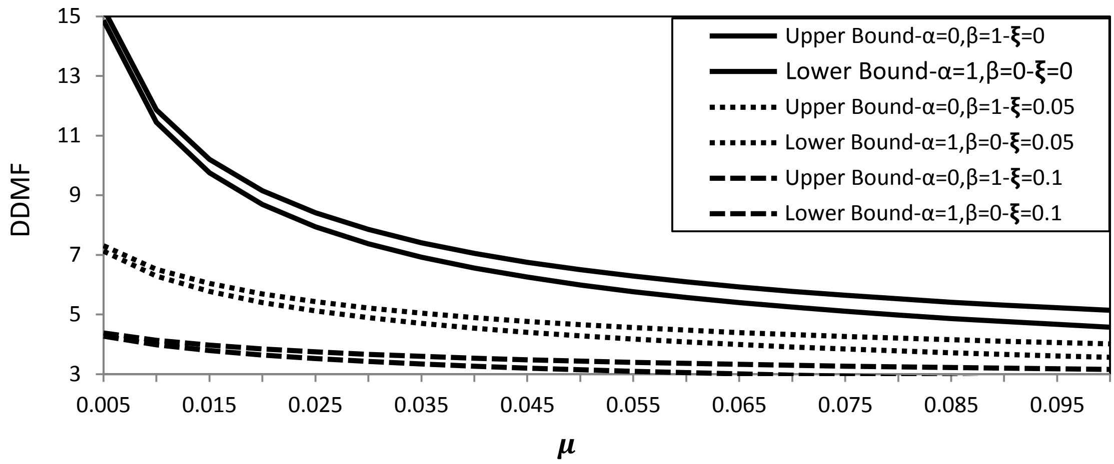

Fig. 3. Optimum DDMF.

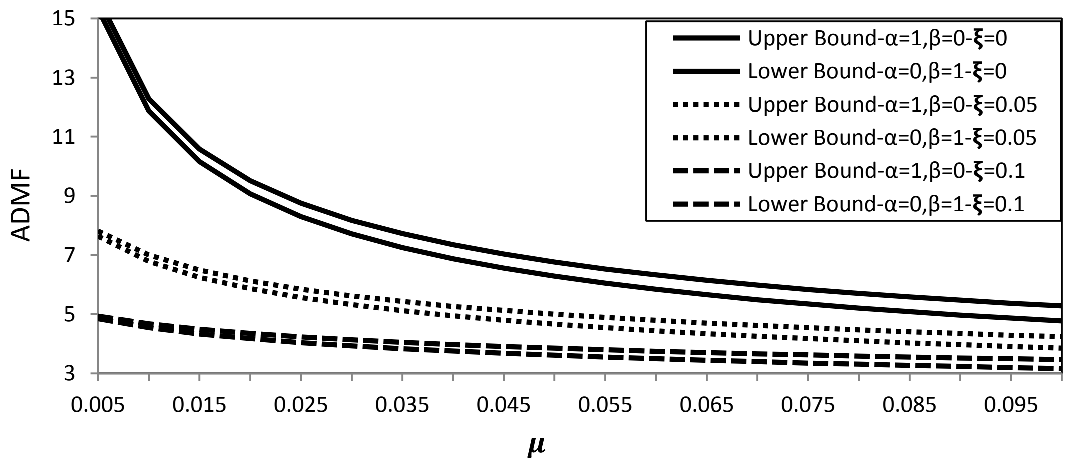

Fig. 4. Optimum ADMF.

The optimum tuning frequency and damping ratio of the TMD system versus mass ratio are plotted in Figures 5 and 6, respectively. The diagrams have been depicted for three structural damping ratios and two cost functions. Figure 5, shows that the optimum tuning frequency ratio of the TMD decreases as the mass TMD ratio increases. It can also be observed that high-level structural damping leads to lower optimum tuning frequencies. It can also be clearly seen that the optimum tuning frequency ratio of the TMD is dependent on the cost function. The higher the damping of the main system and the lager the TMD mass ratio, the greater difference in the optimum tuning frequency ratio for the two control levels is provided. This difference is clearly seen for large mass ratio (more than 3%) and high structural damping (ξ_s=10%). Figure 6 shows that the optimum TMD damping ratio increases as the mass TMD ratio and structural damping increase. For small mass ratio values, the optimum TMD damping ratio is slightly affected by structural damping level. Significant differences in optimum damping are observed for the two cost functions (displacement and acceleration control levels), where a large TMD mass ratio is considered.

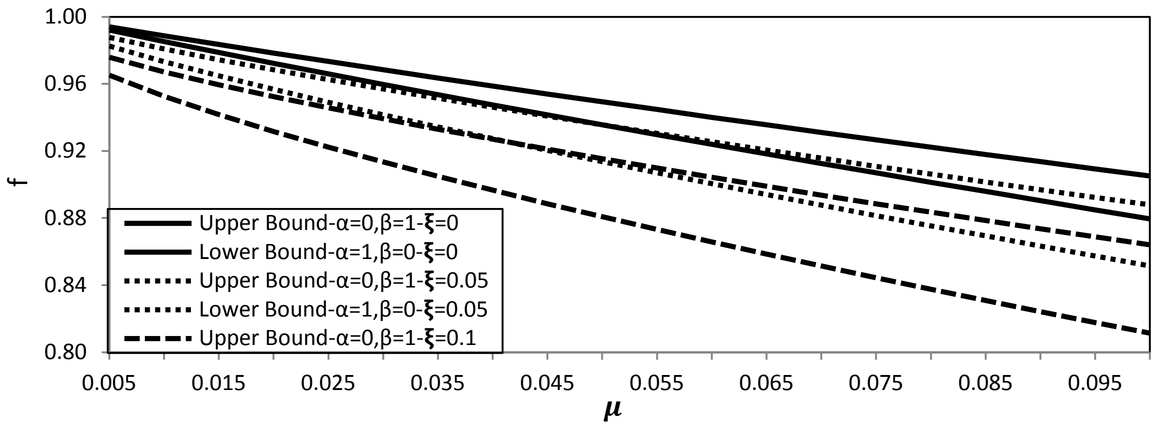

Fig. 5. Optimum tuning frequency ratio of TMD.

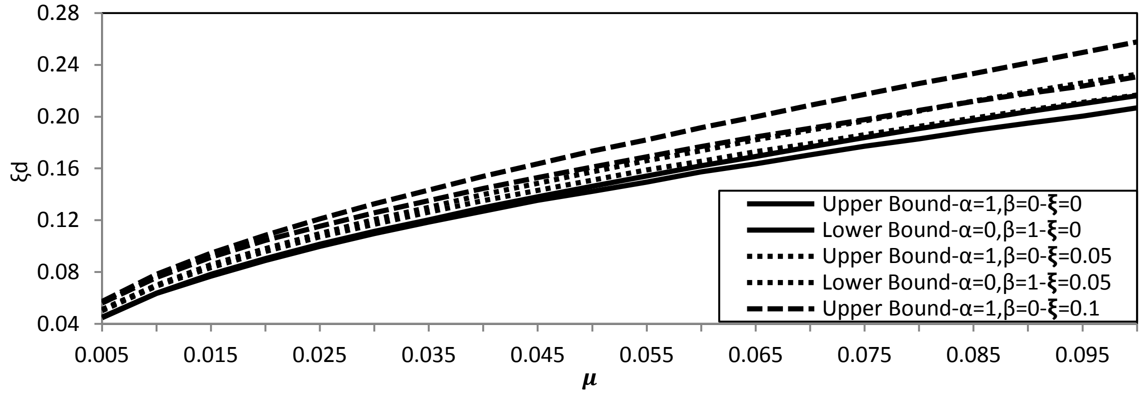

Fig. 6. Optimum damping ratio of TMD.

NUMERICAL EXAMPLE

To examine the optimum TMD parameters obtained by the proposed approach, a 10-storey shear building with a TMD on the top floor is considered. TMD parameters have been optimized for this structure previously by Sadek et al. (1997). The mass and stiffness of the building are provided in Table 2.

Table 2. System Properties.

Storey	Stiffness (kN/m)	Mass (Ton)
1	62470	179
2	52260	170
3	56140	161
4	53020	152
5	49910	143
6	46790	134
7	43670	125
8	40550	116
9	37430	107
10	34310	98

The only information available about the damping of the structure is the value of the first modal damping ratio ($\xi_1 = 0.02$). Hadi and Arfiadi (1998) found that the damping matrix should be proportional to the stiffness matrix (C=0.0129K), which makes the procedure developed in their paper comparable to the work of Sadek et al. (1997). In this study, the damping matrix is considered as that suggested by Hadi and Arfiadi (1998).

Here, TMD is firstly designed to control the first modal displacement response of the 10-storey building $(\alpha = 1, \beta = 0)$. Given the properties of the first mode that need to be controlled, the TMD is designed in the same procedure as designing a TMD for a SDOF structure. The first-mode shape vector of the structure according to the procedure suggested by Rana and Soong (1998) is:

$$\varphi_1 = \{0.1274 \;\; 0.2755 \;\; 0.4053 \;\; 0.5308 \;\; 0.6486 \;\; 0.7550 \;\; 0.8467 \;\; 0.9203 \;\; 0.9724 \;\; 1.0\}^T$$

The first-mode modal mass is calculated as:

$$M_1 = \varphi_1^T M \varphi_1 = 6.0867e5\,kg$$

For the sake of comparison to the previous studies, the TMD mass is assumed to be 4% of the total building mass as in Hadi and Arfiadi (1998).

$$m_d = 0.04 \times 1385e3 = 55400\,kg$$

The mass ratio is defined as the damper mass to the first-mode modal mass.

$$\mu = \frac{55400}{6.0867 \times 10^5} = 0.09$$

The mass ratio (μ) and first-mode modal damping ratio $(\xi_s = \xi_1 = 0.02)$ are now known. Referring to the Table 1, μ and ξ_s are used to obtain the optimum TMD parameters as:

$$\xi_{dopt} = 0.2071,\ f_{opt} = 0.8839$$

Now, the TMD can be controlled the first-mode displacement response.

Similarly, the optimum TMD parameters are obtained for controlling the acceleration response $(\alpha = 0, \beta = 1)$ as:

$$\xi_{dopt} = 0.1974,\ f_{opt} = 0.9096$$

This completes the design process. The TMD is tuned once for displacement control and then again tuned for acceleration control. The optimum damping and tuning frequency ratio of TMD are presented in Table 3 and compared with those values obtained by other approaches.

The results show that the present method provides a smaller optimum tuning frequency ratio for TMD. A lower optimum tuning frequency, a smaller stiffness element is required for installation of the TMD. In this case, the cost of TMD may be decreased. The optimum damping ratios are obtained 20% and 19% for the case of displacement control and acceleration control, respectively. This is a medium and practical value that results in smaller TMD displacement (See Table 4). Therefore; TMD requires little space for movement.

Checking the optimum parameters of TMD for displacement control and acceleration control levels, the structure is then subjected to El-Centro 1940 NS excitation. The maximum displacement and acceleration of the floors obtained by the proposed method are compared with the results of Den Hartog (1956), Sadek et al. (1997) and Hadi and Arfiadi (1998) approaches. They are provided in Table 4 and shown in Figures 7 and 8.

Table 4 shows that when TMD is tuned for displacement control, the maximum displacement of the top floor decreases about 20% with respect to the response of the uncontrolled structure. Table 4 also shows that when TMD is tuned for acceleration control, the maximum acceleration of the top floor decreases about 30% with respect to the response of the uncontrolled structure.

Table 3. Optimum TMD parameters in different methods.

Method	f_{opt}	ξ_{dopt}
Present approach (Displacement Control)	0.88	0.20
Present approach (Acceleration Control)	0.90	0.19
Den-Hartog	0.92	0.18
Hadi and Arfiadi	0.91	0.15
Sadek et al.	0.93	0.32

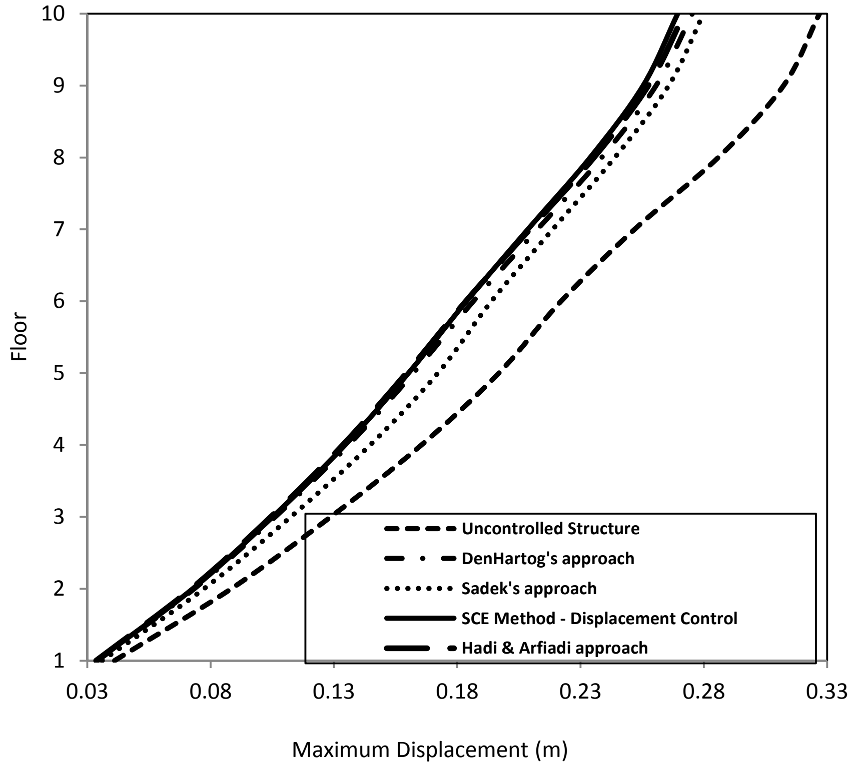

Fig. 7. Peak floors displacement- TMD is tuned for displacement control. $(\alpha = 1, \beta = 0)$.

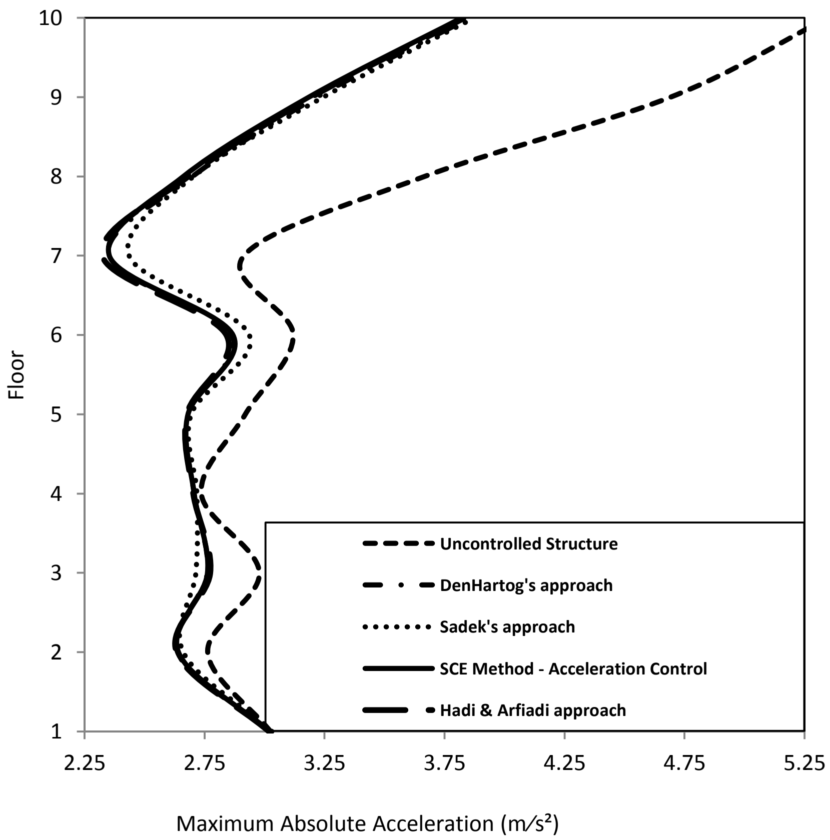

Fig. 8. Peak floors absolute acceleration- TMD is tuned for acceleration control $(\alpha = 0, \beta = 1)$.

Table 4. Maximum displacement and acceleration of a 10-storey building under El-Centro earthquake.

	Uncontrolled		**Den-Hartog**	
Storey	**Displacement(m)**	**Acceleration (m/s²)**	**Displacement (m)**	**Acceleration (m/s²)**
1	0.0411	3.0247	0.0342	3.0198
2	0.0884	2.7633	0.0734	2.6378
3	0.1287	2.9784	0.1062	2.7613
4	0.1655	2.7366	0.1360	2.7064
5	0.1969	2.9199	0.1622	2.6805
6	0.2221	3.1197	0.1866	2.8642
7	0.2519	2.9140	0.2125	2.3510
8	0.2859	3.6667	0.2386	2.7000
9	0.3124	4.6915	0.2607	3.2177
10	0.3269	5.3472	0.2754	3.8547
TMD	-	-	0.6035	4.8067

	Sadek et al.		**Hadi and Arfiadi**	
Storey	**Displacement (m)**	**Acceleration (m/s²)**	**Displacement (m)**	**Acceleration (m/s²)**
1	0.0360	3.0305	0.0336	3.0181
2	0.0773	2.6613	0.0721	2.6335
3	0.1125	2.7138	0.1044	2.7738
4	0.1447	2.7179	0.1336	2.7029
5	0.1722	2.6942	0.1594	2.6766
6	0.1939	2.9365	0.1835	2.8409
7	0.2187	2.4358	0.2094	2.324
8	0.2439	2.7041	0.2355	2.6829
9	0.2658	3.2446	0.2575	3.1954
10	0.2799	3.8731	0.2721	3.8335
TMD	0.4559	3.7376	0.6354	5.0049

	Present Approach (Displacement Control) (a=1, b=0)		**Present Approach (Acceleration Control) (a=1, b=0)**	
Storey	**Displacement (m)**	**Acceleration (m/s²)**	**Displacement (m)**	**Acceleration (m/s²)**
1	0.0339	3.0218	0.0343	3.0211
2	0.0728	2.6415	0.0736	2.6402
3	0.1052	2.7624	0.1065	2.7551
4	0.1345	2.7068	0.1363	2.7071
5	0.1602	2.6811	0.1624	2.6801
6	0.1838	2.8683	0.1866	2.8711
7	0.2087	2.3534	0.2122	2.3613
8	0.2340	2.6618	0.2380	2.6913
9	0.2554	3.1846	0.2599	3.2122
10	0.2694	3.8217	0.2743	3.8491
TMD	0.5424	4.3266	0.5702	4.5601

From Table 4, it is understood that when TMD is tuned for acceleration control, the peak displacement of floors reduces. In this case (acceleration control), the floors' displacement slightly increases when compared to those results that TMD is tuned for displacement control. From Table 4, it is observed that the acceleration responses are somewhat similar in two cases of displacement control and acceleration control. Therefore, it is recommended that TMD parameters are adjusted for displacement control rather than acceleration control. In this case (displacement control), the peak displacement of floors reduces at maximum value and the floors' acceleration decreases as nearly those values when TMD is tuned for acceleration control. In displacement control level, stroke of TMD are smaller than that of the acceleration control level.

Figure 7 shows the maximum displacement of floors obtained by the proposed method and compared with the results of other approaches (Den Hartog, 1956; Sadek et al., 1997 and Hadi and Arfiadi, 1998). Here, TMD is tuned for displacement control $(\alpha = 1, \beta = 0)$. In Den Hartog (1956), Sadek et al. (1997) and Hadi and Arfiadi (1998), the parameters of TMD are also optimized for displacement control. Figure 7 indicates that the results developed by SCE method agree well with those of Hadi and Arfiadi's approach. Although, the peak displacement responses of the building in other methods are almost similar to the proposed approach, using SCE as an optimizer tool decreases the maximum displacement of the top floor with about 7% more than that of the Sadek et al. (1997).

Figure 8 shows the maximum acceleration of floors obtained by the proposed method and compared with the results of other approaches (Den Hartog, 1956; Sadek et al., 1997 and Hadi and Arfiadi, 1998). Here, TMD is tuned for acceleration control $(\alpha = 0, \beta = 1)$. In Den Hartog (1956), Sadek et al. (1997) and Hadi and Arfiadi (1998), the parameters of TMD are already optimized for displacement control. It can be understood that the results obtained by SCE method for acceleration control can be matched to the other approaches in which TMD is tuned for displacement control. Finally, it is seen that although tuning a TMD for acceleration control decreases the peak displacement of the floors; it increases the floor displacement slightly over tuning TMD for displacement control (Table 4). Therefore; it is concluded that the best performance of a TMD is achieved by adjusting its parameters for only displacement control. In this case, both displacement and acceleration decrease to an appropriate level.

COMPARISON BETWEEN SCE AND OTHER META-HEURESTIC OPTIMIZATION ALGORITHMS

To compare the SCE algorithm with other meta-heuristic algorithms (GA, PSO, HS), all methods are coded using Matlab 2009 program on a 1.5 GHz dual core laptop machine. The performance of SCE is compared with the GA, PSO and HS algorithms for convergence speed and computational processing time.

Convergence speed is the rate at which an algorithm finds an optimum solution. It also shows the number of trails required for an objective function to reach an optimum value. Figure 9 shows the results of objective function versus number of iteration. As seen, SCE is reached to the optimum value faster than all the other methods and GA has the slowest convergence rate among all algorithms. Furthermore, SCE method provides the lowest cost function. It is also seen from Figure 9, that SCE found the minimum value of an objective function in less than 10 iterations which is 6 times less than PSO, 8 times less than HS and 14 times less than GA.

The computational processing time required to attain the target for each algorithm is observed in Figure 10. It can be seen that the SCE algorithm required the lowest convergence time and HS required the most convergence time. The results show that SCE algorithm outperformed GA, PSO and HS algorithms in finding an optimum solution of a problem in terms of convergence rate and computational processing time.

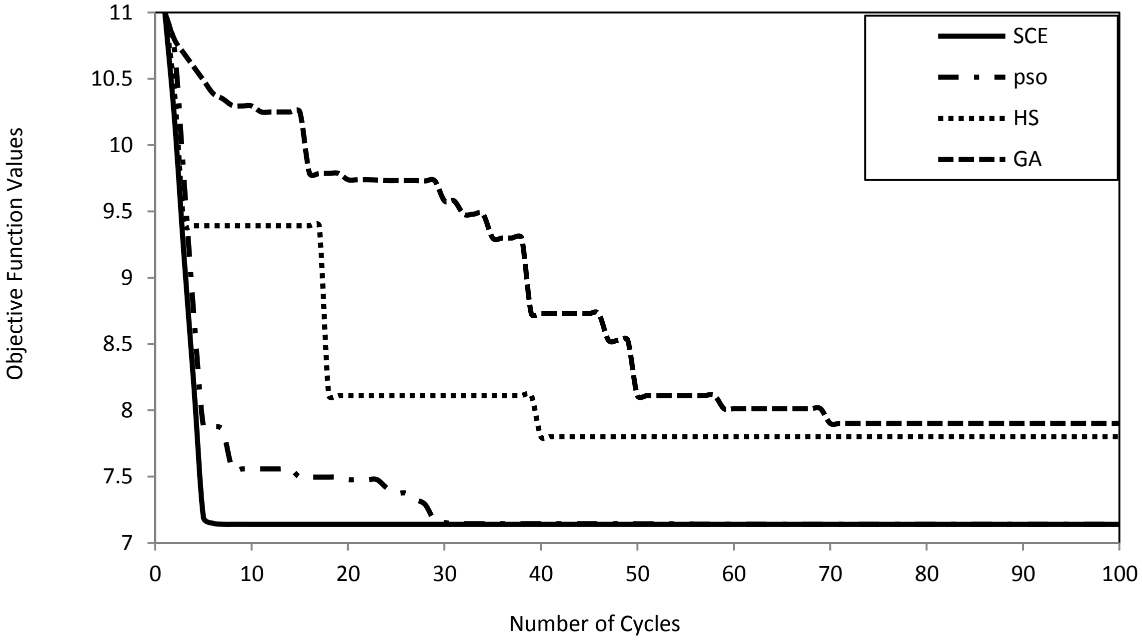

Fig. 9. Convergence rate for different optimization methods.

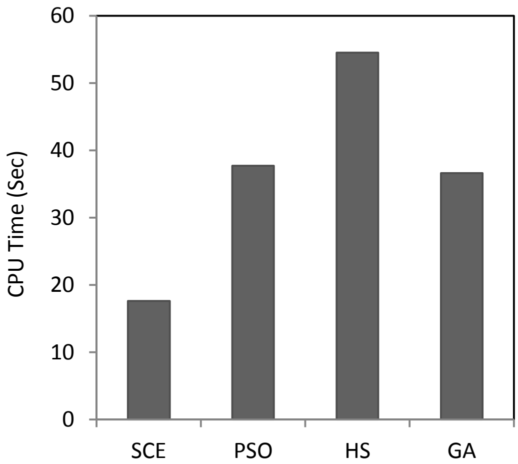

Fig. 10. Processing time to reach the target.

CONCLUSIONS

This is the first study on the application of shuffled complex evolution (SCE) algorithm to find optimum parameters for a TMD. SCE is used to find the optimum TMD parameters by minimizing the DDMF and ADMF parameters for a single-mode model subjected to base excitation. The final optimum TMD parameters are checked for a 10-storey shear building subjected to El-Centro earthquake excitation. The numerical results revealed that the TMD system is very effective in decreasing the displacement and acceleration of building's floors. Also, it is found that the optimum tuning frequency ratio of TMD obtained by the proposed procedure is smaller than that of the Den Hartog (1956), Sadek et al. (1997) and Hadi and Arfiadi (1998) approaches. A comparison of optimization procedures indicated that SCE is utterly effective in terms of convergence rate and computational processing time. Based on this study some conclusions can be drawn as follows:

- The optimum TMD parameters, i.e. tuning frequency ratio and damping ratio, differ depending on displacement control or acceleration control. This difference in optimum parameters becomes clearer for large mass ratio (more than 3%) and high structural damping ratio ($\xi_s = 10\%$).
- The maximum storey displacements of building obtained by SCE method are smaller than those obtained by the other optimization methods. This reduction is about 7% more than that of the Sadek et al. (1997) approach for the top floor displacement.
- SCE successfully solved this specific problem. This situation has very beneficial effect in which the computational time significantly decreased and cost of TMD reduced due to required elements with smaller stiffness.
- Since the maximum floors' acceleration in acceleration control level is approximately same with that of the displacement control level, it is suggested that TMD parameters are set for displacement control rather than the acceleration control.

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