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VPP decision making in power markets using Benders decomposition

S. Seyyed Mahdavi*,† and M. H. Javidi

Department of Electrical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran

SUMMARY

A group of energy resources (including energy storage units) physically distributed in the network which are placed under a unified and integrated management with a central control system is called a Virtual Power Plant (VPP). The bidding strategy of a VPP, as a participant in energy and reserve markets, has significant role in maximizing its profit. This paper proposes a new mathematical approach based on a comprehensive model for bidding of a VPP in energy and reserve markets. In our proposed model network topology, VPP security constraints, constraints of distributed energy resources (DER) composing the VPP, power loss in the VPP and the balance between supply and demand are considered. The method determines the amount of energy and reserve that should be bought or sold in day-ahead markets, commitment of DER units, charge or discharge status of storage units and the amount of load curtailments. We have used Benders decomposition (BD) method for solving the problem. The results obtained using BD technique is compared with those obtained using genetic algorithm (GA). Simulation results confirm that using BD is more advantageous for solving the problem. It is also shown that BD can be applied to bidding problem of large-scale VPPs considering all the constraints. It is also observed that computational time for solving the problem using GA and the optimality of the solution are major obstacles for applying GA to solve large-scale VPPs. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS: distributed generator (DG); virtual power plant; bidding strategy; Benders decomposition; energy market; reserve market

1. INTRODUCTION

Recently, increasingly attention has been paid to distributed generation as well as renewable energy resources. This is mainly because of recent advances in distributed generation and renewable energy technologies, pollution reduction and their positive environmental impacts [1–3].

Due to the development and expansion of distributed generators (DGs) technology, electric power system is gradually experiencing decentralized and distributed structure, especially in operation and control strategies. Wide spread usage of DGs may seriously affect on operation and control of power systems. Therefore, ignoring their special features may endanger stability, reliability and power quality of power systems [4–6]. The owners of DGs not only like to sell their energy surplus in the market but also want to be able to receive energy from the market when they need it or in cases it is cheaper than their own generated energy. Thus, the power flow through the network may frequently change at different hours during a day. Therefore, power exchange between DG and power network should be managed properly. On the other hand, while all power transactions should be traced in the market environment, independent participation of renewable energy resources, such as solar and wind, due to their low capacity and uncertain nature, may be difficult. In order to overcome these difficulties, DGs may be placed under a unified and integrated management with a central control system. This can be achieved by Virtual Power Plant (VPP) concept [7–9].

*Correspondence to: Saeed Seyyed Mahdavi, Electrical Engeeniering, Ferdowsi University of Mashhad, Mashhad, Iran.
†E-mail: saeed.seyyedmahdavi@gmail.com

Some researches have been carried out to manage and control transactions between VPP and energy market. In most studies, it has been assumed that the VPPs do not directly participate in energy market and energy is traded between VPPs and market players based on market prices via bilateral contracts [10–13]. In these studies, the purchase from the wholesale market is not considered.

A framework for a VPP in power market has been presented in [14–16]. In this proposed structure, a VPP can participate in the market based on the market rules. Distributed generation units offer their bids to the operator of the VPP. The operator of a VPP is responsible for estimating the market price, participating in the market and/or contracting with the customers on behalf of the VPP. After determining the bidding strategy, the operator of the VPP participates in the market. At the end of the negotiation period, the operator specifies the market share of each DG.

In [7,8], a VPP is addressed as a market player who can bid in both energy and reserve markets. A bidding strategy for a VPP is defined through an un-equilibrium model based on price-based unit commitment (PBUC), which considers constraints of distributed energy resources (DER), the balance between supply and demand and the security constraints of the VPP. The model uses genetic algorithm (GA) for solving PBUC problem.

In this paper, we have proposed a new approach based on a more developed model of a VPP for its participation in the energy and reserve markets. The proposed approach is advantageous to previously proposed methods because it not only considers VPP security constraints, constraints of DER composing the VPP and the balance between supply and demand but also includes the network topology and the power loss in VPP. In the model, according to the direction of power flow between the VPP and the network, the VPP can play a dual role. In other words, it may be considered either as a consumer or a producer. The bidding model of the VPP determines the amount of energy and reserve that will be bought or sold in day-ahead markets, commitment of DER units, charge or discharge status of storage units and the amount of load curtailments. Furthermore, we have used Benders decomposition (BD) method for solving the problem. The results obtained using BD technique compared with those obtained using GA confirm that using BD is more advantageous for solving the problem.

The paper is organized as follows: In section II, the characteristics and constraints of a VPP in energy markets are presented. Section III formulates the decision-making model for a VPP in the energy and reserve markets. The BD technique and its application to the bidding problem of the VPP are introduced in Section IV. Simulation results are presented in Section V. Finally, Section VI presents conclusions.

2. VPP IN ENERGY MARKETS

A VPP may be considered as a participant in energy and reserve markets. Although, similar to conventional power plants, it can participate in the market, there are significant differences between bidding strategy of a VPP and that of a conventional power plant. These differences essentially arise due to the following reasons [7]:

- A VPP may have a dual role (producer or consumer) as a participant in power market while a conventional unit only stands as a power producer.
- While, there is no constraint for conventional units to take care of their own demands, it is essential for a VPP as a participant, composed of both production and consumption, to supply its own demand.
- A VPP may be connected to the network through more than one point. Therefore, its bidding strategy may be affected by the topology and the constraints of the network connecting these points. As a result, a VPP should consider network constraints and DER constraints simultaneously.

Considering above mentioned differences, network security constraints such as line power flow or voltage limits, should be considered in the model of a VPP in power markets. Moreover, the model should include the constraints related to DER units and the balance between supply and demand.

To determine the bidding strategy of a VPP, accurate information about the status of its DER units is required. The summation of production capability of DER units of a VPP defines its total capacity.

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Moreover, the demand of end consumers of a VPP and the prices of both energy and reserve markets, for the next 24 h, should be forecasted.

Based on market prices and the production cost of DERs, the share of each DER in the market and the total bid of the VPP are defined. Cost functions for various DERs may be categorized as below:

- The cost for 1 h of load curtailment P_{cl} can be modeled as[17]:

$$c_{cl}(P_{cl}) = a_{cl}.P_{cl}^2 + b_{cl}.P_{cl} \tag{1}$$

- The cost curve of DG units are modeled similar to [7]:

$$c_{DG}(P_{DG}) = a_{DG}.P_{DG}^2 + b_{DG}.P_{DG} \tag{2}$$

- The cost curve of electrochemical storage units can be assumed as [18]:

$$c_{es}(P_{es}) = a_{es}.\left|P_{es}\right| + b_{es} \tag{3}$$

In addition to the revenue obtained from providing energy for end user consumers, a VPP can earn money by selling energy and reserve in the market. On the other hand, costs imposed on the VPP are mainly related to start-up and operation of DGs, curtailing interruptible loads, operation of electrochemical storage units and buying energy from the energy market. Considering the costs and the revenues, the proposed model for the VPP in the energy and reserve markets will define the strategy for its bidding in the market. It should be noted that the security constraints of the VPP, the limitations of DER units, the power loss in the network and the balance between supply and demand are included within the model.

Based on our proposed model, a VPP tends to bid in energy and reserve markets based on the economic and technical aspects of its components, the forecasted price and network parameters. The bidding strategy of a VPP determines the amount of energy and reserve that should be bought or sold in day-ahead markets, commitment of DER units, charge or discharge status of storage units and the amount of load curtailments.

3. PROPOSED BIDDING MODEL FOR VPP IN POWER MARKETS

3.1. Objective function

The objective of a VPP for participating in energy and reserve markets is to maximize its total profit through supplying energy to end users and exchanging power in energy market and selling reserves in the reserve market. Based on the above explanation, the objective function of the bidding problem for the VPP in the market is given by Equation (4).

In Equation (4), the first part stands for the revenue from exchanging power with power markets and also supplying energy for end consumers. The second, third and the fourth parts of Equation (4) represent the costs of DG units, the costs of electromechanical storage units and the costs of the load curtailment, respectively. In this model, the energy market can be cleared either in uniform or pay as bid clearing mechanisms. The reserve market is settled based on bids for capacity. Similar to [7], our proposed model is formulated based on forecasted prices of energy and reserve in the market. It is also assumed that the capacities of winner participants submitted in the reserve market are really provided.

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$$
\begin{aligned}
profit = & \sum_{t=1:24} \left(\lambda_{E,t} \times P_t + \lambda_{R,t} \times R_t + \lambda_{L,t} \times L_t\right) \\
& - \sum_{t=1:24} \left(\sum_{i \in N_{DG}} \begin{pmatrix} c_{DG,i,t}\left(P_{DG,i,t} + R_{DG,i,t}\right) \\ + c_{stDG,i,t} + c_{shDG,i,} \end{pmatrix} \right) \\
& - \sum_{t=1:24} \left(\sum_{j \in N_{es}} \left(c_{es,j,t}\left(P_{es,j,t}\right)\right) \right) \\
& - \sum_{t=1:24} \left(\sum_{k \in N_{us}} \left(c_{us,k,t}\left(P_{us,k,t} + R_{us,k,t}\right)\right) \right)
\end{aligned}
\tag{4}
$$

3.2. *Supply–demand balancing constraints*

The power balance constraints of the system for energy and reserve are given by Equations (5) and (6), respectively. In both these equations, power losses of VPP network are also included.

$$
\begin{aligned}
& P_t + \sum_{i \in N_{DG}} P_{DG,i,t} + \sum_{j \in N_{es}} \left(\eta_{es,j} \times P_{es,j,t}\right) + \sum_{k \in N_{us}} P_{us,k,t} \\
& = Load_t + PLoss_t
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
& R_t + \sum_{i \in N_{DG}} R_{DG,i,t} + \sum_{j \in N_{es}} \left(\eta_{es,j} \times R_{es,j,t}\right) + \sum_{k \in N_{us}} R_{us,j,t} \\
& = \mathrm{Re}s_t + RLoss_t
\end{aligned}
\tag{6}
$$

3.3. *DER constraints*

The DERs constraints include constraints of the DGs, Equations (7) to (12), requirements of the electrochemical storage, Equations (13) and (14), and curtailed load limits, Equation (15). Constraints (9) to (12) represent minimum up/down time limits of the DGs. Ramping limits for DGs and storage units are given by Equations (8) and (14), respectively.

$$P_{DG,i}^{\min} \le P_{DG,i,t} + R_{DG,i,t} \le P_{DG,i}^{\max} \tag{7}$$

$$R_{DG,i,t} \le \min\left\{(10 \times MSR_i), \left(P_{DG,i}^{\max} - P_{DG,i,t}\right)\right\} \tag{8}$$

$$\left[X^{on}(i,t) - T^{on}(i)\right] \times \left[y(i,t-1) - y(i,t)\right] \ge 0 \tag{9}$$

$$\left[X^{off}(i,t-1) - T^{off}(i)\right] \times \left[y(i,t-1) - y(i,t)\right] \le 0 \tag{10}$$

$$X^{on}(i,t) = \left[X^{on}(i,t-1) + 1\right] \times \left[y(i,t-1)\right] \tag{11}$$

$$X^{off}(i,t) = \left[X^{off}(i,t-1) + 1\right] \times \left[1 - y(i,t-1)\right] \tag{12}$$

$$P_{es,j}^{\min} \le P_{es,t,j} \le P_{es,j}^{\max} \tag{13}$$

$$\left|P_{es,t,j}\right| \le R_{CH,j} \tag{14}$$

$$0 \le P_{us,t,k} + R_{us,t,k} \le P_{us,k}^{\max} \tag{15}$$

3.4. *VPP network security constraints*

Figure 1 shows a single-line diagram for a radial distribution system. The power flow equations for a radial distribution system can be presented by Equations (16) to (19) as discussed in [19]. Moreover, the generalized network constraints such as Equations (20) and (21) refer to VPP network security constraints.

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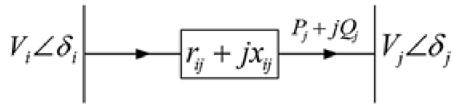

Figure 1. Simple model of radial distribution system.

$$V_j^2 = -\left[r_{ij}P_j + x_{ij}Q_j - \frac{V_i^2}{2}\right] + \sqrt{\left[r_{ij}P_j + x_{ij}Q_j - \frac{V_i^2}{2}\right]^2 - \left[r_{ij}^2 + x_{ij}^2\right]\left[P_j^2 + Q_j^2\right]} \tag{16}$$

$$\sin(\delta_i - \delta_j) = \frac{x_{ij}P_j - r_{ij}Q_j}{V_iV_j} \tag{17}$$

$$P_{loss-ij} = r_{ij}\frac{\left(P_j^2 + Q_j^2\right)}{V_j^2} \tag{18}$$

$$Q_{loss-ij} = x_{ij}\frac{\left(P_j^2 + Q_j^2\right)}{V_j^2} \tag{19}$$

$$-P_{ij}^{\max} \le P_{ij}(t) \le P_{ij}^{\max} \tag{20}$$

$$V^{\min} \le V \le V^{\max} \tag{21}$$

4. APPLICATION OF BD TO VPP BIDDING MODEL

BD was introduced by J.F. Benders for solving large-scale mixed-integer programming problems. This technique, as a very common optimization approach, is widely used in power system problems including network design, system planning and system operation [20,21]. In this approach, as shown in Figure 2, the original problem is first decomposed into a master problem and some subproblems. Usually, the master problem is an integer problem and subproblems are linear ones. Then, relaxing some of the problem constraints, the master problem is easily solved considering only some of the constraints. The subproblems use this solution to examine whether the remaining constraints are satisfied or violated. In the subsequent iteration of the problem, a new objective function is formed to calculate the upper bound solution of the master problem. If any of subproblems is not feasible, an infeasibility cut will be introduced to the master problem. Following iterations will be carried out to calculate lower and upper bounds of the solutions considering more constraints.

Continuing these repetitions, the final solution of the original problem will be achieved when the difference between the upper and lower bounds becomes less than the permitted error [20,21].

The bidding problem for a VPP can be solved using BD algorithm, considering the binary variables y which show the status of DER. The operation of the production facilities may be represented by these binary variables. If $y_i = 0$, then $P_i = 0$. This implies that the production facility i is not working. The optimal solution for this problem is the set of productions that minimizes the costs satisfying the corresponding constraints.

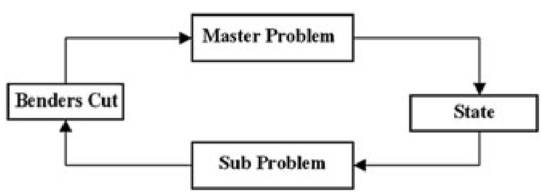

Figure 2. Benders decomposition technique.

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Applying BD technique to VPP bidding problem, the solution algorithm proceeds as follows:

Step 0: Initialization. Initial values at the first iteration for the binary variables (y) are set equal to 1, i. e. initially we consider that all DER units are operating. The lower bound of the objective function in the first iteration $\left(z_{down}^{(1)}\right)$ is set equal to α_{down}.

Step 1: Subproblem solution. In our problem, we have only one subproblem which is formulated as minimizing the cost subject to the constraints such as real power balance, reserve requirements, DER constraints and network security constraints. The subproblem is solved based on binary variable defined at the previous step:

$$\begin{aligned} &\min \ \text{cost} \\ &s.t. \ \ \text{supply} - \text{demand constraints (5) and (6)} \\ &\qquad \text{DER constraints (7) to (15)} \\ &\qquad \text{VPP network security constraints (16) to (21)} \end{aligned} \tag{22}$$

In each iteration, the upper bound of the objective function optimal value is updated at the end of step 1 as $z_{up}^{(Iteration)} = \text{cost}(\hat{y})$. Moreover, using simplex multipliers (π) calculated solving the subproblem, the Benders cut as Equation (23) is introduced and added to the master problem to eliminate the violations.

$$w(y) = \text{cost}(\hat{y}) + \sum_{i=1}^{n} \pi_i (y_i - \hat{y}_i) \tag{23}$$

Step 2: Convergence checking. As long as $\left|z_{up}^{(Iteration)} - z_{down}^{(Iteration)}\right| \leq \varepsilon$, the procedure continues to Step 3, and the iteration counter is increased by one. Otherwise, the optimal solution is obtained.

Step 3: Master problem solution. Based on the results obtained from the subproblem, the following master problem is solved:

$$\begin{aligned} &\min \ \alpha \\ &s.t. \ \ \alpha \geq \alpha_{down} \\ &\qquad w(y) \leq \alpha \\ &\qquad _{yi} \in \{0, 1\} \end{aligned} \tag{24}$$

In the master problem shown as Equation (24), α is a function that provides the optimal objective function value for VPP bidding problem and $w(y)$ is the set of infeasibility cuts presented by the subproblem at different iterations.

The solution of the master problem defines the new value of the binary variables (y). The lower bound of the objective function optimal value is updated as $z_{down}^{(Iteration)} = \alpha^{(Iteration)}$. The procedure continues to Step 1. The flowchart of BD technique for solving the VPP bidding problem is illustrated in Figure 3.

5. EXPERIMENTAL RESULTS

5.1. Case study

In this section, the test system used for evaluation of the proposed model is introduced. The test system used is chosen to be the same as the one used in [8]. In the test system, a VPP together with eight distributed generations form a market player in energy and reserve markets. The network configuration of VPP and its total forecasted load are shown in Figures 4 and 5, respectively. In order to illustrate the effects of network topology on bidding strategy, the total load is assumed to be distributed at six different nodes in the network. Parts of the loads located at buses 4 and 7 are considered as interruptible loads. The loads at buses 4 and 7 can be curtailed up to 30 kW and 40 kW, respectively. The cost functions of load curtailment at these two buses are chosen to be in the forms of Equations (25) and (26), respectively.

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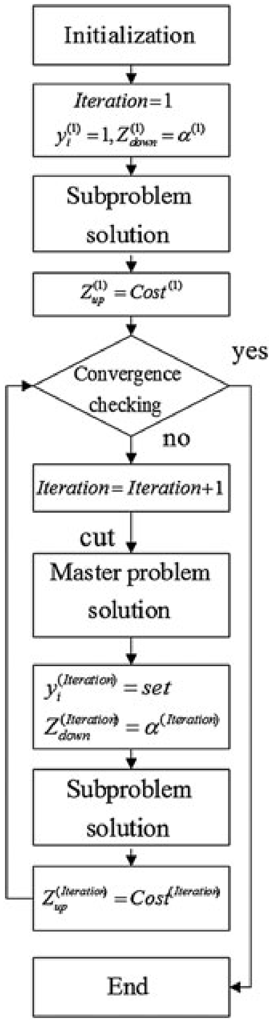

Figure 3. Flowchart of Benders decomposition technique for solving the VPP bidding problem.

$$c_{us}(P_{us}) = 0.01 \times P_{us}^2 + 3 \times P_{us} \tag{25}$$

$$c_{us}(P_{us}) = 0.01 \times P_{us}^2 + 1.5 \times P_{us} \tag{26}$$

The characteristics and constraints of distributed generations such as generation limits, cost function coefficients, minimum up/down time limits and ramping capability for reserve are shown in Table I.

Figures 6 and 7 show the retail rates for end users, the forecasted price of energy and the forecasted price of reserve in the market. DER, including DG1, DG4, DG8 and interruptible loads in nodes 4 and 7 are assumed to provide the reserve service.

5.2. Model evaluation

In this section, the test system is used to investigate BD and GA methods for solving the VPP bidding problem. In order to show the efficiency of BD, the results of using BD method for solving VPP bidding problem is compared with the results obtained using GA. In our investigation, the same as in [7] and [8], the network topology is not considered, and the losses of power in VPP are assumed

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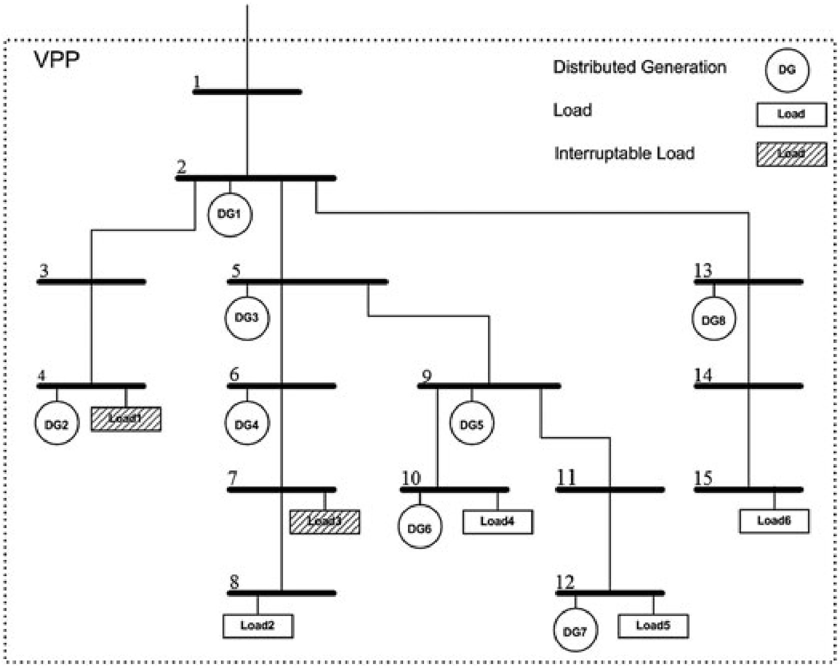

Figure 4. The test system.

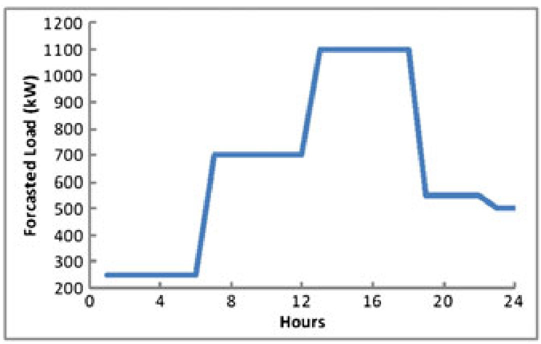

Figure 5. The total forecasted load for VPP.

Table I. Characteristics and constraints of distributed generations.

	Pmin (KW)	Pmax (KW)	a	b	MUT (h)	MDT (h)	MSR (KW/min)
DG1	20	85	0.01	10.5			3
DG2	35	115	0.01	8.5	4	4	
DG3	30	110	0.01	9.2	3	3	
DG4	20	75	0.01	12.6			2.5
DG5	25	80	0.01	7.2	2	2	
DG6	30	90	0.01	7	3	3	
DG7	30	100	0.01	10.1			
DG8	20	90	0.01	12.7			3.5

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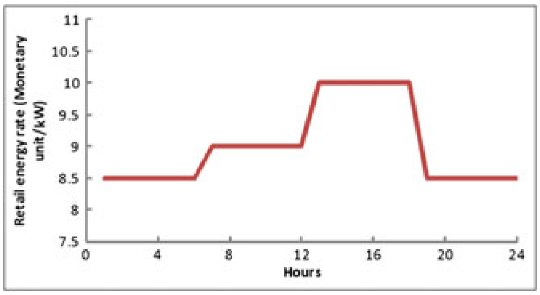

Figure 6. The forecasted Retail rate for end customers in VPP.

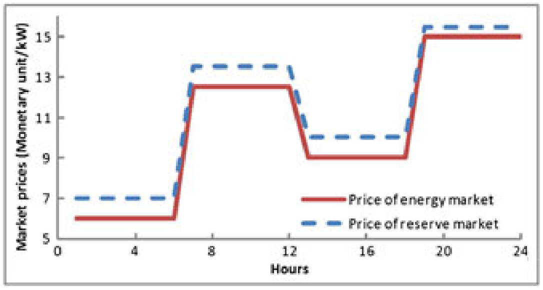

Figure 7. The forecasted prices of wholesale energy and reserve market.

to be negligible. Figures 8 and 9 show the results obtained by solving the VPP bidding problem using BD and GA methods.

As Figure 6 shows, during hours 1 to 6, the prices for both energy and reserve in the market are lower than marginal costs of DG units. Therefore, the VPP tends to buy energy from the market. The total demand of VPP is provided from the energy market in this time period while all DERs are off. In the time period between hours 7 to 18, despite the fact that some of DG units are providing energy, VPP is still a consumer in the energy market. However, after reduction in energy price at hour 12, VPP turns off expensive units including DG1 and DG7. During the time between the hours 19 and 24, the VPP is providing energy for the market and tends to sell its surplus energy to the network. The shared capacities of DG units in each market solved using BD and GA algorithms are shown in Figures 8.a and 9.a, respectively. The results of model utilizing GA show that DG4 and DG8 are on between hours 6 and 12 and operate at their minimum levels. These units can also be used for providing reserve in the reserve market. The solution to the model using BD recommends that DG4 and DG8 be off during the period between hours 6 and 12. The price of reserve market is relatively high at time period between hours 6 and 12, but it is not sufficient, so the benefit from providing for energy and reserve market cannot overcome the cost of production. A comparison between results and execution time of solving the VPP bidding problem with BD and GA algorithm is made in Table II. It shows that the GA computation time and solution optimality would be the major obstacles for applying GA to large-scale VPPs.

The master problem and the subproblem of the VPP bidding problem using BD technique are solved through an iterative process. Figure 10 shows the total number of iterations, values of the lower and upper bounds of the objective function at each iteration and the convergence of upper and lower bounds.

5.3. *Power loss in bidding model of VPP*

In this paper, the model for bidding problem in the market is solved considering the power loss in VPP, which has a significant role in maximizing the profit of VPP. The test system introduced in previous

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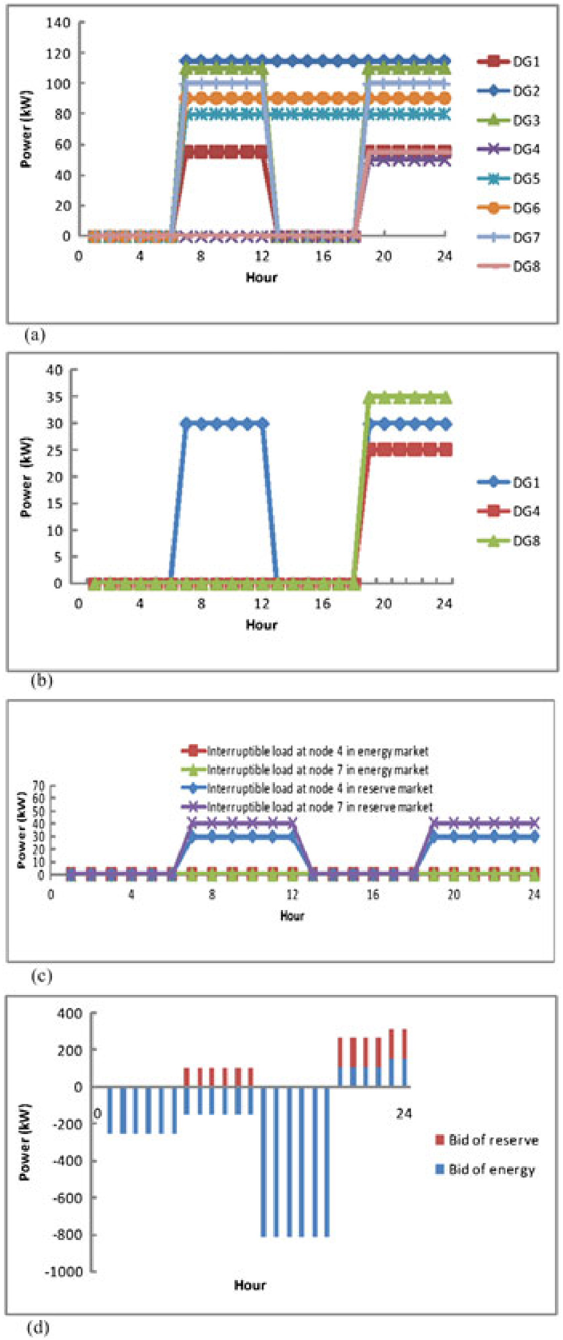

Figure 8. VPP bidding problem solved by Benders decomposition without considering the VPP network. (a) Capacity of DG units for energy market. (b) Capacity of DG units for reserve market. (c) Interruptible loads in energy market. (d) Bids for energy and reserve markets.

sections is used for studying the integrated model of VPP in the market considering VPP network and power losses. The line data of the test system is given in Table III.

In order to demonstrate the influence of power loss on bidding strategy, the results from the model over 24 h, with and without considering the power loss in VPP, are illustrated in Table IV and Table V.

Table IV shows the bids for energy and reserve markets of VPP during 24 h with and without considering power loss inside the VPP. While the commitments of DG units are not necessarily

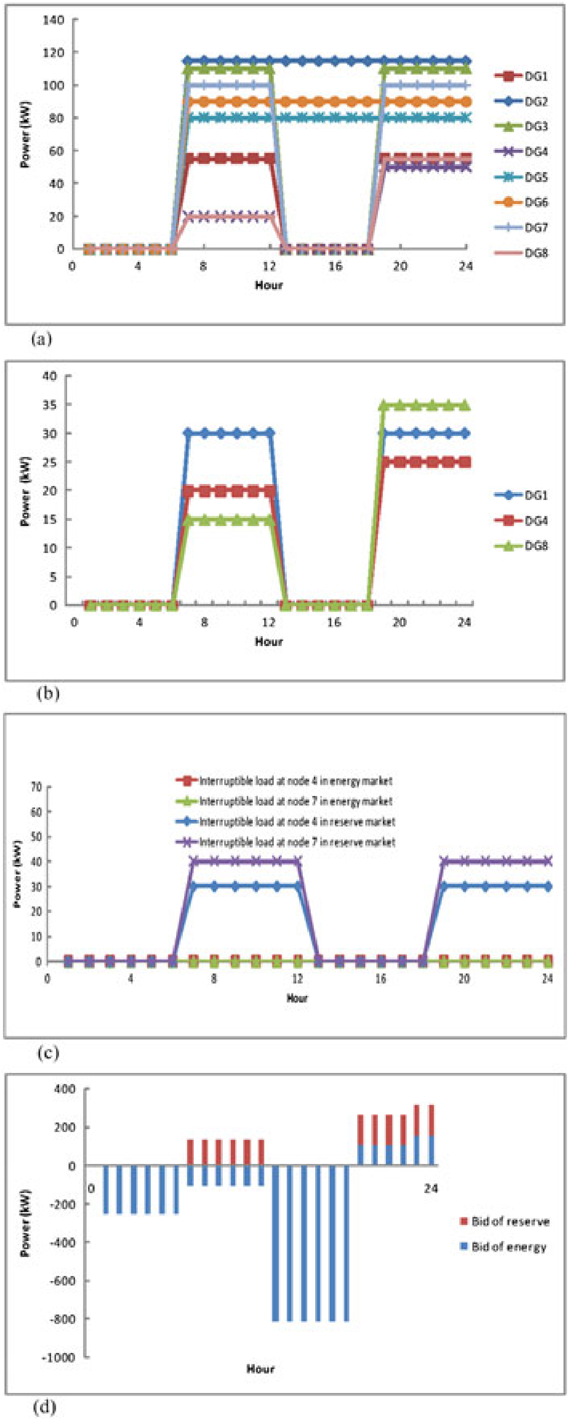

Figure 9. VPP bidding problem solved by genetic algorithm without considering the VPP network. (a) Capacity of DG units for energy market. (b) Capacity of DG units for reserve market. (c) Interruptible loads in energy market. (d) Bids for energy and reserve markets.

affected by power losses in VPP, the amounts of bids in energy market change such that they rise in buying hours and fall at selling periods. The increase or decrease in bids is equal to the value of power losses inside the VPP at different time periods. Cost of buying energy or revenue of selling power is changed due to power losses in VPP, which results in reduction of the total benefit of VPP. Table V

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Table II. Results of solving the VPP bidding model with BD and GA.

	Model without considering VPP network solved by Benders cut	Model without considering VPP network solved by genetic algorithm [8]
Total production (KW)	8940	9180
Total load (KW)	15 456	15 456
Total bid to energy market (KW)	6516	6276
Total loss of power in VPP (KW)	0	0
Total curtailed load (KW)	840	840
Revenue from reserve market (Monetary unit)	22 440	25 254
Cost of exchanging power from energy market (Monetary unit)	52 824	49 824
Revenue from end costumers (Monetary unit)	135 996	135 996
Cost of production (Monetary unit)	94 098	99 958.5
Benefit (Monetary unit)	11 514	11 467.5
Execution Time (s)	5.84	2269

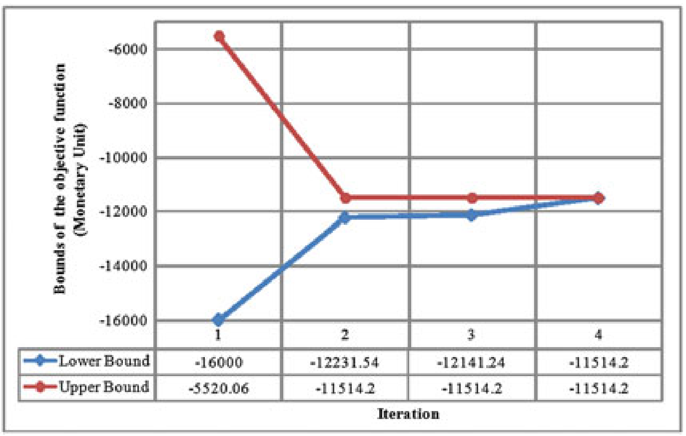

Figure 10. Evolution of the upper and lower bounds of the objective function of the master problem for VPP bidding problem.

shows the costs, revenues and total benefit of VPP from bidding in power markets during 24 h for both cases of considering and neglecting power losses in VPP.

5.4. Considering distribution lines power limit

Network congestion and line capacity of VPP are important factors in bidding strategy of VPP in energy and reserve market. Line power limit may prevent VPP from providing the appointed energy to the market. Moreover, delivering energy from the market to end consumer could be stopped due to line congestion. This can seriously affect the total profit of VPP. The proposed model for VPP in power markets considers VPP network constraint such as line power limit and network congestion, in addition to other constraints.

In order to show the effects of network constraints on bidding strategy of VPP, it is assumed that the test system has power limitation for distribution lines. It is supposed that the branch between buses

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Table III. Line data of the test system.

Line number	Sending bus	Receiving bus	Resistance (Ω)	Reactance (Ω)
1	1	2	1.587	1.587
2	2	3	0.1587	0.1587
3	2	5	2.38	1.587
4	2	13	1.587	1.587
5	3	4	0.1587	0.1587
6	5	6	2.38	1.587
7	5	9	1.587	1.587
8	6	7	2.38	2.38
9	7	8	1.587	1.587
10	9	10	0.1587	0.1587
11	9	11	1.587	1.587
12	11	12	2.38	2.38
13	13	14	1.587	1.587
14	14	15	1.587	1.587

Table IV. Results of solving the VPP bidding model with and without considering the loss of power in VPP.

	Model without considering loss of power		Model considering loss of power	
Hour	Bid to energy market (+ sell) (− buy) (KW)	Bid to reserve market (KW)	Bid to energy market (+ sell) (− buy) (KW)	Bid to reserve market (KW)
1	−252	0	−253.5	0
2	−252	0	−253.5	0
3	−252	0	−253.5	0
4	−252	0	−253.5	0
5	−252	0	−253.5	0
6	−252	0	−253.5	0
7	−146	100	−147.7	100
8	−146	100	−147.7	100
9	−146	100	−147.7	100
10	−146	100	−147.7	100
11	−146	100	−147.7	100
12	−146	100	−147.7	100
13	−813	0	−831.9	0
14	−813	0	−831.9	0
15	−813	0	−831.9	0
16	−813	0	−831.9	0
17	−813	0	−831.9	0
18	−813	0	−831.9	0
19	109	160	107.33	160
20	109	160	107.33	160
21	109	160	107.33	160
22	109	160	107.33	160
23	109	160	155.01	160
24	109	160	155.01	160

2 and 13 has a capacity of 150 kW. In this case, during hours 13–19, VPP is not able to deliver all the energy needed for load 6 from the energy market. Therefore, to provide the remaining demand for load 6, the VPP would use DG8, which is not economically profitable. Considering the line power limit leads to lower profit for VPP. Changes in costs, revenues, bid to energy market, power losses, and the total profit of VPP from bidding in power markets during 24 hours considering the line power limit are illustrated in Table VI.

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Table V. Costs, revenues and total benefit of VPP with and without considering the loss of power in VPP.

	Model without considering loss of power	Model considering loss of power
Total production (KW)	8940	8940
Total load (KW)	15 456	15 456
Total bid to energy market (KW)	6516	6659.3
Total loss of power in VPP (KW)	0	143.26
Total curtailed load (KW)	840	840
Revenue from reserve market (Monetary unit)	22 440	22 440
Cost of exchanging power from energy market (Monetary unit)	52 824	54 185
Revenue from end costumers (Monetary unit)	135 996	135 996
Cost of production (Monetary unit)	94 098	94 098
Benefit (Monetary unit)	11 514	10 153

Table VI. Costs, revenues and total benefit of VPP with and without considering line power limit in VPP.

	Model without considering line power limit	Model considering line power limit
Total production (KW)	8940	9138
Total load (KW)	15 456	15 456
Total bid to energy market (KW)	6659.3	6425.995
Total loss of power in VPP (KW)	143.26	107.995
Total curtailed load (KW)	840	840
Revenue from reserve market (Monetary unit)	22 440	22 440
Cost of exchanging power from energy market (Monetary unit)	54 185	52 085.73
Revenue from end costumers (Monetary unit)	135 996	135 996
Cost of production (Monetary unit)	94 098	96 677.94
Benefit (Monetary unit)	10 153	9672.32

6. CONCLUSION

In this paper, a mathematical model for VPP bidding problem in energy and reserve markets is presented which considers the network topology of VPP, VPP security constraints, DER constraints, power loss in VPP and supply–demand balance. The proposed model determines the commitment of DER units, charge or discharge status of storage units, the amount of curtailed loads and the amount of energy to buy or sell during the day-ahead energy and reserve markets. BD method is utilized for solving the VPP bidding model. Compared to the GA-based solution, the BD-based method exhibits optimal solution with much less computation time.

It is shown that considering the congestion and power losses of VPP network in modeling has a significant role in maximizing the profit of VPP. Line congestion in VPP may change the commitments of DG units, costs, revenues, bid to energy market, power losses and total benefit of VPP from bidding in the market. The results confirm that the losses in VPP network affects the cost of buying energy, the revenue obtained through selling power and consequently the income of VPP.

7. LIST OF SYMBOLS AND ABBREVIATIONS

$c_{DG,i,t}\,(P_{DG,i,t})$	Cost curve of DG unit i
$c_{es,j,t}\,(P_{es,j,t})$	Cost curve of electrochemical storage unit j
$c_{shDG,i,}$	Shut down cost of a DG unit i
$c_{stDG,i,t}$	Start up cost of a DG unit i

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$c_{us,k,t}(P_{us,k,t})$	Cost of load curtailment for load k per hour
L_t	Supplied load of VPP for end customers
$Load_t$	Supplied load of VPP when the accepted bid for reserve is not called on to produce
MSR_i	Ramp rate for reserve of unit i
N_{DG}	Set of distributed generators
N_{es}	Set of electromechanical storage units
N_{us}	Set of interruptible loads
$P_{DG,i,t}$	Generation of a distributed generator i in energy market
$P_{DG,i}^{\min}, P_{DG,i}^{\max}$	Lower and upper limits of generation of DG i
$P_{es,j,t}$	Charged or discharged capacity of an electromechanical storage unit j
$P_{es,j}^{\min}, P_{es,j}^{\max}$	Generation lower and upper limits of storage unit j
$P_{ij}^{\min}, P_{ij}^{\max}$	Lower and upper limits of flow in line ij
$P_{us,k,t}$	Unserved load in energy market
$P_{us,k}^{\max}$	Upper limit of interruptible load k
P_t	Total bid of VPP in energy market (buy or sell)
$PLoss_t$	Losses of VPP when the accepted bid for reserve is not called on to produce
$R_{CH,j}$	Maximum charge or discharge rate of electromechanical storage unit j
$R_{DG,i,t}$	Generation of distributed generator i in reserve market
Res_t	Supplied load of VPP when the accepted bid for reserve is called on to produce
R_t	Total bid of VPP in reserve market(sell)
$R_{us,k,t}$	Unserved load in reserve market
$RLoss$	Losses of VPP when the accepted bid for reserve is called on to produce
$T^{on}(i), T^{off}(i)$	Minimum up and down time limits of unit i
$V^{\min}, V^{\max}$	Lower and upper limits of node voltage
$w(y)$	Constraints of master problem
$X^{on}(i,t)$	The time duration before time t that unit i has been on at t
$X^{off}(i,t)$	The time duration before time t that unit i has been off at t
$y(i,t)$	Commitment status of unit i at time t
$Z_{down}^{(it)}, Z_{up}^{(it)}$	Lower and upper bound of the solution in iteration it
α	Objective function of master problem
ε	Permitted error
π	simplex multipliers
$\lambda_{E,t}$	Price of energy in the market
$\lambda_{L,t}$	Retail rate of VPP for end customers
$\lambda_{R,t}$	Price of reserve in the market

REFERENCES

1. Bakari KE, Kling WL. Virtual power plants: An answer to increasing distributed generation. *in Proc. 2010 Innovative Smart Grid Technologies Conf.*, Gothenburg; 2010, 1–6.
2. Pudjianto D, Ramsay C, Strbac G, Durstewitz M. The virtual power plant: Enabling integration of distributed generation and demand. FENIX Bulletin 2, Feb. 2008.
3. Braun M, Strauss P. A review on aggregation approaches of controllable distributed energy units in electrical power systems. *International Journal of Distributed Energy Resource* 2008; **4**:297–319.
4. Yuan Y, Qian K, Zhou C. The effect of Distributed Generation on distribution system reliability. *in Proc. 2007 Universities Power Engineering Conf.*; 2007 911–916.
5. Hemdan NGA, Kurrat M. Distributed generation location and capacity effect on voltage stability of distribution networks. *in Proc. 2008 Annual IEEE Conf.*; 2008, 1–5.
6. Ochoa LF, Padilha-Feltrin A, Harrison GP. Evaluating distributed generation impacts with a multiobjective index. *IEEE Trans. Power Delivery* 2006; **21**:1452–1458.
7. Mashhour E, Moghaddas-Tafreshi SM. Bidding Strategy of Virtual Power Plant for Participating in Energy and Spinning Reserve Markets—Part I: Problem Formulation. *IEEE Trans. Power Systems* 2011; **26**:949–956.
8. Mashhour E, Moghaddas-Tafreshi SM. Bidding Strategy of Virtual Power Plant for Participating in Energy and Spinning Reserve Markets—Part II: Problem Formulation. *IEEE Trans. Power Systems* 2011; **26**:957–964.
9. Mashhour E, Moghaddas-Tafreshi SM. A review on operation of micro grids and Virtual Power Plants in the power markets. *in Proc. 2009 Adaptive Science & Technology International Conf.*; 2009, 273–277.

DOI: 10.1002/etep

10. Caldon R, Rossi PA. Turri R. Optimization algorithm for a virtual power plant operation. *in Proc. 2004 IEEE-39th International Universities Power Engineering Conf., Bristol, UK*; 2004, 1058–1062.
11. Caldon R, Rossi PA, Turri R. Optimal control of a distribution system with a virtual power plant. *Bulk Power System Dynamics and Control Vl, Cortina d'Ampezzo 2004, Italy*; 2004, 278–284.
12. Salmani MA, Tafreshi SMM. Operation Optimization for a Virtual Power Plant. *in Proc. 2009 IEEE PES/IAS Sustainable Alternative Energy Conf.*, Valencia, Spain; 2009, 1–6.
13. Celli G, Pilo F, Pisano G, Soma GG. Optimal participation of a microgrid to the energy market with an intelligent EMS. *in Proc. 2005 IEEE-7th International Power Engineering Conf.*, Singapore; 2005, 663–668.
14. Morais H, Cardoso M, Castanheira L, Vale ZA, Praca I. A decision-support simulation tool for virtual power producers. *in Proc. 2005 IEEE International Future Power Systems Conf.*, Amsterdam, Netherlands; 2005, 1–6.
15. Morais H, Cardoso M, Castanheira L, Vale ZA, Praca I. VPP information needs for effective operation in competitive electricity markets. *in Proc. 2007 IEEE International Industrial Informatics Conf.* , Viennu, Austria; 2007, 1183–1188.
16. You S, Træholt C, Poulsen B. A Market-Based Virtual Power Plant. *in Proc. 2009 International Clean Electrical Power Conf.*, Capri, Italy; 2009, 460–465.
17. Palma-Behnke R, Cerda JL, Jofre A. A distribution company energy acquisition market model with integration of distributed generation and load curtailment options. *IEEE Trans. Power System* 2005; **20**:1718–1727.
18. Mashhour E, Moghaddas-Tafreshi SM. Mathematical modeling of electrochemical storage for incorporation in methods to optimize the operational planning of an interconnected micro grid. *Journal of Computer and Electronics Journal Zhejiang Univesity.* 2010; **11**:737–750.
19. Baran ME, Wu FF. Optimal sizing of capacitors placed on a radial distribution system. *IEEE Transation Power Delivery* 1989; **4**:735–743.
20. Shahidehpour M, Yamin H, Li Z. Market Operations in Electric Power Systems: Forecasting, Scheduling, and Risk Management. IEEE-Wiley-Interscience: Piscataway, NJ, 2002.
21. Conejo AJ, Castillo E, Mínguez R, García-Bertrand R. Decomposition Techniques in Mathematical Programming: Engineering and Science Applications. Springer: Heidelberg, 2006.

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