



## Numerical investigation of Hydro-magnetic flow of air in three different geometries of open cavity

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### Abstract

MHD or Magnetic hydro dynamics of an open cavity with three different depths is investigated numerically in this paper. The SIMPLE algorithm is utilized in order to solve the equations of conservation of momentum and energy. Isothermal lines and streamlines of the flow are presented for 3 Ha numbers: Ha=10, 20, and 30 for each of the three cavity depths: L, 2L, and 3L. Then the Nu number of the heated wall has been studied and compared for each Ha number and cavity depth. It was observed that in the case of depth L and Ha=30 the maximum heat transfer is achieved while the minimum heat transfer rate is for the case of depth 2L and Ha=20.

**Keywords:** MHD, open cavity, mixed convection

### Introduction

Generally, cavities can be classified into two major groups; namely closed and open cavities. The open cavities' heat transfer and mass flow have been investigated widely due to their significant applications in many industries such as: solar concentrators, lakes and reservoirs, refrigeration, cooling process of electric devices and fire researches [1-5]. Also, the problems of natural convection and hydro magnetic flow have been major topics of research during the last decade. Their wide occurrence in many industrial cases like: crystal growth, oil extraction, electronic cooling and solar collectors make them undeniable [6-9].

In 1992, Braga and Viskanta [10] reported an experimental and theoretical investigation of transient natural convection in a rectangular cavity. Al-Nimr obtained analytical solutions for MHD fully developed upward (heating) or downward (cooling) natural convection in open ended porous annuli (1995) [11]. Later on in 1999, Al-Nimr and Hader modified their solution for more general thermal boundary conditions [12].

Ishikawa et al. (2000) numerically investigated the natural convection with density inversion in a square cavity with variable fluid properties [13]. Hossain and Rees in 2005 studied unsteady laminar natural convection flow of water in a rectangular cavity and found that the heat generation rate and the mean temperature of solid walls play an important role in the flow and temperature fields [14]. Kandaswamy et al. [15] investigated the heat transfer rate of partially active walls of a square cavity. They found that the

heat transfer rate is maximum for middle – middle thermally active location and it is very poor for the top – bottom locations (2008).

In 2011, Sivasankaran et al. numerically investigated the magneto-convection of cold water in an open cavity with variable fluid properties. In their work it was observed that the convection heat transfer is enhanced by thermo capillary force when buoyancy force is weakened [16]. Oztop et al. made a numerical study on the MHD mixed convection in a lid driven cavity with corner heater. They applied a finite volume technique to observe the fluid flow and temperature fields under different Grashof and Hartmann numbers. They have taken the Joule effect under account in their study [17].

In 2012, the LBM (Lattice Boltzmann Method) method was applied to solve the fluid flow and energy equations. Nemati et al. used LBM to study the Magnetic field effects on natural convection flow of nanofluid in a rectangular cavity. They found that the average Nusselt number increases for nanofluid when increasing the solid volume fraction, while, in the presence of a strong magnetic field, this effect decreases [18]. Kefayati et al. used the same method to simulate the MHD mixed convection in a lid driven cavity with linearly heated wall [19]. They studied the influence of Richardson and Hartmann Numbers as well as the inclination angle of the magnetic field on the convection heat transfer rate.

Also in this year, Prakash et al. investigated the natural convection of open cavities numerically. Different shapes of open cavities were considered in their work such as: cubical, hemispherical and spherical. They presented the 3D results of the convective loss and Nu number of these cavities [20]. Rahman et al. (2012) have made a computational study on the mixed convection of an open square cavity heated from different sides [21]. They reported their results for fixed Ha and Re numbers. In this work the mixed convection of an open cavity is investigated for different Ha numbers and different cavity depths.

### Geometry and governing equations

#### Studied model geometries

The geometries and boundary conditions which are considered in this work are presented in figure 1(a) – (c). The open cavity includes an open channel from which the flow enters and exits, and a cavity with the depths of L, 2L, and 3L. The length of the channel and the bottom wall of cavity are fixed to 2L and L, respectively. The bottom wall of the cavity is assumed to have a fixed temperature

of  $T_h$  while all other walls are adiabatic. The flow enters the channel at a uniform velocity of  $U_{in}$  and temperature of  $T_{in}$ . A magnetic field is acting on the  $x$  direction with variable intensities due to different Ha numbers. Finally, the gravity is acting downward in the vertical direction.

### Governing equations and boundary conditions

The equations of two dimensional steady, incompressible fluid flow were solved numerically. In this work the effects of Joule heating and dissipation were neglected. The dimensionless equations of motion under Boussinesq approximation are as follows:

$$\begin{aligned} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= -\frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} &= -\frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \\ &+ \frac{Ra}{Re^2 Pr} \theta + \frac{Ha^2}{Re} V \\ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \end{aligned}$$

In the above equations the dimensionless numbers are defined as:

$$\begin{aligned} Re &= \frac{u L}{\nu}, \quad Pr = \frac{\nu}{\alpha} \\ Ra &= \frac{g \beta (T_h - T_c) L^3}{\nu \alpha}, \quad Ha^2 = \frac{\sigma B_0^2 L^2}{\mu} \end{aligned}$$

Where the non-dimensionalizing quantities are:

$$\begin{aligned} X &= \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{u}{u_{in}}, \quad V = \frac{v}{u_{in}} \\ P &= \frac{(p + \rho g y) L^2}{\rho u_{in}^2}, \quad \theta = \frac{T - T_{in}}{T_h - T_{in}} \end{aligned}$$

The boundary conditions which were considered for this case are as follows:

Inlet:  $U = 1, V = 0, \theta = 0$

Outlet:  $\frac{\partial U}{\partial X} = 0, V = 0, \frac{\partial \theta}{\partial X} = 0$

All solid walls:  $U = 0, V = 0, \frac{\partial \theta}{\partial N} = 0$

Heated wall:  $U = 0, V = 0, \theta = 1$

In the above equations  $g, \beta, B_0, \sigma, \mu$  are the gravity acceleration, thermal expansion coefficient, Magnetic induction, electrical conductivity and dynamic viscosity of the fluid, respectively.

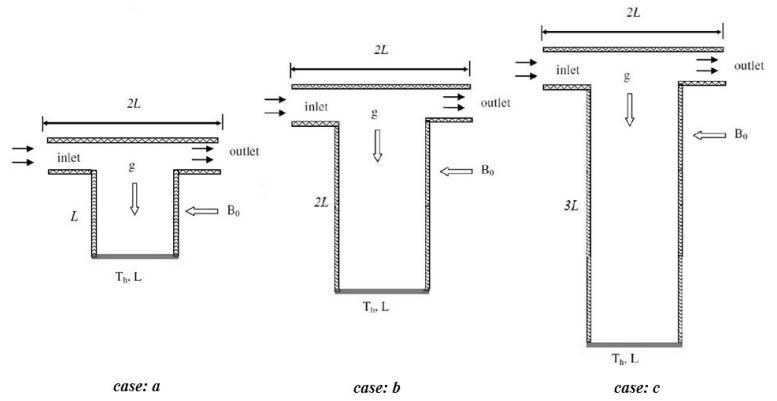


Figure 1: the 3 considered geometries and their boundary conditions

### Solution procedure and validity

In this work the Control Volume method was applied to discretize the dimensionless equations. Then the SIMPLE algorithm was used to solve the discretized equations. The convergence limit was set to:

$$|\phi^{m+1} - \phi^m| \leq 10^{-6}$$

Where  $\phi$  can be any general dependent variable. The result of the local Nu number of the geometry 1 are compared to the results of Rahman et al. [21] and shows a good agreement as can be seen in figure 2.

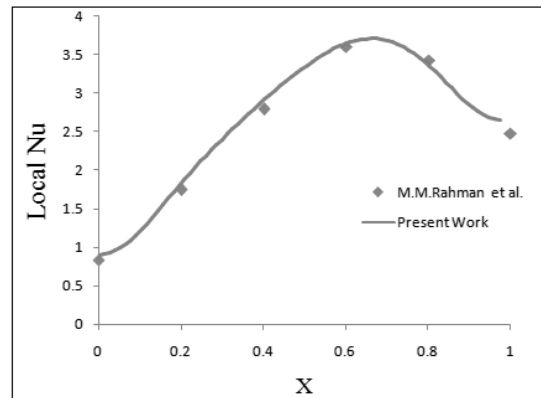


Figure 2: the local Nu number of the heated wall in geometry 1 compared to the results of [21]

### Results and discussion

In this study, physical properties are constant and the  $Re = 100, Ra = 10^5$  for all cases and geometries. The Ha number is changed from 10 to 30 and the cavity depth for each case is defined as in the figure 1. The Pr number is also assumed to be fixed at 0.7 for 27°C.

Figure 3 shows the streamlines and isothermal lines for case a geometry ( $L \times L$ ) for three different Ha numbers: 10, 20, 30. As it can be seen, in the case of  $Ha=10$  the magnetic field do not influence greatly on the streamlines and as it is expected a clockwise vortex is formed. However, in the case of  $Ha=20$  two main vortices are formed. This is due to the strength and direction of the magnetic field. This direction of magnetic field is





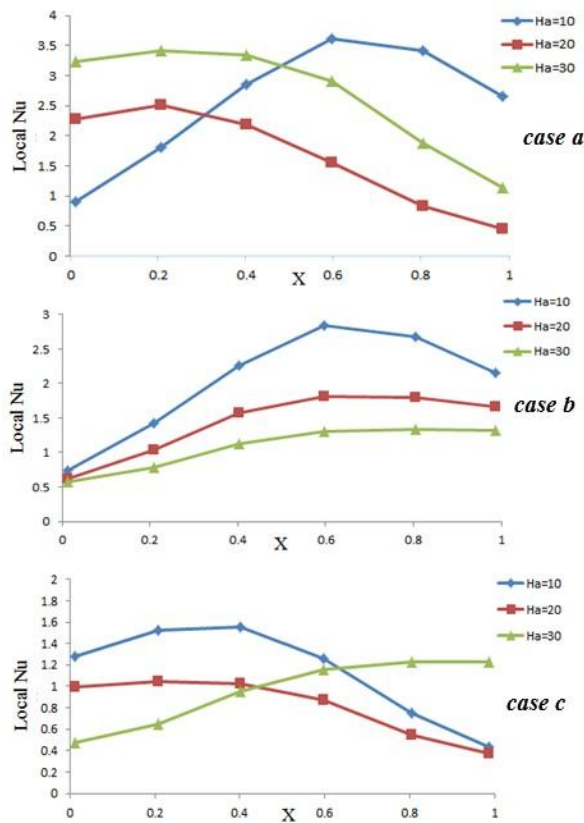


Figure 6: Local Nusselt number of the heated wall for the three geometries: case 1, case 2 and case 3

The mean Nusselt number of each case is demonstrated in figure 7-a. As can be seen, the highest Nu number is observed in the geometry of L\*L and Ha=30 and the minimum Nu number is in the geometry of 3L\*L and Ha=20. Also one can see that there is a relative minimum on the Nu plot for the cases of L\*L and 3L\*L while the case of 2L\*L experiences an absolute decrease of Nu by increasing Ha.

Figure 7-b demonstrates the temperature of the flow at the outlet boundary. It is seen that the outlet temperature for each Ha number directly follows the form of Nu plot. It shows that the conduction heat transfer can be neglected with respect to the convective form of heat transfer, which is expected.

Finally, the mean pressure change of the domain is demonstrated in figure 7-c. As it is seen that in the case of 2L\*L geometry the mean pressure gradient is higher than other cases but for Ha=30 it has a drastic decrease but the geometry of L\*L has a different style so that it has a sever increase at Ha=30.

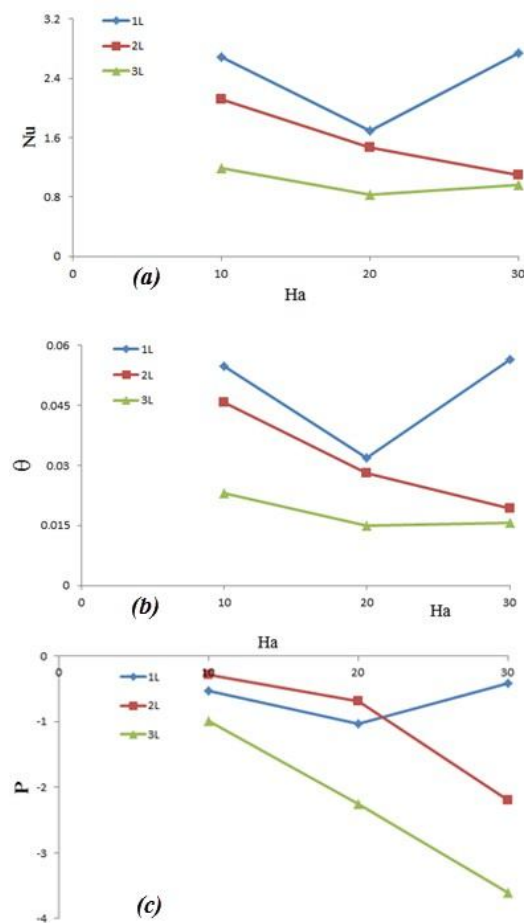


Figure 7: a) Mean Nusselt number of the heated wall for the three geometries b) The outlet temperature of the flow c) The mean pressure change in the domain.

## Conclusion

The Hydromagnetic fluid flow in a rectangular open cavity with variable depths has been investigated for 3 different geometries and 3 different Ha numbers. It was seen that the highest temperature gradient at the outlet boundary occurs in the case of geometry L\*L and Ha=30. This condition is also relevant for the highest Nu number on the heated wall. Also it was seen that the mean Nu numbers for the case of 2L\*L and 3L\*L at Ha=30 are almost the same so one can deduce that the change of cavity's depth at this Ha is effect less. Finally, a relative minimum is observed on the curve of Nu versus Ha for the L\*L and 3L\*L geometries while the case of 2L\*L experiences an absolute decrease of Nu by increasing Ha.

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