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Research Article

Robust fault-tolerant tracking control design for spacecraft under control input saturation

Danyal Bustan [*,1], Naser Pariz [1], Seyyed Kamal Hosseini Sani [1]

Department of Electrical Engineering, Ferdowsi University of Mashhad, P.O. Box 9177948974, Mashhad, Iran

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ABSTRACT

In this paper, a continuous globally stable tracking control algorithm is proposed for a spacecraft in the presence of unknown actuator failure, control input saturation, uncertainty in inertial matrix and external disturbances. The design method is based on variable structure control and has the following properties: (1) fast and accurate response in the presence of bounded disturbances; (2) robust to the partial loss of actuator effectiveness; (3) explicit consideration of control input saturation; and (4) robust to uncertainty in inertial matrix. In contrast to traditional fault-tolerant control methods, the proposed controller does not require knowledge of the actuator faults and is implemented without explicit fault detection and isolation processes. In the proposed controller a single parameter is adjusted dynamically in such a way that it is possible to prove that both attitude and angular velocity errors will tend to zero asymptotically. The stability proof is based on a Lyapunov analysis and the properties of the singularity free quaternion representation of spacecraft dynamics. Results of numerical simulations state that the proposed controller is successful in achieving high attitude performance in the presence of external disturbances, actuator failures, and control input saturation.

1. Introduction

One of the challenging problems in the field of aerospace engineering is designing a spacecraft attitude tracking controller to maintain stability and performance in the presence of actuator failures, external disturbances, uncertainty in inertial matrix and control input saturation.

A conventional feedback control design for a complex system may result in an unsatisfactory performance, or even instability, in the event of malfunctions in actuators, sensors or other system components. To overcome such weaknesses, new approaches to control system design have been developed in order to tolerate component malfunctions while maintaining desirable stability and performance properties. These types of control systems are often known as fault-tolerant control systems (FTCS). Generally speaking, FTCS can be classified into two types: passive (PFTCS) and active (AFTCS). In PFTCS, controllers are fixed and are designed to be robust against a class of presumed faults. This approach needs neither fault detection and isolation (FDI) schemes nor controller reconfiguration. Compared to the passive approach, the active FTC approach requires a FDI mechanism to detect and identify the faults in real time, and then a mechanism to reconfigure the controllers according to the online fault information from the FDI. Compared to the passive approach, the AFTCS need significantly more computational power to implement. Furthermore, there is a time delay between the detection of faults and the reconfiguration of the controller in this approach [1]. These drawbacks motivate us for the investigation of a passive fault-tolerant controller for a spacecraft attitude control system with the occurrence of unexpected faults.

Numerous research results are available for the passive fault-tolerant controller design with different approaches, such as linear matrix inequalities (LMIs) schemes [2], H_∞ [3], adaptive control [4], sliding mode control [5], fuzzy logic [6] and neural networks [7]. Authors in [8] proposed four different controllers for certain and uncertain plants based on the absolute stability theory to handle the loss of actuator effectiveness; however, the designed controllers performed unsatisfactorily for systems with actuator failures [9].

In [2] a reliable robust fault-tolerant controller based on an LMI approach is designed. Model matching is the method used in [10] for actuator fault tolerant control and in [11] a robust fault tolerant control which is capable of attenuating both bounded and unbounded disturbances is proposed. In [12], performance indices are explicitly considered while stabilizing the attitude control of

* Corresponding author.
E-mail addresses: bustan.danyal@stu.um.ac.ir (D. Bustan), n-pariz@um.ac.ir (N. Pariz), k.hosseini@um.ac.ir (S.K. Hosseini Sani).
[1] Tel./fax: +98 5118763302.

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spacecraft. However, most of these controllers can only be applied to linear systems and are not applicable for non-linear dynamics.

Although in [13–16] a range of controllers to effectively handle the limited actuator output have been developed, actuator failures have not been taken into account in them. Considering actuator failures, there have been a number of results in the literature on attitude control of the spacecraft [7,17–22]. However, the issue of control input constraints has not been dealt within these approaches. Also Refs. [23–25] considered these two issues concurrently, but the disturbance effect is not taken into account.

To deal with the problem of actuator failures in presence of actuator saturation and external disturbances, Ref. [9] developed a robust fault-tolerant controller for spacecraft attitude control subsystem. But the proposed method fails in the situation in which any maneuver is required. Also robustness to uncertain inertial matrix is not considered.

To achieve high attitude performance, several issues including external disturbances, actuator failures, uncertainty in inertial matrix and control input saturation are required to be explicitly taken into account in the attitude controller design, which makes the controller design much more difficult.

To address this problem, a robust fault tolerant attitude tracking controller based on the variable structure approach is proposed for attitude control of the spacecraft with explicit consideration of external disturbances, actuator failures, and control input saturation. A key feature of the proposed strategy is that the design of the FTC is independent of the information about the faults.

Also the unit quaternion is employed to describe the attitude of a rigid spacecraft because of its global representation without singularities. The asymptotic stability of the closed-loop system is guaranteed by the Lyapunov direct approach and numerical simulations are carried out on the governing non-linear system equations of motion to show the performance of the proposed controller.

This paper is organized as follows. In Section 2, the spacecraft attitude dynamics are introduced. Fault-tolerant controllers are derived in Section 3. The results of numerical simulations are presented in Section 4. Finally, the paper is completed with some concluding remarks.

2. System model and equations of motion

2.1. Spacecraft attitude dynamics

The spacecraft is modeled as a rigid body with actuators that provide torques about three mutually perpendicular axes that defines a body-fixed frame ($\mathbf{B}$). The equations of motions are given by [26]

$$\dot{q}_0 = -\frac{1}{2}\mathbf{q}^T\boldsymbol{\omega} \tag{1}$$

$$\dot{\mathbf{q}} = \frac{1}{2}(\mathbf{q}^\times + q_0\mathbf{I}_3)\boldsymbol{\omega} \tag{2}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} = \mathbf{u} + \mathbf{d} \tag{3}$$

where $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)^T$ is the spacecraft angular velocity with respect to an inertial frame ($\mathbf{I}$) and expressed in the body-fixed frame ($\mathbf{B}$), the unit quaternion $\mathbf{Q} = (q_0, \mathbf{q}^T) \in \mathbb{R} \times \mathbb{R}^3$ describes the attitude orientation of the spacecraft in ($\mathbf{B}$) with respect to ($\mathbf{I}$), and satisfies $\mathbf{q}^T\mathbf{q} + q_0^2 = 1$, $\mathbf{I}_3$ denotes the 3×3 identity matrix, $\mathbf{J} \in \mathbb{R}^{3\times3}$ represents the positive definite spacecraft inertial matrix which has the property $J_m\|\mathbf{x}\|^2 \le \mathbf{x}^T\mathbf{J}\mathbf{x} \le J_M\|\mathbf{x}\|^2, \quad \forall \mathbf{x} \in \mathbb{R}^3$ where J_m and J_M are the positive constants, $\mathbf{u} = (u_1, u_2, u_3)^T \in \mathbb{R}^3$ is the control torque input generated by actuators, and $\mathbf{d} = (d_1, d_2, d_3)^T \in \mathbb{R}^3$ denotes the disturbance torque. For, $\boldsymbol{\xi} = (\xi_1, \xi_2, \xi_3)^T$ the notation $\boldsymbol{\xi}^\times$ denotes the following skew symmetric matrix:

$$\boldsymbol{\xi}^\times = \begin{pmatrix} 0 & -\xi_3 & \xi_2 \\ \xi_3 & 0 & -\xi_1 \\ -\xi_2 & \xi_1 & 0 \end{pmatrix} \tag{4}$$

Remark 1. Eqs. (1)–(3) are the dynamics equations of a rigid spacecraft equipped with thrusters as attitude control actuators and star tracker as attitude determination sensor.

For the development of the control laws, the following assumptions are made:

Assumption 1. All three components of the control torque, $\mathbf{u}$, are constrained by a bounded value, expressed by

$$|u_i| \le u_{\max}\, \forall t > 0 \quad i = 1, 2, 3 \tag{5}$$

Assumption 2. The disturbance, $\mathbf{d}$, is bounded, and for all elements of d_i there exists a positive but known constant, $\overline{d}$ such that $|d_i| \le \overline{d}$.

2.2. Desired dynamics

The desired motion of the spacecraft is specified by the attitude of a frame ($\mathbf{D}$) whose orientation with respect to ($\mathbf{I}$) is described by the unit quaternion $\mathbf{Q}_d = (q_{0d}, \mathbf{q}_d^T) \in \mathbb{R} \times \mathbb{R}^3$ that satisfy the constraint $\mathbf{q}_d^T\mathbf{q}_d + q_{0d}^2 = 1$. Let $\boldsymbol{\omega}_d = (\omega_{d1}, \omega_{d2}, \omega_{d3})^T$ denote the angular velocity of ($\mathbf{D}$) with respect to ($\mathbf{I}$), which is equivalent to the desired angular velocity of the spacecraft expressed in the frame ($\mathbf{D}$). The following assumption is made about $\boldsymbol{\omega}_d$ and $\dot{\boldsymbol{\omega}}_d$:

Assumption 3. There exist constants $\overline{\boldsymbol{\omega}}_d \ge 0$ and $\overline{\dot{\boldsymbol{\omega}}}_d \ge 0$, such that $|\omega_{di}| \le \overline{\omega}_d, \quad i = 1, 2, 3$ and $|\dot{\omega}_{di}| \le \overline{\dot{\omega}}_d, \quad i = 1, 2, 3$ for all $t \ge 0$.

2.3. Spacecraft attitude error dynamics

To address the attitude tracking problem, the attitude tracking error $\mathbf{Q}_e = (q_{0_e}, \mathbf{q}_e^T)^T$ is defined as the relative orientation between the body frame ($\mathbf{B}$) and the desired frame ($\mathbf{D}$) and it is computed by the quaternion multiplication rule [26] as

$$\mathbf{q}_e = q_{0_d}\mathbf{q} - q_0\mathbf{q}_d + \mathbf{q}^\times\mathbf{q}_d \tag{6}$$

$$q_{0_e} = q_{0_d}q_0 + \mathbf{q}_d^T\mathbf{q} \tag{7}$$

The corresponding rotation matrix is given by

$$\mathbf{C}(\mathbf{Q}_e) = (q_{0_e}^2 - \mathbf{q}_e^T\mathbf{q}_e)\mathbf{I}_3 + 2\mathbf{q}_e\mathbf{q}_e^T - 2q_{0_e}\mathbf{q}_e^\times \tag{8}$$

Note that $\|\mathbf{C}\| = 1$ and $\dot{\mathbf{C}} = -\boldsymbol{\omega}_e^\times\mathbf{C}$, where the relative angular velocity $\boldsymbol{\omega}_e$ of ($\mathbf{B}$) with respect to ($\mathbf{D}$) is defined as $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \mathbf{C}\boldsymbol{\omega}_d$.

Now, the governing differential equations for the attitude tracking error, $\mathbf{q}_e$, are stated as follows:

$$\dot{q}_{0_e} = -\tfrac{1}{2}\mathbf{q}_e^T\boldsymbol{\omega}_e \tag{9}$$

$$\dot{\mathbf{q}}_e = \tfrac{1}{2}(\mathbf{q}_e^\times + q_{0_e}\mathbf{I}_3)\boldsymbol{\omega}_e \tag{10}$$

$$\mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} + \mathbf{u} + \mathbf{d} - \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d - \boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d) \tag{11}$$

When the spacecraft has three actuators and some of them partially fail, the attitude dynamics of the spacecraft is expressed as

$$\mathbf{J}\dot{\boldsymbol{\omega}}_e = -\boldsymbol{\omega}^\times\mathbf{J}\boldsymbol{\omega} + \boldsymbol{\Gamma}\mathbf{u} + \mathbf{d} - \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d - \boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d) \tag{12}$$

where $\boldsymbol{\Gamma} = \text{diag}\{\Gamma_1, \Gamma_2, \Gamma_3\}$, $0 < \Gamma_i \le 1$ is the actuation effectiveness matrix. The case in which $\Gamma_i = 1$ implies that the ith actuator is healthy, and $0 < \Gamma_i < 1$ corresponds to the case in which the ith actuator partially fails.

For a spacecraft attitude control system, in the presence of actuator saturation, external disturbances and the partial loss of control effectiveness, the control objectives can be stated as follows [9]:

1. All of the internal signals in the closed-loop system are bounded and continuous.
2. The attitude error, $\mathbf{q}_e$, and relative angular velocity, $\boldsymbol{\omega}_e$ converge asymptotically to zero.
3. The performance index $\mathbf{I}_p = \lim \frac{1}{t}\int_0^t \|\mathbf{S}\|^2 dt$ is bounded, where $\mathbf{S}$ is an auxiliary variable defined as

$$\mathbf{S} = \boldsymbol{\omega}_e + k^2(t)\mathbf{q}_e \tag{13}$$

and $k(t)$ is a time-varying function that will be defined later.

3. Fault tolerant controller design

Theorem 1. *The control laws (14), globally asymptotically stabilize the system described by Eqs. (9)–(11) and satisfies the above control objectives.*

$$u_i = -\frac{u_{\max} s_i}{|s_i| + k^2\delta} \quad i = 1,2,3 \tag{14}$$

where s_i is the ith element of $\mathbf{S}$ defined in (13), and δ is a positive control constant.

Proof. To establish stability, a candidate Lyapunov function given by (15) is considered.

$$V = \frac{1}{2}\boldsymbol{\omega}_e^T \mathbf{J}\boldsymbol{\omega}_e + k^2[\mathbf{q}_e^T\mathbf{q}_e + (1-q_{0e})^2] + \frac{k^2}{2\gamma} \tag{15}$$

where γ is a positive constant to be determined.

The derivative of (15) is computed, and simplified using (3), (9), (10), (12), and (13), and the property of the unit quaternion, $\mathbf{q}_e^T\mathbf{q}_e + q_{0e}^2 = 1$. After some algebraic manipulation it can be shown that

$$\begin{aligned}
\dot V &= \boldsymbol{\omega}_e^T\mathbf{J}\dot{\boldsymbol{\omega}}_e + k^2\left[2\mathbf{q}_e^T\dot{\mathbf{q}}_e - 2(1-q_{0e})\dot q_{0e}\right] + 2k\dot k\left[\mathbf{q}_e^T\mathbf{q}_e + (1-q_{0e})^2\right] + \frac{k\dot k}{\gamma} \\
&= \boldsymbol{\omega}_e^T\mathbf{J}\dot{\boldsymbol{\omega}}_e + k^2\left[2\mathbf{q}_e^T(\mathbf{q}_e^\times + q_{0e}\mathbf{I}_3)\boldsymbol{\omega}_e + (1-q_{0e})\mathbf{q}_e^T\boldsymbol{\omega}_e\right] \\
&\quad + 2k\dot k\left[1 - q_{0e}^2 + (1-q_{0e})^2\right] + \frac{k\dot k}{\gamma} \\
&= \boldsymbol{\omega}_e^T\mathbf{J}\dot{\boldsymbol{\omega}}_e + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + 4k\dot k(1-q_{0e}) + \frac{k\dot k}{\gamma} \\
&= \boldsymbol{\omega}_e^T\mathbf{J}\dot{\boldsymbol{\omega}}_e + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= \boldsymbol{\omega}_e^T(-(\boldsymbol{\omega}_e + \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}(\boldsymbol{\omega}_e + \mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d) + \boldsymbol{\Gamma}\mathbf{u} + \mathbf{d} \\
&\quad + \mathbf{J}(\boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d)) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= \boldsymbol{\omega}_e^T(-\boldsymbol{\omega}_e^\times\mathbf{J}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\boldsymbol{\omega}_e \\
&\quad - (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d + \boldsymbol{\Gamma}\mathbf{u} + \mathbf{d} + \mathbf{J}(\boldsymbol{\omega}_e^\times\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d - \mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d)) \\
&\quad + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= \boldsymbol{\omega}_e^T(-(\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\boldsymbol{\omega}_e - (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d + \boldsymbol{\Gamma}\mathbf{u} \\
&\quad + \mathbf{d} - \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\boldsymbol{\omega}_e - \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= \boldsymbol{\omega}_e^T(-((\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J} + \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times)\boldsymbol{\omega}_e - (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d \\
&\quad - \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d + \boldsymbol{\Gamma}\mathbf{u} + \mathbf{d}) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= -\boldsymbol{\omega}_e^T((\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J} + \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times)\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^T((\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d \\
&\quad + \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d) + \boldsymbol{\omega}_e^T(\boldsymbol{\Gamma}\mathbf{u} + \mathbf{d}) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= -\boldsymbol{\omega}_e^T\mathbf{H}\boldsymbol{\omega}_e - \boldsymbol{\omega}_e^T\mathbf{g} + \boldsymbol{\omega}_e^T(\boldsymbol{\Gamma}\mathbf{u} + \mathbf{d}) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&= \boldsymbol{\omega}_e^T(\boldsymbol{\Gamma}\mathbf{u} + \mathbf{d} - \mathbf{g}) + k^2\mathbf{q}_e^T\boldsymbol{\omega}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right)
\end{aligned} \tag{16}$$

where $\mathbf{H} = (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J} + \mathbf{J}(\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times$ and $\mathbf{g} = (\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d)^\times\mathbf{J}\mathbf{C}(\mathbf{Q}_e)\boldsymbol{\omega}_d + \mathbf{J}\mathbf{C}(\mathbf{Q}_e)\dot{\boldsymbol{\omega}}_d$ as $\mathbf{H} = -\mathbf{H}^T$, so $\boldsymbol{\omega}_e^T\mathbf{H}\boldsymbol{\omega}_e = 0$.

Suppose that $\lambda_{\min}$ and $\lambda_{\max}$ are two positive constants satisfying $0 < \lambda_{\min} \le \min_{i=1,2,3}\{\Lambda_i\}$ and $\lambda_{\max} \ge \max_{i=1,2,3}\{\Lambda_i\}$, then

$$\begin{aligned}
-\Lambda_i\frac{u_{\max}\omega_{ie}^2}{|s_i| + k^2\delta} &\le -\lambda_{\min}u_{\max}\frac{\omega_{ie}^2}{|s_i| + k^2\delta} \\
&\le -\lambda_{\min}u_{\max}\frac{\omega_{ie}^2}{|\omega_{ie}| + k^2(\delta+1)} \\
&= -\lambda_{\min}u_{\max}|\omega_{ie}|\left(1 - \frac{k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)}\right)
\end{aligned} \tag{17}$$

After using (17) we have

$$\begin{aligned}
\dot V &= -\sum_{i=1}^{3}\Lambda_i\frac{u_{\max}\omega_{ie}s_i}{|s_i| + k^2\delta} + \boldsymbol{\omega}_e^T(\mathbf{d}(t) - \mathbf{g}) \\
&\quad + k^2\mathbf{q}_e^T\mathbf{S} - k^4\mathbf{q}_e^T\mathbf{q}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
\dot V &= -\sum_{i=1}^{3}\Lambda_i\frac{u_{\max}\omega_{ie}(\omega_{ie} + k^2 q_{ie})}{|s_i| + k^2\delta} + \boldsymbol{\omega}_e^T(\mathbf{d}(t) - \mathbf{g}) \\
&\quad + k^2\mathbf{q}_e^T\mathbf{S} - k^4\mathbf{q}_e^T\mathbf{q}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&\le -\sum_{i=1}^{3}|\omega_{ie}|(\lambda_{\min}u_{\max} - \overline{\mathbf{d}} - \overline{\mathbf{g}}) + \sum_{i=1}^{3}|\omega_{ie}|\frac{\lambda_{\min}u_{\max}|\omega_{ie}|k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} \\
&\quad - \sum_{i=1}^{3}\frac{\Lambda_i u_{\max}k^2\omega_{ie}q_{ie}}{|s_i| + k^2\delta} + k^2\mathbf{q}_e^T\mathbf{S} - k^4\mathbf{q}_e^T\mathbf{q}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right) \\
&\le -\sum_{i=1}^{3}|\omega_{ie}|(\lambda_{\min}u_{\max} - \overline{\mathbf{d}} - \overline{\mathbf{g}}) + \sum_{i=1}^{3}|\omega_{ie}|\frac{\lambda_{\min}u_{\max}|\omega_{ie}|k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} \\
&\quad + \sum_{i=1}^{3}\frac{\lambda_{\max}u_{\max}k^2|\omega_{ie}||q_{ie}|}{|s_i| + k^2\delta} + k^2\mathbf{q}_e^T\mathbf{S} - k^4\mathbf{q}_e^T\mathbf{q}_e + k\dot k\left(4(1-q_{0e}) + \frac{1}{\gamma}\right)
\end{aligned} \tag{18}$$

where $\overline{g} = \mathbf{J}_M\overline{\boldsymbol{\omega}}_d^2 + \mathbf{J}_M\overline{\dot{\boldsymbol{\omega}}}_d$. Based on (18), the updating law for k is designed to satisfy

$$\begin{aligned}
\dot k &= \frac{-\gamma}{k(4\gamma(1-q_{0e}) + 1)}\left\{\sum_{i=1}^{3}|\omega_{ie}|\frac{\lambda_{\min}u_{\max}|\omega_{ie}|k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + k^2\mathbf{q}_e^T\mathbf{S}\right. \\
&\quad \left. + \sum_{i=1}^{3}\lambda_{\max}\frac{u_{\max}k^2|\omega_{ie}||q_{ie}|}{|s_i| + k^2\delta}\right\}
\end{aligned} \tag{19}$$

By using the updating law for $k(t)$ as (19), (18) is simplified to

$$\dot V \le -|\omega_{ie}|(\lambda_{\min}u_{\max} - \mathbf{d}(t)) - k^4\mathbf{q}_e^T\mathbf{q}_e \tag{20}$$

To ensure $\dot V \le 0$ in (20), the following assumption is made:

Assumption 4. The bound value $\overline{d}$ of the disturbance $\mathbf{d}$ satisfies the following inequality [9,14,27]:

$$\lambda_{\min}u_{\max} > 2(\overline{d} + \overline{g}) \tag{21}$$

Remark 2. Loosely speaking, Assumption 4 states that the available control is sufficient to reject disturbances and simultaneously track the desired trajectory.

Under Assumption 4, $\dot V \le 0$ can be obtained. This implies that $\boldsymbol{\omega}_e$ and $k(t)$ are bounded. With $\mathbf{q}_e^T\mathbf{q}_e + q_{0e}^2 = 1$, and $\mathbf{q}_e$ and q_{0e} are bounded. Also $\boldsymbol{\omega}_d$ is bounded under Assumption 3, so V is bounded.

Let $\lambda_{\min} u_{\max} - \overline{d} = c$.

Then, based on (20), by integrating $\dot{V}$ from 0 to ∞, it can be shown as follows:

$$V(0)-V(\infty) \geq (\lambda_{\min} u_{\max} - \overline{d}) \int_0^\infty |\boldsymbol{\omega}_e| dt + \int_0^\infty k^2 \mathbf{q}_e^T \mathbf{q}_e$$
$$\geq c \int_0^\infty |\boldsymbol{\omega}_e| dt + \int_0^\infty k^2 \mathbf{q}_e^T \mathbf{q}_e$$
$$\geq \overline{c}_{\min} \left(\int_0^\infty |\boldsymbol{\omega}_e| dt + \int_0^\infty k^2 \mathbf{q}_e^T \mathbf{q}_e \right) \tag{22}$$

In which $\overline{c}_{\min} = \min\{c, 1\}$. It is clear that $\overline{c}_{\min} > 0$.

Since V is bounded, then $\boldsymbol{\omega}_e \in L_1$ and $k^2 \mathbf{q}_e \in L_2$.

It is clear that $\mathbf{S}$ is bounded because all of its terms are bounded as shown above. Thus, since $\boldsymbol{\omega}_e$, $\boldsymbol{\omega}_d$, $\mathbf{q}_e$, $\mathbf{S}$ and $\mathbf{u}$ (from (5)) are bounded, $\dot{k}$ is bounded. Moreover, $\dot{\boldsymbol{\omega}}_e$ is also bounded from the attitude dynamics in (11), and $\dot{\mathbf{q}}_e$ is bounded from (10), because $\boldsymbol{\omega}_e$, $\mathbf{q}_e$and q_{0e} are bounded. By the Barbalat lemma [28]

$$\lim_{t\to\infty} \boldsymbol{\omega}_e = \lim_{t\to\infty} k^2 \mathbf{q}_e = 0 \tag{23}$$

However, the fact that $k^2 \mathbf{q}_e$ converges to zero does not ensure that $\mathbf{q}_e$ will converge to zero. But it is shown that k is bounded, because $\dot{V} \leq 0$, so it can be concluded that $\lim_{t\to\infty} \mathbf{q}_e = 0$.

It is seen that the potential problem with the above algorithm is that $k(t)$ could converge to zero before $\boldsymbol{\omega}_e$, and, therefore, cause chattering of the signals in the system because, for $k=0$, the control laws become $u_i = -\text{sign}(s_i) u_{\max}$. If there is a positive lower bound for $k(t)$ for all t, the above potential problem has been resolved.

Lemma 1. *There is a lower bound for k(t) for all $t \geq 0$.*

Proof. Based upon (19), it follows that

$$\dot{k} \geq \frac{-\gamma}{k(4\gamma(1-q_{0e})+1)} \left\{ \sum_{i=1}^{3} |\omega_{ie}| \frac{\lambda_{\min} u_{\max} |\omega_{ie}| k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + k^2 \|\mathbf{q}_e\| \|\mathbf{S}\| + \sum_{i=1}^{3} \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}| |q_{ie}|}{|s_i| + k^2 \delta} \right\}$$
$$\dot{k} \geq \frac{-\gamma}{k} \left\{ \sum_{i=1}^{3} |\omega_{ie}| \frac{\lambda_{\min} u_{\max} |\omega_{ie}| k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + k^2 \|\mathbf{S}\| + \sum_{i=1}^{3} \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}|}{|s_i| + k^2 \delta} \right\} \tag{24}$$

Considering the terms within the summation sign, it can be shown that

$$\frac{\lambda_{\min} u_{\max} |\omega_{ie}| k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}|}{|s_i| + k^2 \delta}$$
$$\leq \frac{\lambda_{\min} u_{\max} |\omega_{ie}| k^2(\delta+1)}{|s_i| + k^2 \delta} + \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}|}{|s_i| + k^2 \delta}$$
$$= u_{\max} |\omega_{ie}| k^2 \frac{\lambda_{\min}(\delta+1) + \lambda_{\max}}{|s_i| + k^2 \delta}$$
$$= u_{\max} k^2 (\lambda_{\min}(\delta+1) + \lambda_{\max}) \frac{|s_i - k^2 q_{ie}|}{|s_i| + k^2 \delta}$$
$$\leq u_{\max} k^2 (\lambda_{\min}(\delta+1) + \lambda_{\max}) \frac{|s_i| + k^2}{|s_i| + k^2 \delta}$$
$$\leq u_{\max} k^2 (\lambda_{\min}(\delta+1) + \lambda_{\max}) \left(1 + \frac{1}{\delta}\right) \tag{25}$$

From (23), it can be first noted that $\lim_{t\to\infty} \mathbf{S}(t) = 0$ due to its definition in (13). Therefore there exist constant $\overline{S}$ such that $|\mathbf{S}(t)| \leq \overline{S}$ for all time. Hence

$$\dot{k} \geq -\frac{\gamma}{k} \left\{ 3 u_{\max} k^2 (\lambda_{\min}(\delta+1) + \lambda_{\max}) \left(1 + \frac{1}{\delta}\right) + \overline{S} \right\} \dot{k} \geq -\gamma k \varepsilon \tag{26}$$

where

$$\varepsilon = 3 u_{\max} (\lambda_{\min}(\delta+1) + \lambda_{\max}) \left(1 + \frac{1}{\delta}\right) + \overline{S} \tag{27}$$

Assume $k(0) = k_0 > 0$ then (26) can be integrated to obtain

$$k(t) \geq \frac{1}{2} k_0 e^{-\gamma \varepsilon t} \tag{28}$$

Thus (28) shows that $k(t) > 0$ for all time and $k(t) = 0$ is possible only at $t = \infty$. On the other hand, since $k(t)$ is bounded, for a given positive constant γ which satisfies above condition, there exists a positive function $\tau(\gamma)$ satisfying $k(t) < \tau(\gamma)$. Then, from (19), we have

$$\dot{k} \geq \frac{-\gamma}{k(4\gamma(1-q_{0e})+1)} \left\{ \sum_{i=1}^{3} |\omega_{ie}| \frac{\lambda_{\min} u_{\max} k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + \sum_{i=1}^{3} \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}| |q_{ie}|}{|s_i| + k^2 \delta} + k^2 \mathbf{q}_e^T \mathbf{S} \right\}$$
$$\geq \frac{-\gamma}{k} \left\{ \sum_{i=1}^{3} |\omega_{ie}| \frac{\lambda_{\min} u_{\max} k^2(\delta+1)}{|\omega_{ie}| + k^2(\delta+1)} + \sum_{i=1}^{3} \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}|}{|s_i| + k^2 \delta} + k^2 \|\boldsymbol{\omega}_e + k^2 \mathbf{q}_e\| \right\}$$
$$\geq \frac{-\gamma}{k} \left\{ \sum_{i=1}^{3} |\omega_{ie}| \frac{\lambda_{\min} u_{\max} k^2(\delta+1)}{k^2(\delta+1)} + \sum_{i=1}^{3} \lambda_{\max} \frac{u_{\max} k^2 |\omega_{ie}|}{k^2 \delta} + k^2 \|\boldsymbol{\omega}_e\| + k^4 \|\mathbf{q}_e^T \mathbf{q}_e\| \right\}$$
$$\geq \frac{-\gamma}{k} \left(\lambda_{\min} u_{\max} + \frac{\lambda_{\max} u_{\max}}{\delta} + \tau^2(\gamma) \right) \|\boldsymbol{\omega}_e\| + k^4 \|\mathbf{q}_e^T \mathbf{q}_e\|$$
$$\geq \frac{-\gamma c_{\max}}{k} \left(\|\boldsymbol{\omega}_e\| + k^4 \|\mathbf{q}_e^T \mathbf{q}_e\| \right) \tag{29}$$

where $c_{\max} = \max\{\lambda_{\min} u_{\max} + (\lambda_{\max} u_{\max}/\delta) + \tau^2(\gamma), 1\}$. Thus

$$\dot{k} k \geq -\gamma c_{\max} (||\boldsymbol{\omega}_e|| + k^2 \mathbf{q}_e^T \mathbf{q}_e) \tag{30}$$

Integrating this inequality from 0 to ∞, gives

$$k^2(\infty) \geq k^2(0) - 2\gamma c_{\max} \int_0^\infty [||\boldsymbol{\omega}_e|| + k^2 \mathbf{q}_e^T \mathbf{q}_e] \, dx$$
$$\geq k^2(0) + \frac{2\gamma c_{\max}}{\overline{c}} [V(\infty) - V(0)]$$
$$\geq k^2(0) - \frac{2\gamma c_{\max}}{\overline{c}} V(0) \tag{31}$$

If the initial value is chosen as $k(0) > \sqrt{(2\gamma c_{\max}/\overline{c}) V(0)}$, then from (31), we obtain

$$k^2(\infty) \geq \frac{\gamma c_{\max}}{\overline{c}} V(0) > 0 \tag{32}$$

Based on (26) and (32), $k > 0$ will hold for all time. Thus, with $\lim_{t\to\infty} k^2 \mathbf{q}_e = 0$ in (23), we must have $\lim_{t\to\infty} \mathbf{q}_e = 0$ since the convergence of $\boldsymbol{\omega}_e$ and $\mathbf{q}_e$ are independent of their initial values; the global asymptotic stability of the closed-loop system is demonstrated. This concludes the proof of Theorem 1. □

4. Simulation example

In this section the application of the proposed controller to the attitude control of a spacecraft is presented. It is assumed that each actuator generates a continuous control torque and its output is limited to 5 Nm. The inertia matrix is obtained from Ref. [9], which is given as

$$\mathbf{J} = \begin{bmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0.9 & 0 & 15 \end{bmatrix} \text{kg m}^2$$

and the external disturbance is assumed to be $\mathbf{d} = (|\omega|^2 + 0.05)[\sin(0.8t), \ \cos(0.5t), \ \cos(0.3t)]^T$ Nm. Also it is assumed that the angular velocity measurements are corrupted with random measurement noise of magnitude 0.1 rad/s. Also the effectiveness for actuators are given by

$$e_i = \begin{cases} 1 & \text{if } f_i > 1 \\ 0.1 & \text{if } f_i < 0.1 \\ f_i & \text{otherwise;} \end{cases}$$

where $f_i = 0.3 + 0.1(\sin(0.5t + i\pi/3) + randn())$, $i = 1, 2, 3$, and $randn()$ is a function which generates a random number with normal distribution.

The parameters for the controller in (14), are $\gamma = 0.002$, $\delta = 0.02$, $k(0) = 2.5$ and $\bar{\lambda}_{\max} = 1.5$.

In order to verify the performance and robustness of the control laws (14) in the presence of actuator faults and external disturbances, tracking the desired attitude is simulated. In the simulation, actuator effectiveness after 150 s changes from normal to faulty condition.

To verify the performance of the control laws (14), tracking the desired attitude is simulated. The initial attitude is set to $q_0 = 0.9631$, $q_1 = -0.1$, $q_2 = 0.15$ and $q_3 = -0.2$, and the angular velocity is assumed to be zero at $t = 0$. The goal is to track the following attitude trajectory:

$$\mathbf{q}_d = \frac{1}{2}\begin{cases} \left[0,\ \cos(0.3t),\ -\sqrt{3},\ \sin(0.3t)\right]^T & t < 100 \\ \left[\sin(0.3t), 0,\ \cos(0.3t),\ -\sqrt{3}\right]^T & 100 \le t < 200 \\ \left[-\sqrt{3},\ \sin(0.3t), 0,\ \cos(0.3t)\right]^T & 200 \le t < 300 \\ \left[\cos(0.3t),\ -\sqrt{3},\ \sin(0.3t), 0\right]^T & 300 \le t < 400 \end{cases}$$

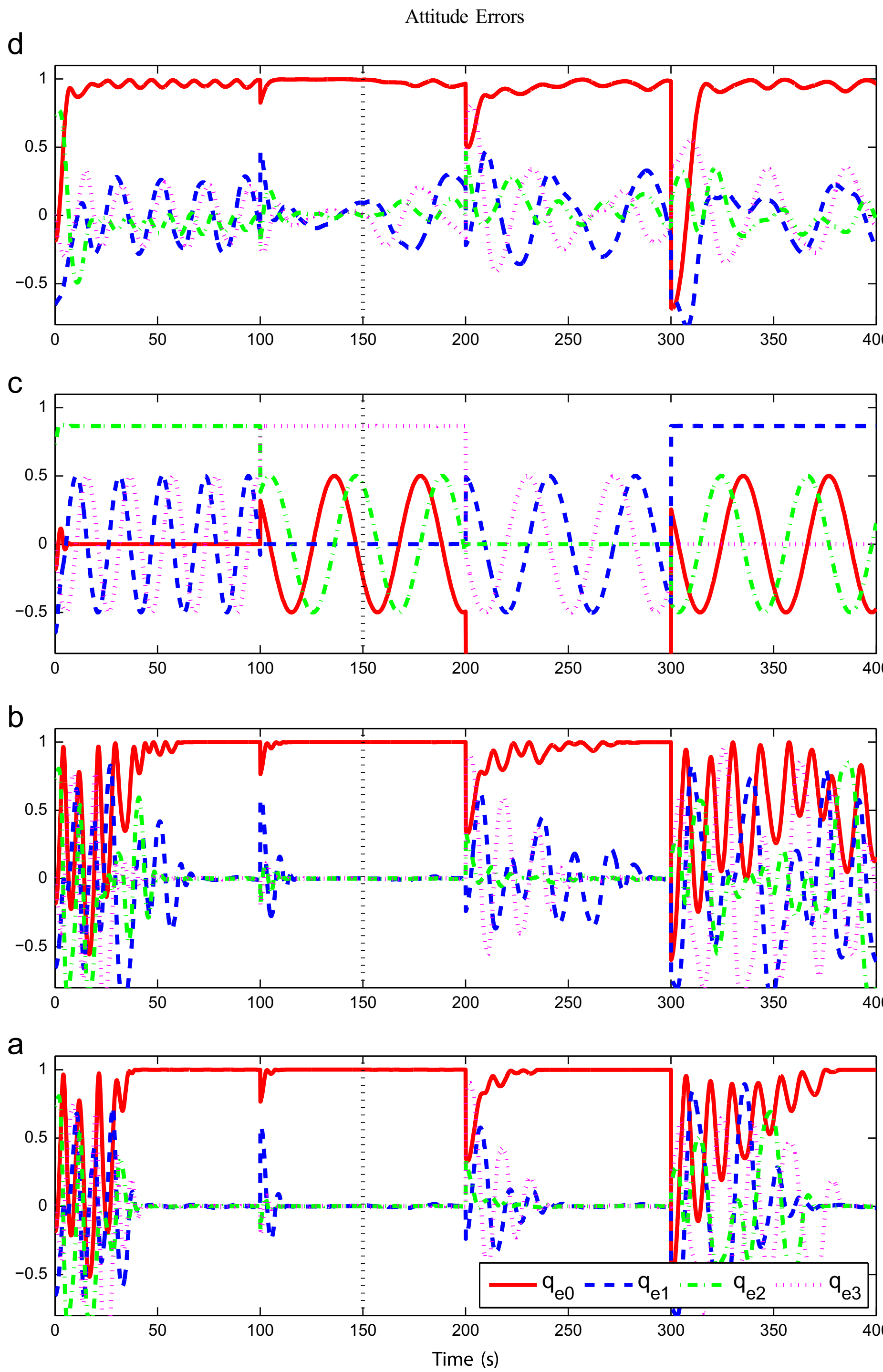

Fig. 1. Attitude errors, (a) proposed controller, (b) proposed method in [14], (c) proposed method in [9], and (d) PD controller. Dotted line: time of switch from normal to faulty condition.

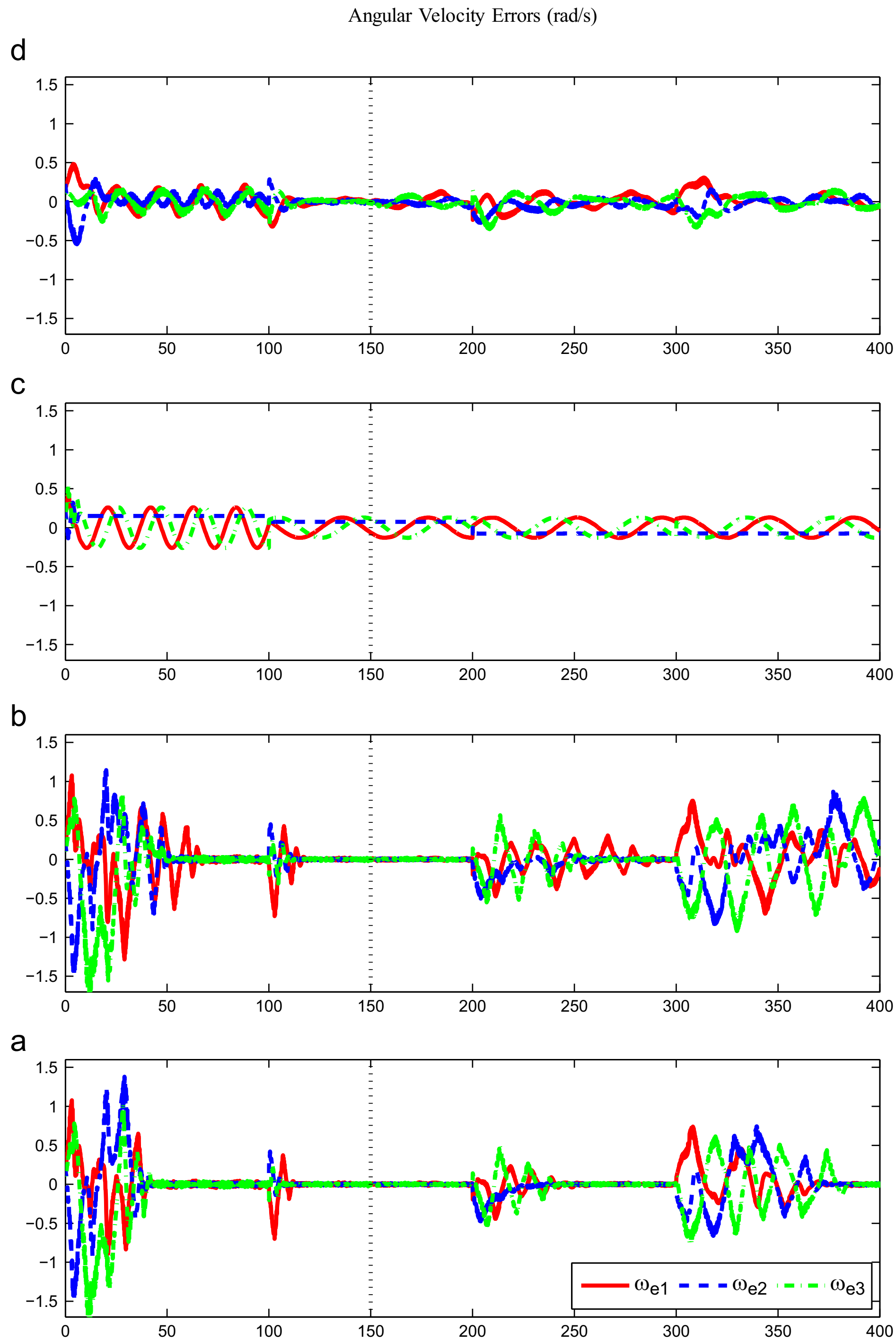

Fig. 2. Angular velocity errors, (a) proposed controller, (b) proposed method in [14], (c) proposed method in [9], and (d) PD controller. Dotted line: time of switch from normal to faulty condition.

For comparison purposes, the results of two comparative papers [9] and [14] with the same parameters are simulated. Although these papers have the same structure for its control laws as (14), the adaptation law for $k(t)$ used in them are different from the proposed one in (19).

Also results of a conventional PD controller with $K_p = 4$ and $K_d = 5$ is added to simulation results. Fig. 1 shows attitude errors, Fig. 2 shows angular velocity error and Fig. 3 shows control effort. Also Fig. 4 shows behavior of adaptive term during simulation.

The absolute value of the sum of the control efforts is calculated for each case. As these papers used thrusters for its actuators too, this parameter can be used to assess the performance of the proposed controllers.

The absolute values of the sum of the control efforts used to achieve desired attitude are summarized in Table 1. It can be concluded that the proposed control scheme has better performance in comparison to the other methods. In Table 1 control effort of controller in [9] and PD controller are not applicable

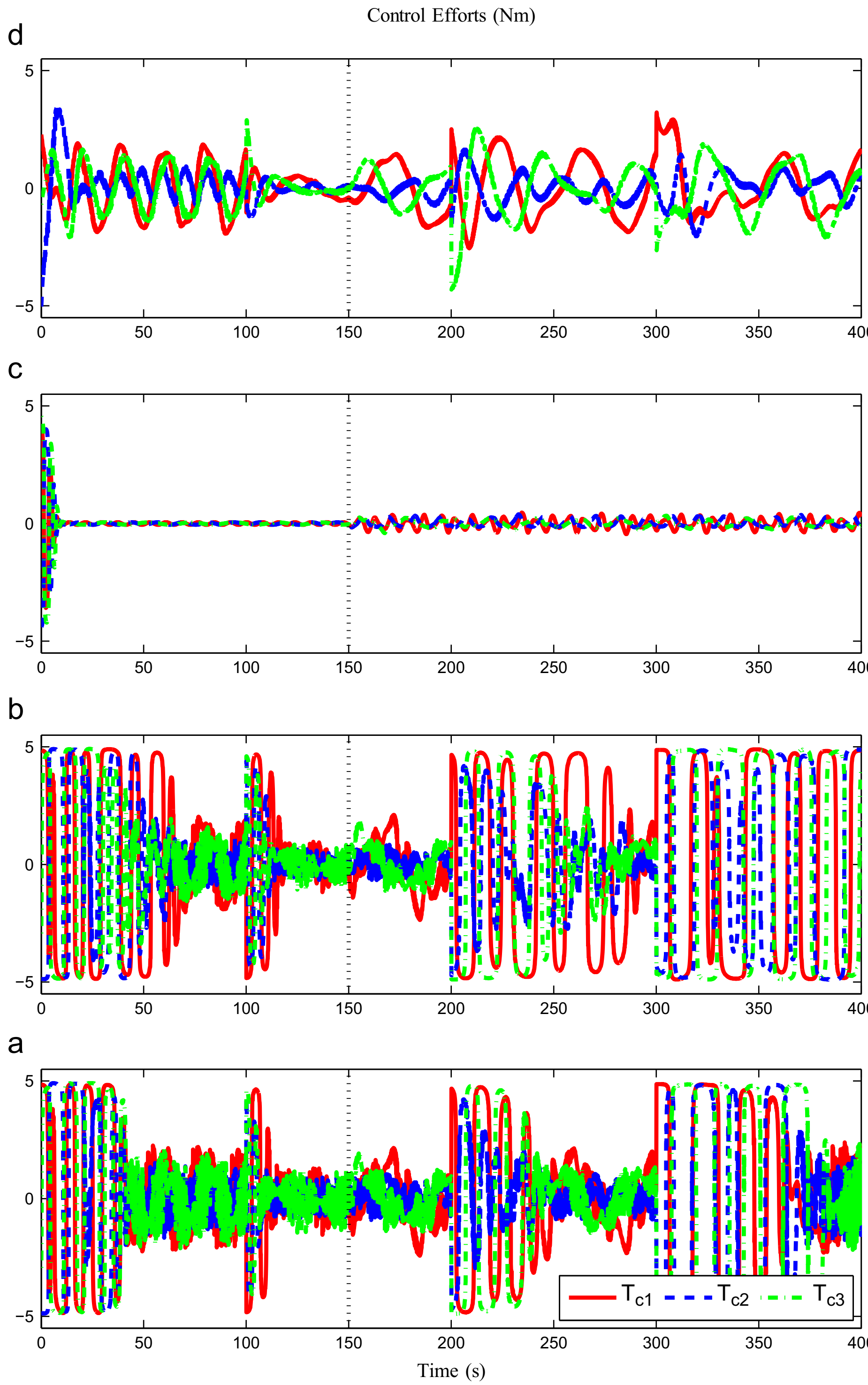

Fig. 3. Control efforts, (a) proposed controller, (b) proposed method in [14], (c) proposed method in [9], and (d) PD controller. Dotted line: time of switch from normal to faulty condition.

because they cannot track the desired attitude as it can be seen in Fig. 1(c) and (d).

It can be seen from the results that, although actuator effectiveness is changed (Fig. 5), there is no dramatic changes in the attitude or angular velocities errors. The price for this immunity is paid by control effort. In both cases, at $t = 150$ the behavior of control effort signals is changed. This is because of switching from normal condition to faulty condition. At this time, controller tries to compensate the effect of faults and this effort causes changes in the behavior of control efforts.

5. Conclusion

A fault-tolerant tracking control scheme based on variable structure control has been developed for spacecraft attitude

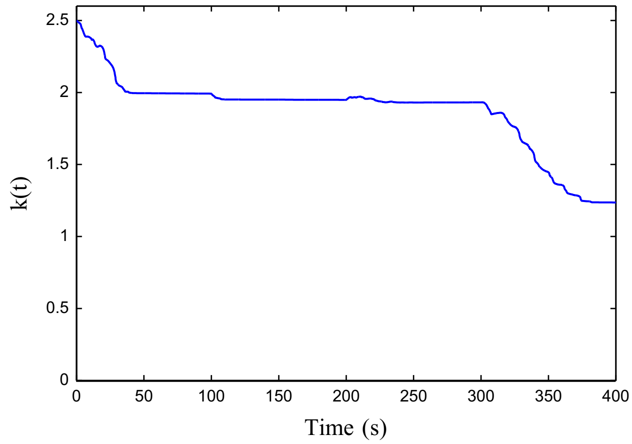

Fig. 4. Adaptive term ($k(t)$).

Table 1
Control effort comparison between three methods.

Control method	Sum of absolute value
Proposed controller in this paper	31125
Proposed controller in [14]	32488
Proposed controller in [9]	Not applicable
PD controller	Not applicable

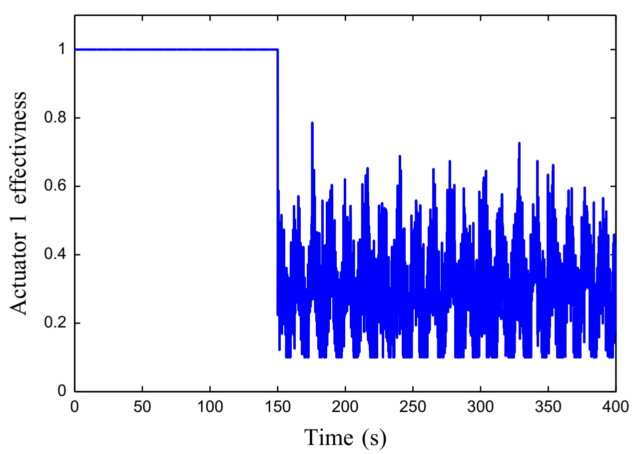

Fig. 5. Actuator 1 effectiveness.

stabilization in the presence of input saturation, external disturbances and unknown actuator faults. The proposed control design methods do not require any system identification process to identify the faults or any method of fault detection and isolation. Although the control law requires the update of the parameter k as (19), this is likely to be more computationally efficient than alternative fault identification schemes. The control formulation is based on Lyapunov's direct stability theorem in the controller synthesis. Evaluating this control scheme using numerical simulations, shows that the robust fault-tolerant attitude controller is able to recover from actuator failure and achieve high precision tracking. Furthermore, the control objective can be achieved even under actuator input constraints. This conclusion is valid with the assumption that the system is controllable with the remaining active controls, such that the control effectiveness of the remaining controls is large enough to counter the undesirable effects produced by external disturbances.

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