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Information Loss for QCD Matter in AdS Black Holes at LHC

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Abstract *In this paper, we find the information loss for QCD matter in AdS black holes at LHC by extending the Gottesman and Preskill methode to AdS black holes. We calculate the information transformation from the collapsing matter to the state of outgoing Hawking radiation for both quarks and gluons. It is noticed that for finite values of quark and gluon energies, information from all emission processes experiences some degrees of loss. Possible explanation for this feature will be presented in this paper.*

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1 Introduction

The production of mini black holes at Large Hadron Collider (LHC) is among the most exciting subjects in physics.[1] It may significantly enhance the signal for the Higgs search at the LHC. In fact, there can be an enormous amount of Higgs particle production from black holes,[2] much more than what is expected from normal pQCD processes.[3] Another important subject, considering Schwarzschild AdS black holes at LHC, has been ignored so far. The present study is the first attempt to investigate this subject. The main question is the possibility of using Schwarzschild AdS metric[4] at LHC. Heavy quark jet quenching in nuclear collisions at LHC was predicted[5] using both AdS/CFT correspondence[6] and Standard Model. Furthermore, the heavy quark drag of a string dangling in a shock metric of AdS space has been derived.[5] Consequently, AdS metric can be applied at LHC. On the other hand, a small AdS black hole, radiates away its mass in a runaway process like an ordinary Schwarzschild black hole in an asymptotically flat spacetime.[7–8]

If AdS black holes are produced at LHC, inside of the black holes will not be accessible and the information will be lost,[1] leading to several problems in analyzing data at LHC. If mini black holes are produced at LHC, the information loss will result in a difference between the observed hadronic cross sections and the predicted cross sections. Since some of QCD matter is not able to come out of the black holes, it's effect on hadronization processes will be decreased. Fortunately, Horowitz and Maldacena have suggested a mechanism to reconcile the unitarily of black hole evaporation.[9] However,Gottesman and Preskill explained how interactions between the collapsing body and infalling Hawking radiation inside the event horizon could cause loss of information. They obtained the amount of this information loss.[10]

The aim of this paper is to study the information loss for QCD matter in AdS black holes at LHC. We also analyze the signature of Higgs boson radiation due to quark and gluon interactions near AdS black holes at LHC. The outline of the paper is as the following. In Sec. 2 we consider the Unruh states for QCD matter near AdS black holes at LHC. Next we study the entangled two-mode squeezed states inside black hole. Hilbert spaces of the evaporating AdS black holes for gluons and quarks are presented in Sec. 3. We calculate the information transformation from collapsing matter to outside of AdS black holes for QCD matter at LHC in Sec. 4. The last section is devoted to summary and conclusion.

2 The Unruh States for QCD Matter near AdS Black Holes at LHC

In this section we extend the results of the derivation of Hawking radiation for Scalar and Dirac fields[11–12] and also the results obtained in Refs. [1–2] for the quarks and gluons in AdS black holes. We show that the ground states for these partons are maximally entangled two-mode squeezed states on outside and inside Hilbert spaces of AdS black holes. We show that more QCD matters are produced near event horizon of AdS black holes compared to Schwarzschild black holes.

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Let us consider the gluon quantization, by using the transverse free field operator:[13]

$$A^a_\mu = \int \frac{\mathrm{d}^3k}{(2\pi)^3\sqrt{2\omega(k)}} \sum_{\lambda=1}^{3} \varepsilon^\lambda_\mu [a^a_\lambda(k)\,\mathrm{e}^{-\mathrm{i}kx} + a^{a\dagger}_\lambda(k)\,\mathrm{e}^{\mathrm{i}kx}], \quad (1)$$

ε^λ_μ are the polarization vectors satisfying the transversality condition

$$k^\mu \varepsilon^\lambda_\mu = 0, \quad (2)$$

which follows directly from the Lorentz condition. The gluon field satisfies the wave equation:[2,11]

$$(-g)^{1/2}\frac{\partial}{\partial x^\mu}\Big[g^{\mu\nu}(-g)^{1/2}\frac{\partial}{\partial x^\nu}\Big]A^a_{\rho,s} = 0. \quad (3)$$

The upper index ($a = 1,\dots,8$) is related to eight color of gluons, the lower index "s" ($s = 1, s_z = +1, -1$) denotes the spin of gluon, ρ ($\rho = 1,\dots,d$) is the vector index, d is the number of dimensions and $g^{\mu\nu}$ is the metric tensor.[2] To obtain the Unruh state for gluons, we solve Eq. (3) in AdS space-time. The AdS–Schwarzschild metric for the black hole in four dimensions is described by:[14]

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2\,\mathrm{d}\Omega^2_{d-1},$$

$$f(r) = 1 + r^2 - \frac{\mu}{r^{d-2}}, \quad (4)$$

where

$$\mu = \frac{16\pi G_{d+1} M_{\mathrm{BH}}}{(d-1)A_{d-1}}, \quad (5)$$

G_{d+1} is Newton's constant in $(d+1)$ dimensions and

$$A_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)}, \quad (6)$$

is the volume of a unit $(d-1)$-sphere.

The black hole horizon is located at:[14]

$$f(r) = 0 \to r^{\mathrm{Ads}}_H = \sqrt{\frac{1}{2}(\sqrt{1+4\mu} - 1)}. \quad (7)$$

The AdS Schwarzschild temperature can be obtained as the following:[14]

$$T_{\mathrm{Ads}} = \frac{f'(r_H)}{4\pi} = \frac{\sqrt{1+4\mu}}{2\pi r_H}. \quad (8)$$

Comparing the horizon of AdS black hole with the horizon of Schwarzschild black hole "$r^{\mathrm{Schwarzschild}}_H \sim 2M_{\mathrm{BH}}$", we observe that AdS black holes are smaller than Schwarzschild black holes of similar mass. In addition, the temperature of AdS black hole is higher than the temperature of Schwarzschild black hole. This will result in many differences in particle distributions near event horizon of these black holes.

In Kruskal coordinates the metric of the AdS Schwarzschild black hole becomes:[11]

$$\mathrm{d}s^2 = -r^{\mathrm{Ads}}_H \frac{\mathrm{e}^{-r/r^{\mathrm{Ads}}_H}}{r}\mathrm{d}\bar{u}\mathrm{d}\bar{v} + r^2\mathrm{d}\Omega^2 + g_{ij}\mathrm{d}x^i\mathrm{d}x^j,$$

$$\bar{u} = -2r^{\mathrm{Ads}}_H\,\mathrm{e}^{-u/2r^{\mathrm{Ads}}_H}, \quad \bar{v} = -2r^{\mathrm{Ads}}_H\,\mathrm{e}^{-v/2r^{\mathrm{Ads}}_H},$$

$$u = t - r^*, \quad v = t + r^*, \quad r^* = r + r^{\mathrm{Ads}}_H \ln\Big|\frac{r}{r^{\mathrm{Ads}}_H} - 1\Big|, \quad (9)$$

r^{Ads}_H is the black hole horizon. Using this fact that $\bar{v} = 0$ on past horizon[11] we may estimate the original positive frequency normal mode on past horizon:

$$A^a_{\mu,s} \propto \varepsilon^a_{\mu,s}(\mathrm{e}^{-\mathrm{i}\omega u}) = \varepsilon^a_{\mu,s}(|\bar{u}|/2r^{\mathrm{Ads}}_H)^{-\mathrm{i}4M_{\mathrm{BH}}\omega}$$

$$= \begin{cases} \varepsilon^a_{\mu,s}(-\bar{u}/2r^{\mathrm{Ads}}_H)^{-\mathrm{i}2r^{\mathrm{Ads}}_H\omega}, & \text{region I}, \\ \varepsilon^a_{\mu,s}(\bar{u}/2r^{\mathrm{Ads}}_H)^{-\mathrm{i}2r^{\mathrm{Ads}}_H\omega}, & \text{region II}, \end{cases} \quad (10)$$

ω is the gluon energy. In Eq. (10), we are using the fact that $(-1)^{-\mathrm{i}2r^{\mathrm{Ads}}_H\omega} = \mathrm{e}^{2\pi r^{\mathrm{Ads}}_H\omega}$. We observe that the states in the horizon satisfy the following condition:

$$(A^a_{\mu,s_{\mathrm{in}}} - \mathrm{e}^{2\pi r^{\mathrm{Ads}}_H\omega}A^a_{\mu,s_{\mathrm{out}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}} = 0, \quad (11)$$

or equivalently

$$(A^a_{\mu,s_{\mathrm{out}}} - \tanh r_\omega A^a_{\mu,s_{\mathrm{in}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}} = 0,$$

$$\tanh r_\omega = \mathrm{e}^{-2\pi r^{\mathrm{Ads}}_H\omega}, \quad (12)$$

which actually constitutes a boundary state. Using the expansion in modes for gluons (Eq. (1)) we write:

$$(\alpha^a_{\lambda,s_{\mathrm{out}}} - \tanh r_\omega \alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}} = 0,$$

$$\text{for } n \neq 0, \quad \tanh r_\omega = \mathrm{e}^{-2\pi r^{\mathrm{Ads}}_H\omega}. \quad (13)$$

Now, we assume that the Kruskal vacuum

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}$$

for AdS black hole depends on the Schwarzschild vacuum $|0\rangle_S$ by

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}} = F(\alpha^a_{\lambda,s_{\mathrm{out}}},\alpha^a_{\lambda,s_{\mathrm{in}}})|0\rangle_S, \quad (14)$$

F is some function to be determined later.

From

$$[\alpha^a_{\lambda,s_{\mathrm{out}}},\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}}] = 1,$$

we obtain

$$[\alpha^a_{\lambda,s_{\mathrm{out}}},(\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}})^m] = \frac{\partial}{\partial\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}}}(\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}})^m$$

and $[\alpha^a_{\lambda,s_{\mathrm{out}}},F] = \partial F/\partial\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}}$.[2] Using Eqs. (13) and (14), we get the following differential equation for F.

$$\Big(\frac{\partial F}{\partial\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}}} - \tanh r_{\omega_n,\lambda}\alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}F\Big) = 0, \quad (15)$$

and the solution is given by

$$F = \mathrm{e}^{\tanh r_\omega \alpha^\dagger_{\lambda,s_{\mathrm{out}}}a\alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}}. \quad (16)$$

By substituting Eq. (16) into Eq. (14) and also by properly normalizing the state vector, we get

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}} = \mathrm{e}^{\tanh r_\omega\alpha^{a\dagger}_{\lambda,s_{\mathrm{out}}}\alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}}|0\rangle_S = \frac{1}{\cosh r_\omega}\sum_n \tanh^n r_\omega |n,\lambda,a,s\rangle_{\mathrm{in}} \otimes |n,\lambda,a,s\rangle_{\mathrm{out}}. \quad (17)$$

Summing over transversal (physical) degrees of freedom we obtain:

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}=\sum_{\lambda=1}^{3}\varepsilon_\mu^\lambda|\mathrm{BH},\lambda,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}=\frac{1}{\cosh r_\omega}\sum_n\tanh^n r_\omega|n,\mu,a,s\rangle_{\mathrm{in}}\otimes|n,\mu,a,s\rangle_{\mathrm{out}}\,,\tag{18}$$

where $|n,\mu,a,s\rangle_{\mathrm{in}}$ and $|n,\mu,q,s\rangle_{\mathrm{out}}$ are orthonormal bases (normal mode solutions) for H_{in} and H_{out} of AdS black hole respectively. We observe that the ground state for gluons near AdS horizon is a maximally entangled with two-mode squeezed states on outside and inside Hilbert spaces of AdS black holes. Equation (18) shows that different number of gluons (n) produced with different probabilities inside and outside of AdS black holes at LHC. These probabilities depend both on black hole mass and on the energy of gluon

$$P_{n,\omega}\approx|{}_{\mathrm{out}}\langle n,\mu,a,s|\otimes_{\mathrm{in}}\langle n,\mu,a,s|\mathrm{AdsBH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}|^2=\frac{\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}{\cosh^2 r_\omega}\,.$$

It seems that more gluons are produced near AdS horizons compared to Schwarzschild horizons due to variety in their horizon and temperature. We obtain the thermal distribution for these groups of gluons as the following:

$$\begin{aligned}N^{\mathrm{gluon}}_{\omega,\mathrm{color,spin}}&={}_{\mathrm{in}\otimes\mathrm{out}}\langle\mathrm{Ads\,BH},\mu,a,s|\alpha^{a\dagger}_{\mu,s_{\mathrm{out}}}\alpha^{a}_{\mu,s_{\mathrm{out}}}|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}\\&={}_{\mathrm{out}}\langle n,\mu,a,s|\,{}_{\mathrm{in}}\langle n,\mu,a,s\Big|\frac{1}{\cosh^2 r_\omega}\alpha^{a\dagger}_{\mu,s_{\mathrm{out}}}\alpha_{\mu,s^a_{\mathrm{out}}}\sum_{n=0}^{\infty}\tanh^{2n}(r_\omega)\Big|n,\mu,a,s\rangle_{\mathrm{in}}|n,\mu,a,s\rangle_{\mathrm{out}}\\&={}_{\mathrm{out}}\langle n-1,\mu,a,s|_{\mathrm{in}}\langle n,\mu,a,s\Big|\frac{1}{\cosh^2(r_\omega)}\sum_{n=0}^{\infty}\tanh^{2n}(r_\omega)(n)|n,\mu,a,s\rangle_{\mathrm{in}}\Big|n-1,\mu,a,s\rangle_{\mathrm{out}}\\&=\frac{1}{\cosh^2 r_\omega}\sum_{n=0}^{\omega}\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}(n)=\frac{1}{\cosh^2 r_\omega}\frac{\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}{(1-\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega})^2}=\frac{\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}{1-\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}\,,\end{aligned}\tag{19}$$

where $\alpha^{a\dagger}_{\mu,s_{\mathrm{out}}}\alpha^{a}_{\mu,s_{\mathrm{out}}}$ are creation and annihilation operators that act on black hole outside states of gluons. $N^{\mathrm{gluon}}_{\omega,\mathrm{color,spin}}$ is the thermal distribution for gluons with energy ω and with a one special color and spin. Comparing this distribution with the gluon distribution near Schwarzschild black hole in Ref. [1], we observe that more gluons are produced near AdS black holes compared to Schwarzschild black holes at LHC.

At this stage we study quark production via Unruh effect near AdS black holes at LHC.

The quark equation in black hole space-time may be calculated as:[12]

$$[\mathrm{i}\gamma^\mu(\partial_\mu+\Gamma_\mu)-m_q]\psi^a_{q,s}=0\,,\tag{20}$$

$a=1,2,3$ is related to the color of quark, s denotes the quark spin, m_q is the quark mass and $q=u,\ t,\ b,\ s,\ c,$ d represents the quark flavors. The affine connection is given by:

$$\Gamma_\mu=-\frac{1}{4}\gamma_\nu(\partial_\mu\gamma^\nu+\Gamma^\nu_{\mu\lambda}\gamma^\lambda)\,.\tag{21}$$

The gamma matrices are defined by $\gamma_\mu=e^a_\mu\bar{\gamma}_a$ where e^a_μ are tetrAdS, $\bar{\gamma}_a$ are the gamma matrices for the inertial frame[15] and the Levi–Civita connection coefficients $\Gamma^\nu_{\mu\lambda}$ can be calculated by the Lagrange method.[12] Using the Kruskal coordinate we obtain the following solution for Eq. (20):

$$\psi^a_{q,s_z}\propto\begin{cases}u^{+a}_{r_q}(-\bar{u}/2r_H^{\mathrm{Ads}})^{-\mathrm{i}2r_H^{\mathrm{Ads}}\omega}\,, & \text{region I}\,,\\ v^{-\bar{a}}_{r_{\bar{q}}}(\bar{u}/2r_H^{\mathrm{Ads}})^{-\mathrm{i}2r_H^{\mathrm{Ads}}\omega}\,, & \text{region II}\,,\end{cases}\tag{22}$$

$u^{+a}_{r_q}$, and $v^{-\bar{a}}_{r_{\bar{q}}}$ denote the quark and antiquark spin wave functions respectively. Following the calculations in Ref. [12] we obtain

$$\begin{aligned}&|\mathrm{Ads\,BH},q,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}=\cos r_\omega\sum_{n=0,1}\tan^n r_\omega|n,q,a,s\rangle_{\mathrm{in}}\otimes|n,\bar{q},\bar{a},s\rangle_{\mathrm{out}}\,,\\&\tan(r_\omega)=\mathrm{e}^{-2\pi r_H^{\mathrm{Ads}}\omega},\quad\cos(r_\omega)=(1+\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega})^{-1/2}\,,\end{aligned}\tag{23}$$

where $|n,q,a,s\rangle_{\mathrm{in}}$, $|n,\bar{q},\bar{a},s\rangle_{\mathrm{out}}$ are quark and antiquark states inside and outside of black hole. The thermal distribution for this quark production is:

$$\begin{aligned}N^{\mathrm{quark}}_{\omega,\mathrm{color},s}&={}_{\mathrm{in}\otimes\mathrm{out}}\langle\mathrm{Ads\,BH},q,a,s|c^{a\dagger}_{q,s,\mathrm{out}}c^{a}_{q,s,\mathrm{out}}|\mathrm{Ads\,BH},q,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}\\&={}_{\mathrm{out}}\langle 0,\bar{q},\bar{a},s|\,{}_{\mathrm{in}}\langle 1,q,a,s|\sin^2(r_\omega)|1,q,a,s\rangle_{\mathrm{in}}\,|0,\bar{q},\bar{a},s\rangle_{\mathrm{out}}=\sin^2(r_\omega)=\frac{\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}{1+\mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}}\,.\end{aligned}\tag{24}$$

$c^{a\dagger}_{q,s,\mathrm{out}}$, $c^{a}_{q,s,\mathrm{out}}$ are creation and annihilation operators that act on AdS black hole outside states of quarks. $N^{\mathrm{quark}}_{\omega,\mathrm{color},s}$ is the thermal distribution for quarks with energy ω and a special color and spin. In a similar manner to the gluons, more quarks are produced near AdS black holes compared to Schwarzschild black holes at LHC.[1]

3 Gravitational Collapse of AdS Black Hole

Now we discuss Gravitational collapse and black hole state for quarks and gluons. It had been previously investigated that the field inside the event horizon can be decomposed into the collapsing matter field, in addition, the advanced wave incoming from infinity has a similar form as the Hawking radiation.[11] We extend these calculations to the excited quarks and gluons and derive the stationary state inside the black hole for quarks and gluons. The advanced collapsing shell metric[12] in d dimensions is given by:

$$\mathrm{d}s^2 = \begin{cases} -\mathrm{d}\tau'^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2 + g_{ij}\,\mathrm{d}x^i\mathrm{d}x^j\,, & r < R(\tau')\,, \\ -\Big(1 - \dfrac{r_H^{\mathrm{Ads}}}{r}\Big)\mathrm{d}t^2 + \dfrac{\mathrm{d}r^2}{(1 - r_H^{\mathrm{Ads}}/r)} + r^2\mathrm{d}\Omega^2 + g_{ij}\,\mathrm{d}x^i\mathrm{d}x^j\,, & r > R(\tau')\,. \end{cases} \tag{25}$$

We assume the metric g_{ij} is flat for both collapsing matter and the inside of the black hole. The shell radius $R(\tau')$ is defined by:[12]

$$R(\tau') = \begin{cases} R_0\,, & \tau' < 0\,, \\ R_0 - v\tau'\,, & \tau' > 0\,, \end{cases} \tag{26}$$

where τ' is the proper time. The advanced coordinates has been defined as:[15]

$$V = \tau + r - R_0\,, \quad U = \tau - r + R_0\,. \tag{27}$$

Now we consider the gluon modes inside the black hole, which are given by:

$$A_\mu^a \propto \begin{cases} (-1)^{-2\mathrm{i}r_H^{\mathrm{Ads}}M_s}\Big(1 - \dfrac{vV}{(1-v)(R_0 - r_H^{\mathrm{Ads}})}\Big)^{-2\mathrm{i}r_H^{\mathrm{Ads}}M_s}\,, & V > \dfrac{(1-v)(R_0 - r_H^{\mathrm{Ads}})}{v} \quad \text{(matter wave function)}\,, \\ \Big(1 - \dfrac{vV}{(1-v)(R_0 - r_H^{\mathrm{Ads}})}\Big)^{-2\mathrm{i}r_H^{\mathrm{Ads}}M_s}\,, & V < \dfrac{(1-v)(R_0 - r_H^{\mathrm{Ads}})}{v} \quad \text{(inside wave function)}\,, \end{cases} \tag{28}$$

ω is the gluon energy. In Eq. (28), we use the fact that $(-1)^{-\mathrm{i}2r_H^{\mathrm{Ads}}\omega} = \mathrm{e}^{2\pi r_H^{\mathrm{Ads}}\omega}$. Using Eq. (28) we observe that the states in the horizon satisfy the following condition:

$$(A^a_{\mu,s_{\mathrm{matter}}} - \mathrm{e}^{2\pi r_H^{\mathrm{Ads}}\omega}A^a_{\mu,s_{\mathrm{in}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{matter}} = 0\,, \tag{29}$$

or equivalently

$$(A^a_{\mu,s_{\mathrm{in}}} - \tanh r_\omega A^a_{\mu,s_{\mathrm{matter}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{matter}} = 0\,, \quad \tanh r_\omega = \mathrm{e}^{-2\pi r_H^{\mathrm{Ads}}\omega}\,, \tag{30}$$

which actually constitutes a boundary state. Using the expansion in modes for gluons (Eq. (1)) we may write:

$$(\alpha^a_{\lambda,s_{\mathrm{in}}} - \tanh r_\omega \alpha^{a\dagger}_{\lambda,s_{\mathrm{matter}}})|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{matter}} = 0 \quad \text{for} \quad n \neq 0\,, \quad \tanh r_\omega = \mathrm{e}^{-2\pi r_H^{\mathrm{Ads}}\omega}\,. \tag{31}$$

Now, we assume that the Kruskal vacuum $|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{out}}$ is related to the Schwarzschild vacuum $|0\rangle_S$ by

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{matter}} = F(\alpha^a_{\lambda,s_{\mathrm{matter}}},\alpha^a_{\lambda,s_{\mathrm{in}}})|0\rangle_S\,, \tag{32}$$

F is some function to be determined later.

Using Eqs. (31) and (32), we get the following differential equation for F.

$$\Big(\frac{\partial F}{\partial \alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}} - \tanh r_{\omega_n,\lambda}\alpha^{a\dagger}_{\lambda,s_{\mathrm{matter}}}F\Big) = 0\,, \tag{33}$$

and the solution is given by

$$F = \mathrm{e}^{\tanh r_\omega \alpha^{a\dagger}_{\lambda,s_{\mathrm{matter}}}\alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}}\,. \tag{34}$$

By substituting Eq. (34) into Eq. (33) and by properly normalizing the state vector, we get

$$|\mathrm{Ads\,BH},\mu,a,s\rangle_{\mathrm{in}\otimes\mathrm{matter}} = \mathrm{e}^{\tanh r_\omega \alpha^{a\dagger}_{\lambda,s_{\mathrm{matter}}}\alpha^{a\dagger}_{\lambda,s_{\mathrm{in}}}}|0\rangle_S = \frac{1}{\cosh r_\omega}\sum_n \tanh^n r_\omega|n,\lambda,a,s\rangle_{\mathrm{in}} \otimes |n,\lambda,a,s\rangle_{\mathrm{matter}}\,. \tag{35}$$

Following the calculations in Ref. [12] we obtain

$$|\mathrm{Ads\,BH},q,a,s\rangle_{\mathrm{matter}\otimes\mathrm{in}} = \cos r_\omega \sum_{n=0,1}\tan^n r_\omega|n,q,a,s\rangle_{\mathrm{in}} \otimes |n,\bar q,\bar a,s\rangle_{\mathrm{matter}}\,,$$

$$\tan(r_\omega) = \mathrm{e}^{-2\pi r_H^{\mathrm{Ads}}\omega}\,, \quad \cos(r_\omega) = (1 + \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega})^{-1/2}\,. \tag{36}$$

4 The Information Loss for Quarks and Gluons in Mini Black Holes at LHC

At this stage we calculate the resulting information transformation from the collapsing matter to the state of the outgoing Hawking radiation for both quarks and gluons. First we consider the information transformation for one mode of quarks and gluons and then extend our calculations to all modes of quarks and gluons. Before black hole evaporation, there is an entanglement between the inside and outside of the horizon, but matter itself is not entangled,

$$|\mathrm{Ads\,BH}, \mu, a, s\rangle_{\mathrm{matter}\otimes\mathrm{in}\otimes\mathrm{out}} = |\mathrm{Ads\,BH}, \mu, a, s\rangle_{\mathrm{matter}} \otimes |\mathrm{Ads\,BH}, \mu, a, s\rangle_{\mathrm{in}\otimes\mathrm{out}}\,,$$
$$|\mathrm{Ads\,BH}, q, a, s\rangle_{\mathrm{matter}\otimes\mathrm{in}\otimes\mathrm{out}} = |\mathrm{Ads\,BH}, q, a, s\rangle_{\mathrm{matter}} \otimes |\mathrm{Ads\,BH}, q, a, s\rangle_{\mathrm{in}\otimes\mathrm{out}}\,. \tag{37}$$

After evaporation we describe the state of the black hole as an entangled state of both matter and the inside of the black hole,

$${}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}', \mu, a, s| = {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}, \mu, a, s|(S\otimes I)\,,$$
$${}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}', q, a, s| = {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads}\,BH, q, a, s|(S\otimes I)\,. \tag{38}$$

To describe the unknown effects of quantum gravity, an additional unitary transformation S is introduced.[9] Also Gottesman and Preskill introduced a unitary transformation $U^{[4]}$ to describe interactions between matter and the inside of the horizon.

$${}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}', \mu, a, s| = {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}, \mu, a, s|(S\otimes I)U\,,$$
$${}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}', q, a, s| = {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}, q, a, s|(S\otimes I)U\,. \tag{39}$$

By extending Gottesman and Preskill method to one mode of quarks and gluons, we calculate the information loss for this mode at LHC.

$$\begin{aligned}
T_g &= {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}, \mu, a, s|\mathrm{Ads\,BH}, \mu, a, s\rangle_{\mathrm{in}\otimes\mathrm{out}} \\
&= \frac{1}{\cosh^2(r_\omega)}\sum \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}\,{}_{\mathrm{matter}}\langle m, n|\,{}_{\mathrm{in}}\langle m, n|S\otimes U|m, n\rangle_{\mathrm{in}}|m, n\rangle_{\mathrm{out}}\,, \\
f_g &= |T_g|^2 = \frac{1}{\cosh^4(r_\omega)}\sum_{m,m'} \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega(m+m')} \\
&\quad \times {}_{\mathrm{matter}}\langle m, n|\,{}_{\mathrm{in}}\langle m, n|\,{}_{\mathrm{in}}\langle m', n|\,{}_{\mathrm{out}}\langle m', n|SS^t\otimes UU^t|m', n\rangle_{\mathrm{in}}|m', n\rangle_{\mathrm{matter}}|m, n\rangle_{\mathrm{in}}|m, n\rangle_{\mathrm{out}} \\
&= \frac{1}{\cosh^4(r_\omega)}\sum_{m,m'} \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega(m+m')}\,{}_{\mathrm{matter}}\langle m, n|m', n\rangle_{\mathrm{matter}}\;{}_{\mathrm{in}}\langle m, n|m', n\rangle_{\mathrm{in}}\;{}_{\mathrm{in}}\langle m', n|m, n\rangle_{\mathrm{in}}\;{}_{\mathrm{out}}\langle m', n|m, n\rangle_{\mathrm{out}} \\
&= \frac{1}{\cosh^4(r_\omega)}\sum_{m} \mathrm{e}^{-8\pi r_H^{\mathrm{Ads}}\omega(m)} = 1 - \mathrm{e}^{-8\pi r_H^{\mathrm{Ads}}\omega(m)}\,, \quad SS^\dagger = 1\,, \quad UU^\dagger = 1\,,
\end{aligned} \tag{40}$$

$$\begin{aligned}
T_q &= {}_{\mathrm{matter}\otimes\mathrm{in}}\langle \mathrm{Ads\,BH}', q, a, s|\mathrm{Ads\,BH}', q, a, s\rangle_{\mathrm{in}\otimes\mathrm{out}} \\
&= |\cos^2(r_\omega)|\sum \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega}\,{}_{\mathrm{matter}}\langle m, n|\,{}_{\mathrm{in}}\langle m, n|S\otimes U|m, n\rangle_{\mathrm{in}}|m, n\rangle_{\mathrm{out}}\,, \\
f_q &= |T_q|^2 = \cos^4(r_\omega)\sum_{m,m'} \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega(m+m')}{}_{\mathrm{matter}} \\
&\quad \times \langle m, n|_{\mathrm{in}}\langle m, n|\,{}_{\mathrm{in}}\langle m', n|\,{}_{\mathrm{out}}\langle m', n|SS^t\otimes UU^t|m', n\rangle_{\mathrm{in}}|m', n\rangle_{\mathrm{matter}}|m, n\rangle_{\mathrm{in}}|m, n\rangle_{\mathrm{out}} \\
&= \cos^4(r_\omega)\sum_{m,m'} \mathrm{e}^{-4\pi r_H^{\mathrm{Ads}}\omega(m+m')}\,{}_{\mathrm{matter}}\langle m, nm', n\rangle_{\mathrm{matter}}\;{}_{\mathrm{in}}\langle m'n|m, n\rangle_{\mathrm{in}}\;{}_{\mathrm{out}}\langle m'n|m, n\rangle_{\mathrm{out}} \\
&= \cos^4(r_\omega)\sum_{m} \mathrm{e}^{-8\pi r_H^{\mathrm{Ads}}\omega(m)} = 1 + \mathrm{e}^{-8\pi r_H^{\mathrm{Ads}}\omega(m)}\,.
\end{aligned} \tag{41}$$

If information transformation from the collapsing matter to the state of outgoing Hawking radiation is complete, the value of f should be one. Thus we conclude that information transformation for high energy quarks and gluons is approximately complete. However low energy quarks and gluons are not able to come out from event horizon and information for these particles is lost. Evidently for all finite values of particle energy, the value of f is less than unity and so information from all emission processes experiences some degree of loss. Comparing the information loss in AdS black hole with the information loss in Schwarzschild black hole in Ref. [1], we conclude that less information is lost in AdS black holes. This is due to the fact that AdS temperature is higher than the temperature for Schwarzschild black hole with the same mass. Consequently more particles are emitted from AdS black holes.

These results can be extended to the black holes that their horizon and temperature depend on the mass. In these black holes, before the evaporation, there are three different Hilbert spaces that belong to degrees of freedom of matter, incoming, and outgoing radiation. The total state of a black hole is a direct product of matter state and the entangled states of the states inside and outside of the event horizon. When a black hole begins to evaporate, the matter state will be in a maximally entangled state with incoming Hawking radiation. The information loss in black hole is easily obtained by projecting this entangled state on the entangled states inside and outside of the black hole horizon.

5 Summary and Conclusion

In this paper, the amount of information loss in AdS black holes at LHC is calculated. We observe that information transformation for high energy quarks and gluons is approximately complete. However low energy quarks and gluons are not able to come out from event horizon and information for these particles is lost. Comparing the results of this paper with Ref. [1], we conclude that less information is lost in AdS black holes. This is due to the fact that AdS temperature is higher than the temperature for Schwarzschild black hole with the same mass. In addition, the AdS horizon is smaller than Schwarzschild horizon. This causes the thermal distributions of quarks and gluons near AdS black holes to be higher than the distributions near Schwarzschild black holes. Consequently more particles are emitted from AdS black holes.

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