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Hydraulic Analysis of Water Supply Networks Using a Modified Hardy Cross Method

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ABSTRACT

There are different methods for the hydraulic analysis of water supply networks. In the solution process of most of these methods, a large system of linear equationsis solved in each iteration. This usually requires a high computational effort. Hardy Cross method is one of the approaches that do not need such aprocess and may converge to the solution through scalar divisions. However,this method has two short comings: first, initial discharges should satisfy continuity equation at each node; second a large number of iterations are required to converge to solution. In this article an algorithm is suggested for the selection of initial discharges that are close to the final results while the continuity equations are automatically established. This algorithm may be directly implemented in the Hardy Cross method. To reduce the number of iterations the Hardy Cross method is combined with third-order and sixteenth-order methods. The results of some numerical examples demonstrate that the use of the combined approach with the suggested initial guess reduces the number of iterations and hydraulic analysis time and the solutions converge with a high accuracy.

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1. INTRODUCTION

Cross [1], developed a mathematical approach for moment distribution in indeterminate structures. He discovered that this method mayalso be applied for estimating the pressures and discharges in a water distribution network.In the first approach or loop methodinitial pipe discharges, satisfying the continuity equation at ajunction or node, areadjusted to balancethe energy equation at a loop. Nodal heads are then obtained from a reference point by adding or subtracting head losses between adjacent nodes.

In the second approach or nodal head method initial heads at nodes are modified in successive iterations to satisfy the continuity equation at a junction. In this method, pipe discharges are estimated by solving the Hazen–Williamsor Darcy–Weisbach equations where the head lossesare obtained from the head difference between adjacent nodes.

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In practice, the convergence rate of the nodal head method was slow and the choice of initial heads was inconvenient [2, 3]. As a result, the loop method received a greater acceptance in the engineering community. Although the Hardy Cross method was originally developed for hand calculations its overall formulation was imply compatible with computer programming in the following decades. However, depending on the size and complexity of the network, the Hardy Cross method requirestoo much iteration to converge and sometimes it may diverge.

Martin and Peters were the first who used Newton-Raphson method in the analysis of water supply networks[4]. In their method, all equations are written in terms of nodal heads, H. Then the solution is obtained through modifying the head in successive iterations. One of the disadvantages of Martin and Peters' method is the lack of optimal convergence in large-scale networks. To eliminate this problem, some pipes of the network should be temporarily removed in the analysis procedure. Other disadvantage is the high oscillations to achieve optimal convergence. To decrease the oscillations, the value of ΔH is reduced by half; though this will increase the number of iterations.

Epp and Fowler developed a new technique for analyzing pipe networks where the Newton–Raphson method was applied for the simultaneous solution of flow corrections in the Cross loop method [5]. The result proved a considerable improvementon the convergence properties of the original algorithm.

In 1972 Wood and Charles, presented the linear theory method[6]. According to this method, continuity equations at nodesand energy conservation for each loop or path are solved simultaneously and the dischargein each pipe is directly obtained.There is no need to guess initial discharges to satisfy the continuity equations at nodes.Subsequent developments of this algorithm which led to commercial software (like WOODNET, KYPIPE, PIPE2000) was due to the implementation of the Newton–Raphson method[6-8].

Jeppson presented an algorithm based on the loop method. In this method nonlinear equation of energy for each loop or path in the network,is written in terms of the flow corrections. These equations are linearized by Taylor series expansion and repetitively solved using Newton–Raphson method[9].

The global gradient methodof Todini and Pilati is ahighly popular method, implemented in Epanet software [10].In this method, energy equations are combined with the nodal equations and are simultaneously solved to estimate the nodal heads and flow discharge. Here, like the methods of “simultaneous loop” and "linear theory", nonlinear energy equations are linearized using Taylor series expansion. However they are solved using an optimal and reversal scheme which applies the inverse of the coefficients matrix.

Recently, Moosavian and Jaefarzadeh applied a shuffled complex evolution algorithm (SCE) in an optimization model (co-content model) for the hydraulic analysis of pipe networks. This strategy could simulate pressure-driven demand and leakage in networks accurately [11].

The development of the computational efficiency of water system modelling can be achieved by: (i) advanced mathematics, such as, new robust numerical solvers to fast solution of the linear system of equations[12]; (ii) new technology, such as, parallelization of existing algorithms for multi-cores processing or Graphic Processor Units (GPU) [13, 14] and (iii) innovative engineering, such as methodologies for simplifying the topological representation of water distribution networks while preserving the accuracy of the analysis as for example in the referenced works [15-17].

In this paper the Hardy-Cross loop method is reviewed and its matrix formulation is presented. An algorithm is proposed for the initial discharges which is very close to the final solution and satisfies the continuity equation. Application of the proposed algorithm increases the reliability of the method and reduces the number of iterations considerably. Two methods of third- and sixteenth-order are implemented in to the original method to improve the rate and time of convergence and reduce the number of iterations. Solving some examples indicates the capability of this new algorithm in the hydraulic analysis of water supply networks.

2. HARDY-CROSS METHOD

Hardy–Cross proposed a standard algorithm based on systematic approximations and successive corrections. Due to its simplicity, this algorithm is widely used in the pipe network analysis. This method is based on two principle criteria:

The sum of inflows is equal to the sum of outflows at each node (the continuity or mass balance equation)[18].

$$\sum_i Q_i = q_j \text{ for } j = 1, 2, 3, \ldots, NJ \tag{1}$$

where Q_i is the discharge in pipe i meeting at node j, q_j is nodal outflow and NJ is the total number of junctions.
1. The sum of head loss hf_k around a closed loop is equal to zero (the loss or energy balance equation).

$$\sum_{Loop} hf_k = 0 \tag{2}$$

The nonlinear relationship between the head loss hf_k and discharge Q_k in a pipe k connecting nodes i and j may be written as:

$$hf_k = R_k Q_k^n \tag{3}$$

where R_k is a coefficient of resistance depending on pipe roughness, its length and diameter and n is an exponential constant; for Darcy–Weisbach equation $n=2$ and for Hazen–Williams equation $n=1.852$. Substituting for hf_k from Equation (3) into Equation (4), the loss equations may be rewritten as:

$$\sum_{Loop} R_k Q_k^n = 0 \tag{4}$$

The set of Equations (1) and (4) produces a system of nonlinear equations for pipe network analysis. Cross converted this system into a scalar problem that may be solved by hand calculations. Later on, other researchers observed the above criteria as the necessary rules fornet work analysis. In the Cross loop method initial discharges in the pipes should satisfy the continuity equations at the nodes. These discharges are consecutively modified inthe analysis to satisfy the energy equations for each loop with a high accuracy.
If the correction of discharge in a loop L is shown by ΔQ_L, the modified discharge of pipe k in this loop would be $Q_k + \Delta Q_L$. As a result, for $n=2$, Equation (3) may be written as:

$$hf_k = R_k(Q_k + \Delta Q_L)^2 = \\ = R_k(Q_k^2 + 2Q_k\Delta Q_L + \Delta Q_L^2) \approx \\ \approx R_k(Q_k^2 + 2Q_k\Delta Q_L) \tag{5}$$

Assuming ΔQ_L to be small, the value of ΔQ_L^2 may be neglected. Substituting Equation (5) in(2) we have:

$$\sum hf_k = 0 \Rightarrow \\ \sum R_k(Q_k^2) + 2\sum R_k Q_k \Delta Q_L = 0$$

Thus, the discharge correction may be obtained from

$$\Delta Q_L = -\frac{\sum R_k Q_k^2}{2\sum R_k Q_k} = -\frac{\sum hf_k}{2\sum\left(\frac{hf_k}{Q_k}\right)} \tag{6}$$

For the general head loss-discharge relationship of Equation (3), correction is calculated from

$$\Delta Q_L = -\frac{\sum R_k Q_k^n}{n\sum R_k Q_k^{n-1}} = -\frac{\sum hf_k}{n\sum\left(\frac{hf_k}{Q_k}\right)^{n-1}} \tag{7}$$

Thus, according to Equation(7) the value of discharge at each loop is modified by dividing the total energy loss to its derivative. Hardy-Cross formulation may also be presented in matrix notation. The elements of matrix $\boldsymbol{M}_{31}$ is thus defined to clarifythe mutual situation ofloops and pipes

$$\boldsymbol{M}_{13}(k,L) = \begin{cases} +1 & \text{if the flow of pipe k in loop L is clockwise} \\ 0 & \text{if pipe k is not in loop L} \\ -1 & \text{if the flow of pipe k in loop L is counterclockwise} \end{cases}$$

Obviously, we may deduce $\boldsymbol{M}_{31} = \boldsymbol{M}_{13}^T$

The diagonal matrices $\boldsymbol{A}_{11}$ and $\boldsymbol{D}_{11}$ are defined as:

$$\boldsymbol{A}_{11} = \begin{bmatrix} R_1|Q_1|^{n-1} & 0 & 0 & 0 \\ 0 & R_2|Q_2|^{n-1} & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & R_{NP}|Q_{NP}|^{n-1} \end{bmatrix} \tag{8}$$

$$\boldsymbol{D}_{11} = \begin{bmatrix} nR_1|Q_1|^{n-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & nR_{NP}|Q_{NP}|^{n-1} \end{bmatrix} \tag{9}$$

where NP is the total number of pipes in the network. Inmatrix form Equation (6) in the tth iteration may be written as:

$$\Delta\boldsymbol{Q}^{(t)} = -\frac{\boldsymbol{M}_{31}\boldsymbol{A}_{11}^{(t)} diag\left(\boldsymbol{Q}^{(t)}\right)}{\boldsymbol{M}_{31}\boldsymbol{D}_{11}^{(t)}} \tag{10}$$

and the discharge in pipesare corrected in the $t+1$ iteration:

$$\boldsymbol{Q}^{(t+1)} = \boldsymbol{Q}^{(t)} + \boldsymbol{M}_{13}\Delta\boldsymbol{Q}^{(t)} \tag{11}$$

3. INITIAL GUESS

One of the drawbacks of the loop method is its high dependence on an appropriate initial guess for discharges. In other words, when the initial discharge is close enough to the final solution, the convergence rateis high, and when itis far from the solution, there is a possibility of divergence. However, in typical approaches there is no relation between the initial discharge and the final solution. In this section an algorithmis proposed through which a specific discharge is estimated for each pipe, as the initial guess. This is usually very close to the final solution while satisfying the continuity equation. If the nonlinear loss Equations (4) are properly linearized, the linear continuity Equations (1) plus the linearized loss equations may be solved using any classical direct or iterative method. The results provide an acceptable initial discharge to start the analysis of linear-nonlinear system of continuity and loss Equations (1) and (4) with a relatively small percentage of error. This initial guess enters the range of possible solutions even in complex networks. Using this approach to estimate avariable initial dischargeis very appropriate, especially for the Hardy Cross method, because it frees the application from searching in ranges that are far from the actualsolution. Accordingly, assuming n=2, linearized form of loss Equation (3) may be written as:

$$hf_k = R_k . Q_{max} . Q_i \tag{12}$$

where Q_{max} is the maximum discharge which may pass the pipe k. It may be assumed equal to the total nodal demands across the network, because no pipe can passany flow greater than this. In Figure 1 parabolic and linearized loss functions are plotted for comparison.

By solving the equations system of linear continuity and linearized energy loss a proper initial guess to start the calculationsis obtained. One of the advantages of linearization of loss function is its flexibility in various issues. This means that using this method in all water supply networks in any geometric shape, the initial guessis estimated in the area close to the final solution and consequently the convergence process is accelerated. This approach can be used for all network analysis methods including the Hardy-Cross method, as the initial guess. The matrix form of the initial guess selection is as follows:

$$\boldsymbol{Q}^{(0)} = \begin{bmatrix} \boldsymbol{M}_{31} Q_{max} \boldsymbol{R} \\ \boldsymbol{A}_{21} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -\boldsymbol{q} \end{bmatrix} \tag{13}$$

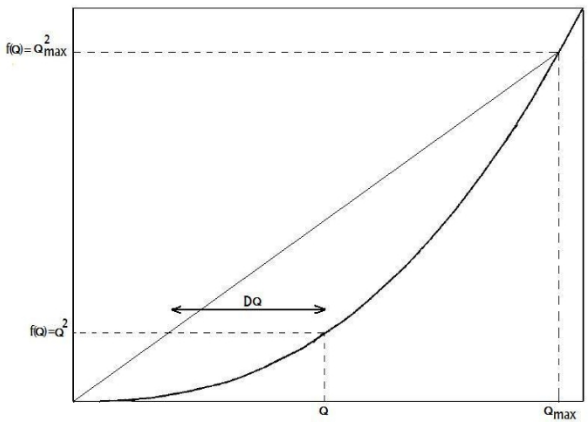

Figure1. Linearization of a non-linear function

In Equation (13) the vector $\boldsymbol{q}$ includes the nodal demands and the diagonal matrix $\boldsymbol{R}$ is as follows:

$$\boldsymbol{R}=\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & R_{NP} \end{bmatrix} \tag{14}$$

After solving the Equation system(13), vector $\boldsymbol{Q}^{(0)}$ is defined as the initial guess. The effect of a proper initial guess is explained in the examples in Section7.

4. THIRD-ORDER NEWTONIAN METHODS

If more terms in Taylor expansion, are considered around the point *x*, higher order derivatives of 2 will arise. The calculation of these derivatives is very time consuming and costly. For this reason, in the Newton–Raphson method these terms are omitted.

But using some tricks we can obtain a higher order of convergence without calculating second-order derivatives. In this section a highly efficient third-order method is introduced. This method was presented by Darvishiand Barati[19] and its algorithm is as follows:

$$\boldsymbol{x}^{(t+1)} = \boldsymbol{x}^{(t)} - [\boldsymbol{J}]^{-1}\left(\boldsymbol{F}(\boldsymbol{x}^{(t)}) + \boldsymbol{F}(\boldsymbol{x}_*^{(t+1)})\right), \tag{15}$$

$$\boldsymbol{x}_*^{(t+1)} = \boldsymbol{x}^{(t)} - [\boldsymbol{J}]^{-1}\boldsymbol{F}(\boldsymbol{x}^{(t)}) \tag{16}$$

Here, as the Newton–Raphson method, inversion of the Jacobian matrix $\mathbf{J}$ is done only once in each iteration. Function $\mathbf{F}$ is also up dated twice.

5. SIXTEENTH-ORDER NEWTONIAN METHOD FOR SOLVING NON-LINEAR EQUATIONS

Sixteenth-order Newtonian method algorithm for solving an on-linear equation was presented by Li et al. [20].

$$\begin{aligned} y^{(t)} &= x^{(t)} - \frac{f(x^{(t)})}{f'(x^{(t)})} \\ z^{(t)} &= y^{(t)} - \frac{2f(x^{(t)}) - f(y^{(t)})}{2f(x^{(t)}) - 5f(y^{(t)})}\frac{f(y^{(t)})}{f'(x^{(t)})} \\ x^{(t+1)} &= z^{(t)} - \frac{f(z^{(t)})}{f'(z^{(t)})} - \\ &\frac{2f(z^{(t)}) - f(z^{(t)} - f(z^{(t)})/f'(z^{(t)}))}{2f(z^{(t)}) - 5f(z^{(t)} - f(z^{(t)})/f'(z^{(t)}))} \times \\ &\times \frac{f(z^{(t)} - f(z^{(t)})/f'(z^{(t)}))}{f'(z^{(t)})} \end{aligned} \tag{17}$$

This method is presented for an on-linear equation and can be used to improve the performance of the Hardy–Cross method.

6. APPLICATION OF HIGHER-ORDER METHODS IN HARDY-CROSS EQUATIONS

As mentioned, the Hardy–Cross algorithm is a very simple method for hydraulic analysis of water supply networks. In this method there is no need to solve linear equation systems at each iteration. However, problems of convergence, dependence on initial guess, and the lack of a systematic structure, made this approach give place to the other algorithms after the entrance of computers. In this section a matrix form for the Hardy-Cross method is provided for the first time, and the convergence problems are fixed using higher-order methods. Thus the Hardy–Cross method turns to a very powerful algorithm for the analysis of water supply networks. The Hardy–Cross loop method is combined using the algorithm (15) and (16) to improve the convergence process. This method is called HCQ1.

$$\begin{aligned} \boldsymbol{M}_1 &= \boldsymbol{M}_{31}\boldsymbol{D}_{11}^{(t)} \\ \boldsymbol{M}_2 &= \boldsymbol{M}_{31}\boldsymbol{A}_{11}^{(t)} diag\left(\boldsymbol{Q}^{(t)}\right) \\ \boldsymbol{y}^{(t)} &= \boldsymbol{Q}^{(t)} - \boldsymbol{M}_{13}\left(\frac{\boldsymbol{M}_2}{\boldsymbol{M}_1}\right) \\ \boldsymbol{M}_3 &= \boldsymbol{M}_{31}\boldsymbol{A}_{11}'^{(t)} diag\left(\boldsymbol{y}^{(t)}\right) \\ \boldsymbol{Q}^{(t+1)} &= \boldsymbol{Q}^{(t)} - \boldsymbol{M}_{13}\left(\frac{\boldsymbol{M}_2 + \boldsymbol{M}_3}{\boldsymbol{M}_1}\right) \end{aligned} \tag{18}$$

In the above equation, the matrix $\boldsymbol{A}_{11}'^{(t)}$ is the matrix $\boldsymbol{A}_{11}^{(t)}$ which is updated using variable $\boldsymbol{y}^{(t)}$.If you use the algorithm (17), the sixteenth-order Hardy-Cross method is obtained. This method is called HCQ3. In the above equation, $\boldsymbol{A}_{11}''^{(t)}$ and $\boldsymbol{D}_{11}''^{(t)}$ are matrixes $\boldsymbol{A}_{11}^{(t)}$ and $\boldsymbol{D}_{11}^{(t)}$ which are updated by variables of $\boldsymbol{z}^{(t)}$. Matrix $\boldsymbol{A}_{11}'''^{(t)}$ is updated by variable $\boldsymbol{x}^{(t)}$. It should be noted that the division operator in these algorithms(e.g. $\frac{\boldsymbol{M}_2}{\boldsymbol{M}_1}$) includes one to one dividing of matrix elements.

$$
\begin{aligned}
&M_1 = M_{31} D_{11}^{(t)} \\
&M_2 = M_{31} A_{11}^{(t)} diag\left(Q^{(t)}\right) \\
&y^{(t)} = Q^{(t)} - M_{13}\left(\frac{M_2}{M_1}\right) \\
&M_3 = M_{31} A_{11}'^{(t)} diag\left(y^{(t)}\right) \\
&z^{(t)} = y^{(t)} - M_{13}\left(\frac{M_3\left(2M_2 - M_3\right)/\left(2M_2 - 5M_3\right)}{M_1}\right) \\
&M_4 = M_{31} D_{11}''^{(t)} \\
&M_5 = M_{31} A_{11}''^{(t)} diag\left(z^{(t)}\right) \\
&x^{(t)} = z^{(t)} - M_{13}\left(\frac{M_5}{M_4}\right) \\
&M_6 = M_{31} A_{11}'''^{(t)} diag\left(x^{(t)}\right) \\
&Q^{(t+1)} = x^{(t)} - M_{13}\left(\frac{M_6\left(2M_5 - M_6\right)/\left(2M_5 - 5M_6\right)}{M_4}\right)
\end{aligned}
\tag{19}
$$

TABLE 1. Information of water supply network related to numerical example 1

Pipe number	Q (l/s)	R	Node number	q (l/s)	H(m)
1	800	1.5625	0	-1000	100
2	200	50	1	100	99
3	100	100	2	200	98
4	400	12.5	3	300	97
5	200	75	4	400	96
6	100	200			
7	100	100			

TABLE 2. The residual of energy equation in network loops (meter) for numerical example 2

Loop number	Energy equation	Loop number	Energy equation
1	-1.39E-16	6	-8.88E-16
2	0.00E+00	7	-2.22E-16
3	0.00E+00	8	0.00E+00
4	-2.22E-15	9	-2.19E-15
5	-2.22E-15	10	-9.85E-16
		11	-1.78E-15

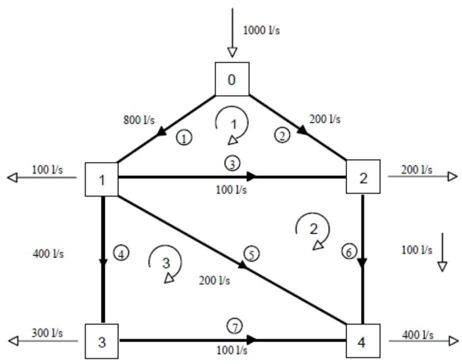

Figure 2. Schematic representation of the three-looped pipe network [21]

7. 1. Numerical Example1 Consider the loop network in Figure 2 having 5 nodes and 7 pipes [21].The pipe resistance coefficient R and with drawal discharge in each node q are shown in Table 1. Node zero is a storage reservoir with ahead of 100 m which enters a discharge of 1,000 liters per second into the network. After the hydraulic analysis, the final values of the pipes' discharges Q and heads H are also listed in Table 1. The equation of head loss is obtained based on the Darcy–Weisbach equation. Hardy-Cross method and its third and sixteenth-order versions are used to analyze the network. The dimensionless parameter of remaining norm is used for the comparison of algorithms:

$$
\|\varepsilon\| = max\left|\varepsilon_i\right| = max\left|\frac{x^{(t+1)} - x^{(t)}}{x^{(t)}}\right| \tag{20}
$$

According to the algorithm in Section 3, the variable initial guess is obtained by linearization of energy loss equations. In Figure 3, the variable initial guess is displayed along with the final solution. According to the figure, the variable initial guess is very close to the final solution and is suitable for embarking on a careful analysis. When the accuracy of the solutions is not important, this initial guess can be taken as the final solution.

TABLE 3. The residual of energy equation in network loops (meter) for numerical example 2 with proposed initial guess

Loop number	Energy equation	Loop number	Energy equation
1	-0.3479686	6	-1.6929067
2	3.9919509	7	2.84489
3	-0.8672334	8	1.1994404
4	3.6113804	9	-8.3988539
5	-1.7529815	10	-0.6786989
		11	-4.2989381

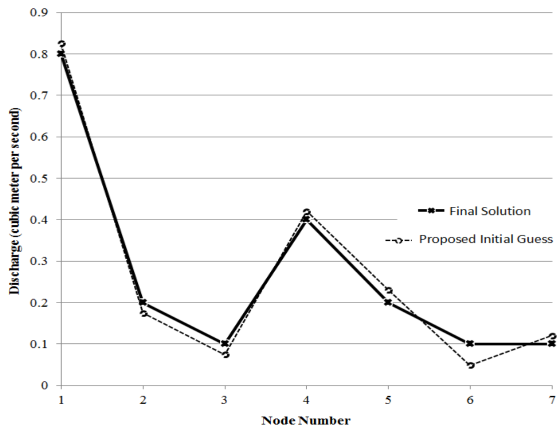

Figure 3. Comparison of proposed initial guess with final solution

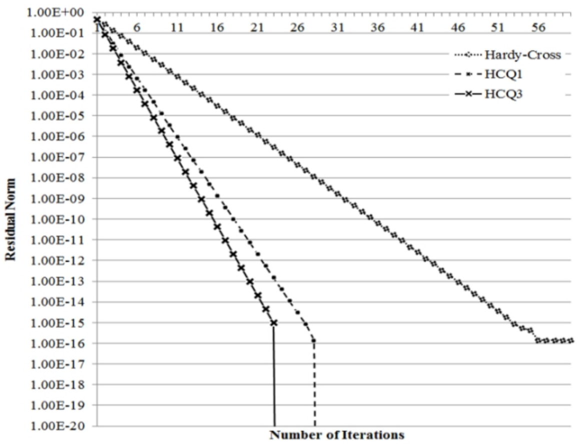

Figure 4. Convergence of different Hardy-Crass algorithms for numerical example 1

In Figure 4 the convergence trend of different algorithms is displayed. The Hardy–Cross method reaches a convergence with the precision of 10^{-16} after about 55 iterations. The third-order method HCQ1 has a steeper slope compared with the Hardy–Cross method and reaches a convergence with the precision of zero at the 27th iteration. The 16th-order method HCQ3, with the maximum slope, converges faster than the other methods. It is worth to note that at each iteration in this method we have only a few scalar divisions and the linear system of equation is not solved. Among the three recent algorithms, the methods of HCQ3 and HCQ1 respectively, have a better performance because they have the lowest computational cost and an appropriate convergence slope.

7. 2. Numerical Example 2 In this example, water supply network of the town of Farhadgard, near the city of Mashhad, is analyzed. The water supply network with 66 pipes, 57 nodes, and twovalves, with a loss coefficient of 10 is displayed in Figure 5. The mentioned valves are in A1 and A2 areas.

The network onlyhas are servoir with a head of 510 m. Since pipes are not crossing over each other and spread over the surface,we can simply determine the relation between pipes and loops in the matrix $\mathbf{M}_{31}$.

In this section, three methods of Hardy-Cross, third-order Hardy-Cross and 16th-order Hardy-Cross, with variable initial guess are used in network analysis. In Figure 6 the convergence trend is displayed based on the residual norm and number of it erations. Hardy-Cross, third-order and 16th-order methods converge at 200, 120 and 65 iterations, respectively. The residual norm in all three methodsis 10^{-16}. Residual of energy equations are illustrated in Tables 2. Solutions are highly accurate, at each iteration of the 16th-order method there is no need to solve the linear system of equations and the computational cost is very small.

In Figure 7, the variable initial discharge is compared with the pipes' final discharge. The solutions are very close.In Table 3 the residual of energy equations of each loop are given for variable initial guess. Continuity equations are well established but the energy equations are less accurate. When accuracyis of low importance, we can consider the initial discharge as the final solution, with a good approximation, by solving the linear system of equations only once.

7. 3. Numerical Example 3 Consider two regular loop networks with N×N nodes (as shown in Figure8) in which *N* (number of nodes in each row or column) is equal to 10 and 25, and the volume of calculations highly increases with the increase of *N*.

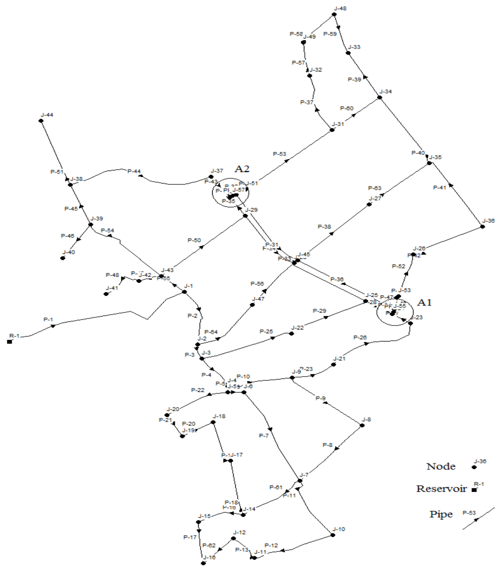

Figure 5. Water supply network of the city of Farhadgard

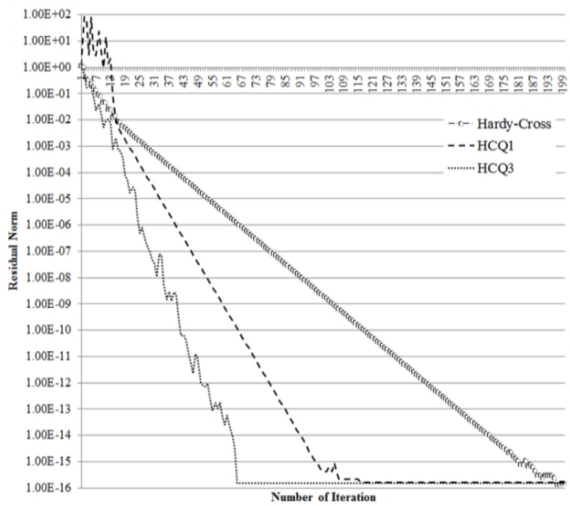

Figure 6. Convergence of different Hardy-Crass algorithms for numerical example 2

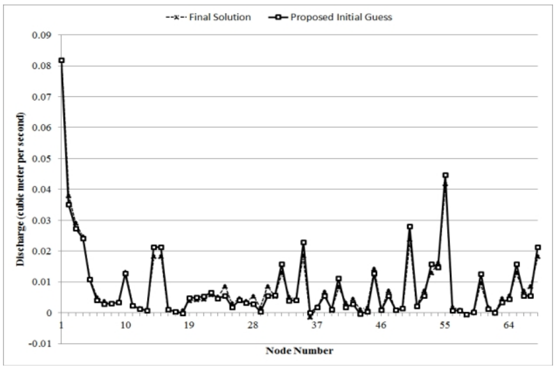

Figure 7. Comparison of proposed initial guess with final solution

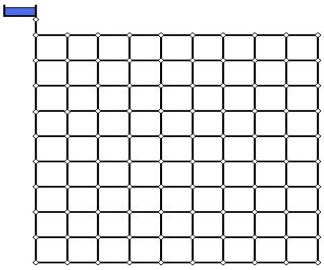

Figure 8. An N×N pipe network

TABLE 4. Comparison of convergence time of different algorithms for numerical example 3

	10x10		25x25	
algorithm	**Time (s)**	**Number of iteration**	**Time (s)**	**Number of iteration**
Newton-Raphson Q	0.08	5	2.991	6
Newton-Raphson Q-H (GGA)	0.03	5	0.609	6
Newton-Raphson ΔQ	0.02	5	0.588	6
Hardy-Cross	0.04	335	0.594	2213
HCQ3	0.04	83	0.158	122

The nodal demand in each node and the pipe resistance coefficient, *R*, in each pipes has been selected using random numbers. The minimum *R* equals 4.63 and the maximum *R* is 996. The minimum nodal demand equals 0.0023 and the maximum nodal demand is 0.996 cubic meters per second. The hydraulic analysis of this network is performed by the Newton–Raphson in equation Q, global gradient algorithm (GGA), Newton–Raphson in equations ΔQ, Hardy-Cross and 16th-order Hardy-Cross methods. The method of Newton-Raphson in equations Q or the improved method of linear theory which was presented by Wood is currently being used in KYPIPE software. The global gradient algorithm is in fact the method of Newton-Raphson in equations Q-H used in the EPANET software. The method of Newton-Raphson in equations ΔQ presented by Epp and Fowler includes the simultaneous modification of discharge in the loops of networks. The algorithm of variable initial guess has been used in each of the methods. The stopping criterion in all methods is fulfilled when the residual norm reaches to10^{-16}. In Table 4, the convergence times in different methods are given for the two networks. In 10×10 network, all methods converge in less than a tenth of a second. The minimum convergence time belongs to the Newton-Raphson method in equations ΔQ and the maximum convergence time is observed in the Newton-Raphson method in equations Q. The convergence time in 25 × 25 network increases due tothe increased number of unknowns. With the increase in the coefficients matrix size in Newton–Raphson methods, computational cost of inversion operation also increases. Minimum convergence time belongs to the 16th-order Hardy-Cross method, since it does not need to solve the linear system of equations. The number of iterations in Newton–Raphson method is less than Hardy-Crossmethod, but the Hardy–Cross method have a much less computational cost in each iteration. Thus, considering the time and volume of calculations, the 16th-order Hardy-Cross method has the best performance.

8. CONCLUSIONS

In the present paper, Hardy-Cross method was reviewed, examined and displayed in matrix form. Disadvantage of Hardy-Cross method is the requirement of establishing continuity equations at the beginning of the analysis, such that the selection of improper initial guess slows down the convergence trend and in some cases divergence occurs. A method for selecting the initial guess was proposed to overcome this disadvantage. The solutions obtained from the variable initial guess areso close to the final solution and the continuity equations are satisfied. The use of this methodat the beginning reduces the number of convergence iterations in Hardy–Cross method. This method was combined with third-order and 16th-order algorithms to speed up the analysis. The combined algorithms converged to solution with fewer iterations. Solving three examples indicated that the convergence time of 16th-order Hardy–Crossmethod is less than Newtonian analysis methods. In this method, there is no need to solve linear system of equations at each iteration.

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Hydraulic Analysis of Water Supply Networks Using a Modified Hardy Cross Method

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چکیده

روش های بسیاری در تحلیل هیدرولیکی شبکه های آبرسانی وجود دارد. در هر تکرار از فرایند حل اکثر این روش ها، یک دستگاه معادله خطی حل می شود که معمولا هزینه محاسباتی زیادی دارد. روش هاردی کراس از جمله روش هایی است که در فرایند تحلیل نیازی به حل دستگاه معادلات خطی ندارد و با تقسیم های اسکالر به جواب همگرا می شود. اما روش هاردی کراس دو عیب دارد. اولا در این روش حدس اولیه باید به گونه ای باشد که معادلات پیوستگی در گره ها برقرار باشد، ثانیا تعداد تکرار برای رسیدن به همگرایی قابل توجه است. در این مقاله راهکاری برای انتخاب حدس اولیه پیشنهاد می شود که به جواب نهایی نزدیک است و در عین حال معادلات پیوستگی بطور خودکار برقرار می شود. بنابراین می تواند در روش هاردی کراس بکار برده شود. همچنین برای کاهش تعداد تکرار همگرایی، روش هاردی کراس را با دو روش مرتبه سه و شانزده ترکیب کردیم. نتایج حاصل از حل چند مثال عددی نشان می دهد روش ترکیبی به همراه انتخاب حدس اولیه پیشنهادی تعداد تکرار و زمان تحلیل هیدرولیکی را کاهش می دهد و جواب ها با دقت بسیار خوبی همگرا می شوند.

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