Case Study

Application of Robust Optimization Approach for Agricultural Water Resource Management under Uncertainty

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Abstract: Optimization models are widely used in agricultural water resources management programs. However, optimization models in most applications use data that is subject to uncertainty. Recently, robust optimization has been used as an optimization model that incorporates uncertainty. This paper proposes a linear programming model with the objective of maximizing the total gross margin for the delivery of water to agricultural areas that cover an irrigation network over a planning horizon. The writers apply uncertain data in this system in the form of a robust optimization approach. The writers also consider analysis of the sensitivity of the total gross margin in accordance with the variations of the degree of conservatism (reliability), irrigation efficiency, and price of irrigation water. The results of changes to these parameters in a wide range of variations caused large changes in the optimal total gross margin of the planning horizon. Application of the proposed model to the case study of the Nekooabad irrigation network in the province of Isfahan, Iran, over a 3-year planning horizon (2012–2014) demonstrates the reliability and flexibility of the model. **DOI:** 10.1061/(ASCE)IR.1943-4774.0000578.

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Introduction

An agricultural irrigation network is comprised of a water supply source (river), a water distribution network (primary and branch irrigation canals), and demand centers (the areas that are covered by the irrigation network). Optimization of an irrigation system for the allocation of available water and cultivable area is one of the best methods of increasing a farmer's income and can be facilitated by an irrigation water distribution network. Irrigation water that is allocated to various crops is usually based on predictive factors such as the water demand for each crop and the availability of water for an irrigation network. Uncertainty is an inherent aspect of predicting the factors under consideration with respect to agricultural water systems. Absolute attention to reducing costs or increasing profits of agricultural irrigation systems, without consideration of uncertainty, can cause many problems with respect to future development. Two possible results of decisions without consideration to uncertainty are the (1) creation of a net benefit that is less than expected and (2) probability of system failure, in which failure is defined as not meeting a given demand or other system constraint (Watkins and McKinney 1997). These results can be adjusted with more costs and the development of flexibility in the system during the design process to allow for this adjustment. Flexibility enhances a system's capacity to help decision-makers with changes to the supply of and demand for water.

Many researchers have used mathematical programming for the management of water resources (Maknoon and Burges 1978; Coe 1990; Karamouz et al. 2007; Pulido-Velazquez et al. 2006; Zarghami et al. 2008; Cheng et al. 2009). Some of these researchers have used fuzzy models (Lu et al. 2010; Maqsood et al. 2005; Guo et al. 2010), genetic algorithms (Kumar et al. 2006; Kuo et al. 2000), positive mathematical programming (Judez et al. 1998; Atance and Barreiro 2006), and quadratic programming (Qin and Huang 2009).

Lu et al. (2009), Maqsood et al. (2005), and Guo et al. (2010) recently used the two-stage programming approach (with different methods for data uncertainty) for water resources management. A multistage programming approach has been applied to water resources management programs, documented by Li et al. (2006, 2008) and Li and Huang (2009).

Mannocchi and Todisco (2006) developed a three-step computational model to integrate multipurpose (domestic/industrial, environmental, and irrigation) reservoir operation for irrigation with water and area allocation for multicrop farms. These researchers applied their proposed model to the Upper Tiber River Basin. Moradi-Jalal et al. (2007) proposed an optimization model for optimal multicrop irrigation areas that are associated with reservoir operation policies in a reservoir-irrigation system. The objective function was to maximize the annual benefits of the reservoir-irrigation system. A sensitivity analysis has been performed to test the effectiveness of the optimization model. The proposed model can be applied to a reservoir-irrigation system that is located in Iran. Considering variable cropping patterns can lead to more benefits from the system because of the flexibility of the system with respect to adapting to different inflow regimes. Cheng et al. (2009) presented a linear programming model to optimal water allocation in conjunction with the use of surface water and groundwater in the

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Aliliao irrigation area, located in Taiwan. The research analyzed an optimal ratio for allocating water to three canals and three scenarios. The minimum quantity of required groundwater and the maximum quantity of excess water in the area can be satisfied by current agricultural practices. However, the previous three research papers did not consider an optimization model under uncertain data.

Uncertainty in the prediction data is an important issue in water resources management. The classical methods of addressing parameter uncertainty include sensitivity analysis and stochastic optimization. However, sensitivity analysis is only a tool for analyzing the goodness of a solution. It is not particularly helpful for generating solutions that are robust to data changes. In stochastic optimization, the chance constraint can destroy the convexity properties and significantly elevate the complexity of the original problem (Bertsimas and Sim 2004). One method of addressing uncertainty is to design a system that is robust to parameter changes (without the greater complexity of the original problem). In other words, the system remains feasible and operates in a near-optimal manner for a variety of values that the uncertain parameters can take (Chung et al. 2009). In this paper, the writers use the robust optimization approach of Bertsimas and Sim (2004), in which the decision-maker must select a strategy without knowing the exact value that is taken by the uncertain parameters.

Numerous practical examples of robust optimization have been used in management issues such as portfolio (Bertsimas and Sim 2004) and inventory (Thiele 2004) theory. Chung et al. (2009) applied this approach to designing a hypothetical water supply system using uncertainty. The degree of conservatism incorporated the probability bound for the constraint violation. The total cost increased as the degree of conservatism increased.

Bohle et al. (2010) used robust optimization for a wine grape-harvesting schedule. They used an iteration solution approach, proposed by Bienstock and Ozbay (2008), to solve this problem. The model demonstrated how the maximum profit of the problem deteriorates as the robustness of the solution increases.

One of the primary causes for rejection of optimal water allocation and cropping pattern by farmers is the enforcement of an optimal answer. However, using different alternatives for water and area allocation (by changes in efficiency, prices of agricultural water, and degree of conservatism) will provide farmers with more choices (Sethi et al. 2006).

The primary aim of this paper is the formulation of a linear programming model for assigning optimal allocation of agricultural irrigation water and areas (simultaneously) for the Nekooabad irrigation network to maximize a farmer's gross margin on a planning horizon. The linear programming model that is presented in this paper has the capacity to change irrigation efficiency, prices paid by farmers for irrigation water, and the degree of conservatism.

Robust Optimization Framework

In the robust framework, optimization is described based on two scientific principles that are fundamental to the practice of modern management which incorporate uncertainty (Nahmias 2005; Sheffi 2005; Simchi-Levi et al. 2004), as follows: (1) point forecasts are meaningless and should be replaced by a range of forecasts, and (2) sets of forecasts are more precise than individual forecasts. Soyster (1973) first introduced this particular form of optimization. However, the optimal solution in Soyster's method created an objective function value that is much worse than the objective function value in a nominal problem (i.e., deterministic problem with mean parameter values). To solve this problem, Ben-Tal and Nemirovski (1999, 2000), El-Ghaoui and Lebret (1997), and El-Ghaoui et al. (1998) expanded on Soyster's method. They introduced a higher degree of nonlinearity (conic quadratic problem) with respect to the nominal problem in real systems. However, the approach of Bertsimas and Sim (2004) considers data uncertainty without the complexity that is associated with the nominal problem.

Consider the following nominal (deterministic) linear optimization problem:

$$\begin{aligned} &\text{Maximize} && Z = cx \\ &\text{Subject to} && Ax \le b, \\ &&& x \ge 0 \end{aligned} \tag{1}$$

In Eq. (1), A, b, and c are coefficient matrices for the technical, right-hand side, and objective function, respectively. Eq. (1) can be reformulated as follows:

$$\begin{aligned} &\text{Maximize} && Z \\ &\text{Subject to} && cx \ge Z, \\ &&& Ax \le b, \\ &&& x \ge 0 \end{aligned} \tag{2}$$

Now suppose that elements in matrix A are subject to uncertainty. If J_i are sets of coefficients in row i for matrix A, then a_{ij}, $j \in J_i$ is modeled as a symmetric and bounded random variable $\tilde{a}_{ij}$ (Ben-Tal and Nemirovski 2000). The element of $\tilde{a}_{ij}$ takes values in $[\bar{a}_{ij} - \varepsilon\bar{a}_{ij}, \bar{a}_{ij} + \varepsilon\bar{a}_{ij}]$, where $0 \le \varepsilon \le 1$ defines a given uncertainty level and $\bar{a}_{ij}$ is the nominal value of the uncertain data.

Suppose that $\tilde{a}_{ij}$ varies with respect to a nominal value $\bar{a}_{ij}$ in a quantity of at most $\varepsilon\bar{a}_{ij}$. The uncertainty set is specified by Eqs. (3) and (4), as follows:

$$\Psi = \{(\tilde{a}_{ij})|\tilde{a}_{ij} = \bar{a}_{ij} + \varepsilon\bar{a}_{ij}\eta_{ij},\ \forall i, j, \eta \in \varphi\} \tag{3}$$

where

$$\varphi = \left\{\eta||\eta_{ij}| \le 1,\ \forall i, j, \sum_{j=1}^{n}|\eta_{ij}| \le \Gamma_i,\ \forall i\right\} \tag{4}$$

where η_{ij} are the scaled deviations of parameter $\tilde{a}_{ij}$ that are symmetrically distributed within the interval of [−1,1]. Bertsimas and Sim (2004) introduced the Gamma parameter for each constraint i (Γ_i) that is not necessarily an integer and takes a value of $[0, |J_i|]$. $|J_i|$ is the number of uncertain data points in constraint i, and the writers considered Γ_i as the budget of uncertainty. Its role was to control the degree of conservatism (uncertainty). When $\Gamma_i = 0$, the η_{ij} values are equivalent to 0, and consequently there is no protection against uncertainty. When $\Gamma_i = |J_i|$, the constraint i is completely protected against uncertainty. When $\Gamma_i \in [0, |J_i|]$, a tradeoff exists between the level of the constraint protection and the degree of conservatism.

The following problem is a robust counterpart that corresponds with the nominal problem [Eq. (1)]:

$$\begin{aligned} &\text{Maxmize} && Z = cx \\ &\text{Subject to} && \textstyle\sum_{j=i}^{n}\bar{a}_{ij}x_{ij} + B_i(x_{ij}, \Gamma_i) \le b_i, \quad \forall_i \\ &&& l \le x \le u, \\ &&& x \ge 0 \end{aligned} \tag{5}$$

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where $B_i(x_{ij}, \Gamma_i)$ is the protection function for each constraint i that includes decision variables and the Gamma parameter; and l and u are bounds of x in another constraint without uncertainty data. The protection function for the ith constraint can then be represented as follows:

$$\begin{aligned} B_i(x_{ij}, \Gamma_i) = \text{Maximize} \quad & \sum_{j=1}^{n} \varepsilon \bar{a}_{ij} |x_{ij}| \eta_{ij} \\ \text{Subject to} \quad & \sum_{j=1}^{n} \eta_{ij} \leq \Gamma_i, \quad \forall i \\ & 0 \leq \eta \leq 1, \quad \forall i, j \end{aligned} \tag{6}$$

In accordance with Eqs. (5) and (6), the linear form for robust optimization can be rewritten as follows:

$$\begin{aligned} \text{Maximize} \quad & Z = cx \\ \text{Subject to} \quad & \sum_{j=1}^{n} \bar{a}_{ij} x_{ij} + z_i \Gamma_i + \sum_{j=1}^{n} p_{ij} \leq b_i, \quad \forall\ i \\ & z_i + p_{ij} \geq \varepsilon \bar{a}_{ij} y_j, \quad \forall i, j \\ & -y_j \leq x_{ij} \leq y_j, \quad \forall i, j \\ & x_{ij}, z_i, p_{ij} \geq 0, \quad \forall i, j \\ & l \leq x \leq u \end{aligned} \tag{7}$$

At optimality, $y_i = |x_{ij}|$ for all j.

The quantities that are represented by z_i and p_{ij} are additional variables for each constraint of the robust problem [Eq. (7)]. Whereas a given uncertainty level is nonzero $(0 < \varepsilon \leq 1)$ and parameter $\Gamma_i = 0$, most values that are allocated to z_i and the zero value are allocated to p_{ij}. Consequently, in this scenario, the ith constraint is equivalent to that of the nominal problem $(\sum_{j=1}^{n} \bar{a}_{ij} x_{ij} \leq b_i)$. It is reasonable that both parameters z_i and p_{ij} are affectless in Eq. (7). Furthermore, although $\varepsilon = 0$, the robust problem changes to a nominal problem. Both parameters z_i and p_{ij} are zero in Eq. (7).

Different values of the parameter Γ allow decision-makers to control the conservatism of a system. The value of Γ is dependent on the maximum probability of conservatism violation (p) and number of uncertain data points in this constraint (n). Assume that x^* is a set of optimal solutions in Eq. (7). The probability that the ith constraint has been violated is bound by the following equation:

$$pr\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_{ij}^* > b_i\right) \leq B_i(n, \Gamma_i) \tag{8}$$

Calculating the Gamma parameter by Bertsimas and Sim (2004) is fully explained.

Agriculture Water Distribution Network

Fig. 1 presents an agricultural water irrigation network with a diversion dam. Two primary canals $(h = 1, 2)$ are located on both sides of the diversion dam. These canals divide irrigation water between branch canals in different regions $(r = 1, \ldots, R)$. The role of the branch canals is the transmission of water from the primary canals to agricultural areas $(j = 1, \ldots, J)$ for each region. The distance between the starting point of the branch canal in region r and the starting point of the primary canal (diversion dam) is marked by $L_{i,r}$.

Mathematical Model Development

The aim of most optimization models in agricultural irrigation systems is to maximize profit by optimal allocation of the available water. Before describing the optimization model for the irrigation water network (Fig. 1), the writers describe the basic assumptions of their model, as follows:

- There are no temporal changes to the physical and chemical properties of the soil;
- The most limiting factor for agricultural production over the total areas that are covered by the irrigation water network is that of available irrigation water;
- Irrigation efficiency is unchanged over the total areas that are covered by the irrigation network; and
- Each unit of land receives the same management practices for a particular crop.

The writers formulated a nominal linear programming model of optimal water allocation and areas in the agricultural irrigation water network as follows:

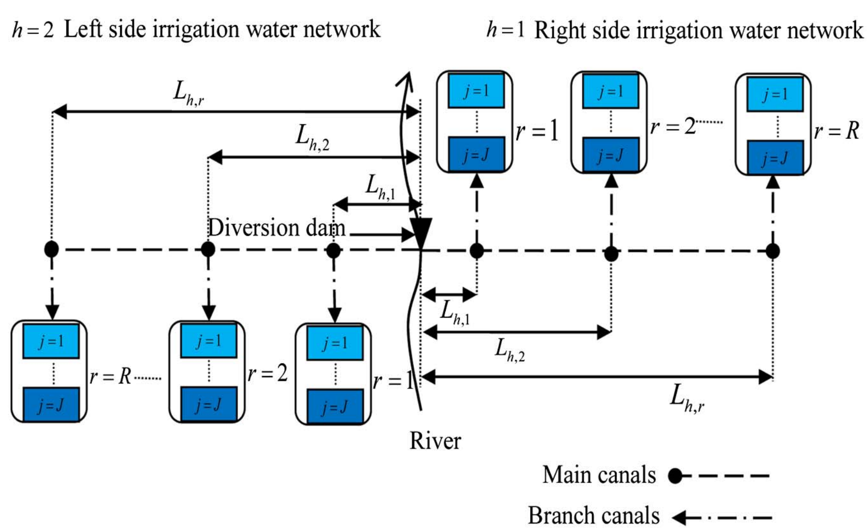

Fig. 1. Schematic of an agricultural irrigation water network

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$$\text{Maximize } f = \left(\sum_{h=1}^{H} \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{t=1}^{T} q_{hrjt} N_{hrjt} - \sum_{h=1}^{H} \sum_{r=1}^{R} \sum_{j=1}^{J} \sum_{t=1}^{T} q_{hrjt} (PW_{hrj}) \right)$$

Subject to

$$\begin{aligned}
&\text{R1: } \sum_{h=1}^{1} \sum_{r=1}^{1} \sum_{j=1}^{J} (1+\delta_{11}) q_{11jt} + \cdots + \sum_{h=1}^{1} \sum_{r=1}^{R} \sum_{j=1}^{J} (1+\delta_{1r}) q_{1rjt} \\
&\quad + \sum_{h=2}^{2} \sum_{r=1}^{1} \sum_{j=1}^{J} (1+\delta_{21}) q_{21jt} + \cdots + \sum_{h=H}^{H} \sum_{r=R}^{R} \sum_{j=1}^{J} (1+\delta_{hr}) q_{hrjt} \leq Q_t, \quad \forall t \\
&\text{R2: } \sum_{r=1}^{1} \sum_{j=1}^{J} (1+\delta_{h1}) q_{h1jt} + \cdots + \sum_{r=R}^{R} \sum_{j=1}^{J} (1+\delta_{hr}) q_{hrjt} \leq C_{ht}, \quad \forall h, t \\
&\text{R3: } \sum_{j=1}^{J} (1+\xi_{hrj}) q_{hrjt} \leq B_{hrjt}, \quad \forall h, r, t \\
&\text{R4: } D_{\min-hrjt} \leq q_{hrjt} \leq D_{\max-hrjt}, \quad \forall h, r, j, t \\
&\text{R5: } w_{hrj} x_{hrjt} = q_{hrjt}, \quad \forall h, r, j, t \\
&\text{R6: } q_{hrjt}, x_{hrjt} \geq 0, \quad \forall h, r, j, t
\end{aligned} \tag{9}$$

where H, J, R, and T = numbers of primary canals (right and left sides of the diversion dam), crops, regions, and planning horizons, respectively; f is the objective function value that represents the total gross margin of farmers for delivering water to agricultural areas that cover an irrigation network over a planning horizon; q_{hrjt} are the decision variables that represent the net flows that are allocated to the primary canal h for total areas of crop j in region r at a particular planning horizon year t; N_{hrjt} = gross margin of crop j per unit of allocated water to the primary canal h for region r at planning horizon year t (US\$/m^3); PW_{hrj} = water price of crop j per unit of allocated water to the primary canal h for region r (US\$/m^3); δ_{hr} is defined to calculate the total quantity of water that is lost in the primary and branch canals; ξ_{hrj} = loss rate that is associated with the branch canal at region r (primary canal h) for total areas of crop j; Q_t = quantity of available water for each planning horizon year t; C_{ht} = capacity of the primary canal h at the planning horizon year t; B_{hrjt} = capacity of branch canal in region r for total areas of crop j at a planning horizon year t; $D_{\max-hrjt}$ and $D_{\min-hrjt}$ = maximum and minimum irrigation water demands, respectively; w_{hrj} = unitary irrigation demand of crop j in region r for the primary canal h (m^3/ha); and x_{hrjt} = total area of crop j in region r at planning horizon year t for primary canal h. The remainder of this section introduces an objective function and constraint sets of the agricultural irrigation water network [Eq. (9)].

Objective Function

As noted, the objective is to maximize a farmer's gross margin on a planning horizon. To calculate the gross margin of crop j per unit of allocated water to the primary canal h for region r at planning horizon year t (N_{hrjt}), suppose that NI_{hrj} is the net irrigation that is need for crop j in region r for the primary canal h (m^3/ha) and $0 < SR \leq 1$ is the indicator for technological improvements to an irrigation system (irrigation efficiency). The writers calculated the unitary irrigation demand of crop j in region r for the primary canal h (m^3/ha) as follows:

$$w_{hrj} = NI_{hrj}/SR \tag{10}$$

The writers calculated the net irrigation need of crop j in region r for the primary canal h per unit of hectare (NI_{hrj}) using a simple soil water balance model (Leenhardt et al. 2004).

N_{hrjt} is calculated as follows:

$$N_{hrjt} = UN_{hrjt}/w_{hrj} \tag{11}$$

where UN_{hrjt} = gross margin of crop j per unit of agricultural area (ha) to the primary canal h in region r at planning horizon year t (US\$/ha). The writers did not deduct the cost of irrigation water (prices paid by farmers for irrigation water) from the gross value. The writers calculated the water price of crop j per unit of allocated water to the primary canal h for region r (PW_{hrj}) as follows:

$$\mathrm{PW}_{hrj} = \mathrm{UPW}_{hrj}/w_{hrj} \tag{12}$$

where UPW_{hrj} = irrigation water price (prices paid by farmers for irrigation water) of crop j per unit of agricultural area to the primary canal h in region r (US\$/ha). The writers fixed this parameter for the planning horizon years.

The writers calculated the annual gross margin for farmers to deliver water to the agriculture areas that are covered by an irrigation network at year t (GM_t) as follows:

$$\begin{aligned}
\mathrm{GM}_t = &\sum_{h=1}^{H} \sum_{r=1}^{R} \sum_{j=1}^{J} q_{hrjt-\mathrm{opt}} N_{hrjt} \\
&- \sum_{h=1}^{H} \sum_{r=1}^{R} \sum_{j=1}^{J} q_{hrjt-\mathrm{opt}} PW_{hrj}, \quad \forall t
\end{aligned} \tag{13}$$

where $q_{hrjt-\mathrm{opt}}$ = optimal net flow that is allocated to the primary canal h for total areas of crop j in region r at planning horizon year t [by solving Eq. (9)].

Constraint Sets

The quantity of available water for each planning horizon year t (Q_t) must be greater than or equal to the total water that is allocated to irrigation network [R1 constraint sets in Eq. (9)]. In other words, the sum of the (1) total water that is allocated to all of the crops and regions and (2) water lost in the primary and branch canals must be

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less than or equal to the quantity of available water for each planning horizon year t. In these constraint sets, the writers define δ_{hr} to calculate the total quantity of water that is lost in the primary and branch canals. This parameter is calculated as follows:

$$\delta_{hr} = S(L_{h,r}) + \xi_{hrj} \tag{14}$$

where S = loss rate per unit of length (km) for the primary canal h that is expressed as a percentage (e.g., 1% loss per km); and L_{hr} = distance between the starting point of branch canal h for region r and the starting point of the primary canal h (diversion dam). To avoid the complexity of Eq. (9), the writers determined the water loss rate for the branch canals (ξ_{hrj}) based on an approximate distance between the total areas of crop j and the starting point of the branch canal extremity to region r.

The sum of the (1) total water that is allocated to the primary canal h and (2) water that is lost in the primary canal h and branch canal in the region r at planning horizon year t must be less than or equal to the capacity of the primary canal h at the planning horizon year t (R2 constraint sets). The final value of this constraint represents the quantity of allocated water to the primary canal h at a planning horizon year t.

The sum of the (1) total water that is allocated to region r for total areas of crop j (primary canal h) at the planning horizon year t and (2) water lost in the branch canal at region r must be less than or equal to the capacity of branch canal in the region r for total areas of crop j at a planning horizon year t (R3 constraint sets). The final value of this constraint represents the quantity of allocation water to the branch canal in region r for total areas of crop j at a planning horizon year t.

The net flows that are allocated to the primary canal h for total areas of crop j in region r at planning horizon year t (q_{hrjt}) must be less than or equal to the maximum irrigation water demands ($D_{\max-hrjt}$) and greater than or equal to the minimum irrigation water demands ($D_{\min-hrjt}$) (R4 constraint sets). To calculate $D_{\max-hrjt}$ and $D_{\min-hrjt}$, suppose that $A_{\min-hrjt}$ is the maximum of total areas of crop j in region r at planning horizon year t (primary canal h). The value of $D_{\max-hrjt}$ can be calculated as follows:

$$D_{\max-hrjt} = A_{\max-hrjt} w_{hrj} \tag{15}$$

and $D_{\min-hrjt}$ is calculated as follows:

$$D_{\min-hrjt} = D_{\max-hrjt} \mathrm{WQ}_{hrjt} \tag{16}$$

where WQ_{hrjt} = water quota for total areas of crop j in region r at planning horizon year t (for primary canal h).

The writers calculated the total area of crop j in region r at planning horizon year t for the primary canal h (x_{hrjt}) in accordance with the R4 constraint sets. Using the unitary irrigation demand of crop j for region r at the primary canal h (w_{hrj}) and the net flow that is allocated to the primary canal h for total area of crop j in region r at planning horizon year t (q_{hrjt}), the total crop area can be calculated (simultaneously with water allocation).

R6 constraint sets require that the decision variables (q_{hrjt} and x_{hrjt}) must be nonnegative.

The allocated agriculture irrigation water and area for perennial plants (such as alfalfa) must be constant during the planning horizon. Thus, the following constraint sets add to Eq. (9) for perennial plants, as follows:

$$q_{hr(je)t} = q_{hr(je)t+1}, \quad \forall h, r, je, t \tag{17}$$

where je = subset of J for perennial plants.

Data Uncertainty and Robust Formulation

The writers used the robust optimization approach to survey the effect of uncertain data on the values of optimal allocation of agricultural irrigation water and areas.

Modified Uncertain Data

There are many uncertain data points that arise from predictions of future water supply and demand in Eq. (9). The writers assume that uncertainties are considered in the parameters of gross margin per unit of allocated water (N_{hrjt}), quantity of available water (Q_t), maximum irrigation water demand ($D_{\max-hrjt}$), and minimum irrigation water demand ($D_{\min-hrjt}$). In accordance with Eqs. (3) and (4), the random form for these variables can be rewritten as follows:

$$\tilde{N}_{hrjt} = \bar{N}_{hrjt} + \varepsilon \bar{N}_{hrjt}\eta_1 \tag{18}$$

$$\tilde{Q}_t = \bar{Q}_t + \varepsilon \bar{Q}_t \eta_2 \tag{19}$$

$$\tilde{D}_{\max-hrjt} = \bar{D}_{\max-hrjt} + \varepsilon \bar{D}_{\max-hrjt}\eta_3 \tag{20}$$

$$\tilde{D}_{\min-hrjt} = \bar{D}_{\min-hrjt} + \varepsilon \bar{D}_{\min-hrjt}\eta_4 \tag{21}$$

where $\bar{N}_{hrjt}$, $\bar{Q}_{hrjt}$, $\bar{D}_{\max-hrjt}$, and $\bar{D}_{\min-hrjt}$ = nominal value of gross margin per unit of allocated water, quantity of available water, maximum irrigation water demand, and minimum irrigation water demand, respectively; and η_1, η_2, η_3, and η_4 are random variables in the interval [1, 1].

The level of uncertainty (i.e., the degree of conservatism) must be controlled to ascertain a certain degree of system reliability in this paper. The writers have already introduced the degree of conservatism (Γ), which relates to that which has been proposed by Bertsimas and Sim (2004).

Aggregated Modified Robust Reformulation

The reformulation of the constraint sets, which contain the uncertain data as robust formulation, are rewritten in Eq. (7). To achieve this reformulation, the writers first converted the objective function of constraint sets in the model. The first constraint sets in the robust formulation using Eqs. (2) and (9) can be written as follows:

$$a1\begin{cases} \left(\sum_{h=1}^{H}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T} q_{hrjt}\bar{N}_{hrjt} - \sum_{h=1}^{H}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T} q_{hrjt}(PW_{hrj})\right) + z_1\Gamma_1 + \sum_{h=1}^{H}\sum_{r=1}^{R}\sum_{j=1}^{J}\sum_{t=1}^{T} p_{hrjt} \geq f, & \forall h,r,j,t \\ z_1 + p_{hrjt} \geq (\varepsilon \bar{N}_{hrjt}) y_{hrjt}, & \forall h,r,j,t \\ -y_{hrjt} \leq q_{hrjt} \leq y_{hrjt}, & \forall h,r,j,t \\ q_{hrjt}, p_{hrjt}, y_{hrjt}, z_1 \geq 0, & \forall h,r,j,t \end{cases} \tag{22}$$

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In Eq. (22), the writers introduced the parameter Γ_1 for the objective function that controlled the degree of conservatism in the constraint set $a1$ (for uncertain data N_{hrjt}). The variables z_1 and p_{hrjt} are additional variables for robust constraint $a1$ [Eq. (22)]. The writers maximized the objective function in accordance with the total gross margin of farmers for delivering water (f) in the robust formulation, considering Eq. (2). Similarly, the writers introduced the parameter Γ_2 for the CR1 constraint sets in Eq. (9). The writers converted it to a robust form as follows:

$$a2\begin{cases} \sum_{h=1}^{1}\sum_{r=1}^{1}\sum_{j=1}^{J}(1+\delta_{11})q_{11jt}+\cdots+\sum_{h=1}^{1}\sum_{r=1}^{R}\sum_{j=1}^{J}(1+\delta_{1r})q_{1rjt} & \\ +\sum_{h=2}^{2}\sum_{r=1}^{1}\sum_{j=1}^{J}(1+\delta_{21})q_{21jt}+\cdots+\sum_{h=H}^{H}\sum_{r=R}^{R}\sum_{j=1}^{J}(1+\delta_{hr})q_{hrjt}+z_2\Gamma_2+p_t-\bar{Q}_t\leq 0, & \forall t \\ z_2+p_t\geq\varepsilon\bar{Q}_t, & \forall t \end{cases} \quad (23)$$

In the constraint sets $a2$, the available irrigation water is uncertain data.

The writers converted the CR4 constraint sets to a robust form in the same manner, as follows:

$$a3\begin{cases} q_{hrjt}-\bar{D}_{\max-hrjt}+z_3\Gamma_3+p_{\max-hrjt}\leq 0, & \forall h,r,j,t \\ z_3+p_{\max-hrjt}\geq\varepsilon\bar{D}_{\max-hrjt}, & \forall h,r,j,t \end{cases} \quad (24)$$

$$a4\begin{cases} \bar{D}_{\min-hrjt}-q_{hrjt}+z_4\Gamma_4+p_{\min-hrjt}\leq 0, & \forall h,r,j,t \\ z_4+p_{\min-hrjt}\geq\varepsilon\bar{D}_{\min-hrjt}, & \forall h,r,j,t \end{cases} \quad (25)$$

The previous problem can be solved in accordance with the linear solution method using the computer software package General Algebraic Model System (*GAMS 23.5*).

Application

The writers collected the data from different agricultural planning units. The sources are as follows: the District Statistical Yearbook (Dept. of Regional Planning and Development 2010), Jehad-Keshavarzi Organization (unpublished results, 2008), Iranian Ministry of Energy (2003), and Isfahan regional water organization (2008).

The Zayandehrood River feeds water to the Nekooabad irrigation network. The writers implemented Monte Carlo simulation to generate a random number for simulating the quantities of available water (Hardaker et al. 2004). The simulated quantity of available water for this irrigation network for a 3-year planning horizon are 470 (Q_1), 343 (Q_2), and 371 (Q_3) in units of millions of cubic meters. The writers calculated the net irrigation need of crop j in region r for the primary canal h per unit hectare (NI_{hrj}) with a simple soil water balance model (Leenhardt et al. 2004). The writers calculated this parameter for the primary crops of the Nekooabad irrigation network using the *NETWAT* software package (the net irrigation water of Iran crops and orchards production); see Table 1.

In this scenario, the capacities of the left- and righthand sides of the primary canals are 1,577 and 473 ($\mathrm{m}^3\times 10^6$/year), respectively, levels that will remain constant over the planning horizon. Table 2 presents the capacities of the branch canals for different regions in addition to the distances between the starting points of these canals and the diversion dam ($L_{h,r}$). The capacities of the branch canals will also remain constant over the planning horizon, assuming that 1% of the water allocated per kilometer is lost ($S=0.01$) to the primary canal h (Rostamian and Abedy 2010).

Table 1. Net Irrigation Needs of Crops within Different Regions in Units of Cubic Meters per Hectare

Regions	Crops						
	Wheat, $j=1$	Barley, $j=2$	Potato, $j=3$	Corn, $j=4$	Rice, $j=5$	Onion, $j=6$	Alfalfa, $j=7$
Mobarake, $r=1$	4,970	4,460	—	—	7,090	—	9,280
Nadjafabad, $r=2$	4,270	3,590	6,240	4,870	7,940	7,010	9,310
Lenjan, $r=3$	4,270	3,590	—	—	6,890	4,970	4,460
Falavarjan, $r=4$	4,910	4,480	5,690	—	6,890	4,970	4,460
Shahinshahr, $r=5$	4,970	4,210	5,690	4,530	—	—	9,280
Borkhar, $r=6$	4,970	4,800	—	4,780	—	—	9,630
Khomeinishahr, $r=7$	4,320	4,210	6,400	—	8,240	—	9,630
Isfahan, $r=8$	4,700	4,610	—	4,870	—	—	9,810

Table 2. Properties of Branch Canal for the Nekooabad Irrigation Network

Primary canal	Mobarakeh	Nadjafabad	Lenjan	Falavarjan	Shahinshahr	Borkhar	Khomeinishahr	Isfahan
Maximum capacity of the branch canal ($\mathrm{m}^3\times 10^6$)								
Left side	79	349	79	349	157	315	159	—
Right side	213	—	—	213	—	—	—	47
Distance between the starting point of the branch canals and diversion dam (km)								
Left side	10.3	52.1	24.2	18.2	80.8	158.7	60.4	—
Right side	30.6	—	—	35.1	—	—	—	36.2

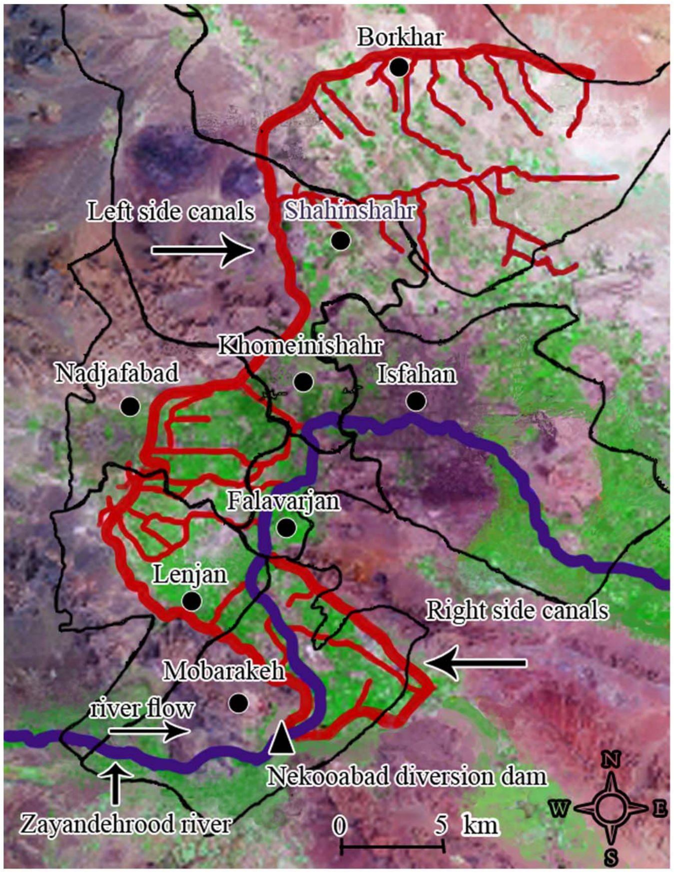

Fig. 2. Nekooabad agricultural irrigation water network in Isfahan province, Iran

Results and Discussion

Results Analysis

The writers formulated the nominal problem and its robust counterpart for the Nekooabad irrigation network, presented in Fig. 2. In accordance with the numbers of primary canals, regions, crops, and planning horizon years, an optimal allocation irrigation water model generates a problem with 297 decision variables (147 optimal net flows and 147 optimal areas for cultivation). Furthermore, the writers generated this problem with 883 and 1,968 constraints for nominal and robust problems, respectively. These problems were solved using the *GAMS* optimization solver.

For all of constraint sets that include uncertain data, the writers considered four levels of given uncertainty ($\varepsilon = 0.01$, 0.05, 0.1, and 1.5) and different values of the Gamma parameter (Γ) that provide choice as a function of the maximum probability of constraint violation, introduced by Bertsimas and Sim (2004). Table 3 provides the values of Γ for a given maximum constraint violation probability. For the similar probability level, the Γ values vary in accordance with the different number of random variables ($n = |j_i|$). The writers also considered four levels of irrigation efficiency, as follows: 35, 45, 55, and 65% (i.e., SR = 0.35, 0.45, 0.55, and 0.65) and three prices at increasing levels for irrigation water with respect to the nominal value (UPW_{hrj}), as follows: 10, 20, and 30% (i.e., 1.1, 1.2, and $1.3 \times \mathrm{UPW}_{hrj}$).

The solutions indicate that most of the decision variables are nonzero. Twenty and 23 annual optimal net flows and areas for nominal and robust problems are zero, respectively, which is attributable to the very small gross margin per unit of agricultural area (UN_{hrjt}).

Table 4 represents some of the important results obtained through the nominal (i.e., all $\Gamma = 0$) and robust ($\varepsilon = 0.05$ and $p = 0.1$) problem, with an irrigation efficiency of 35% and the existing irrigation water price. The most optimal annual allocated net flows in both problems are for rice production in the Lenjan region on the lefthand side of the primary canal in year 1, with values of 28.97 and 27.52 $\mathrm{m}^3 \times 10^6$ (q_{1351}). This is attributable to the large total gross margin per unit of agricultural areas (UN_{hrjt}) for rice production in this region and the short distance from the diversion dam that has a high net irrigation need (Table 1). Because of a large distance from the diversion dam and the Nadjafabad region, optimal annual allocated net flow in both problems are zero for rice production despite the high total gross margin per unit of agricultural area. At nonzero optimal annual allocated net flows, the smallest in both problems are correlated with wheat production in the Isfahan region on the righthand side of the primary canal in year 2, with values 0.16 and 0.17 $\mathrm{m}^3 \times 10^6$ (q_{2812}). The alfalfa crop is a perennial plant in this application. Consequently, the writers used Eq. (20) to impart a constant net flow and area allocation in this scenario. The optimal annual allocation net flows and areas for the alfalfa crop were constant during the planning horizon (for each region). For example, the annual optimal net flows that were allocated to the Borkhar region (lefthand side of the primary canal) are 4.58 and 4.81 $\mathrm{m}^3 \times 10^6$ (q_{1671}, q_{1672}, and q_{1673}) for the nominal and robust problems, respectively, during the 3-year planning horizon.

Model calibration refers to adjusting the model and parameters to bring the model outputs as close to the observed values as possible. To calibrate the proposed model, the writers used time series data of model parameters (18-year time series data) and ran the model using this data. The writers used some statistical criteria to evaluate between the model outputs and observed values of variables, including the coefficient of determination (R^2), root mean square error (RMSE), and average percent error (APE), which the writers calculated as 0.78, 0.046, and 0.005, respectively.

Table 5 provides the optimized annual gross margin for the 3-year planning horizon for nominal and robust problems with the same conditions as those in Table 3. Table 5 indicates a decrease of the annual gross margin at a constant planning horizon year with an increase in the degree of conservatism (i.e., the nominal problem with respect to the robust problem). The total gross margin decreases from US\$338.5 million to US\$302.2 million over the planning horizon with an increase in the degree of conservatism. In the robust optimization approach, a tradeoff exists between the degree of conservatism and the total gross margin. The writers suggest that the optimized allocation for irrigation water of the robust problem,

Table 3. Quantity of Gammas as a Function of the Allowable Probability of Constraint Violation

		Probability of constraint violation									
Γ	$n = \|j_i\|$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Γ_1	147	19.70	16.82	13.34	10.94	7.40	5.32	3.11	0.77	0.00	0.00
$\Gamma_{1\text{–}4}$	1	2.00	2.00	1.82	1.47	1.11	0.76	0.40	0.05	0.00	0.00

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Table 4. Annual Net Flows and Area Allocations for Nominal and Robust Problems

					Net flows ($m^3 \times 10^6$/year)		Areas (ha/year)	
hrjt	Primary canal	Region	Crop	Year	Nominal	Robust	Nominal	Robust
1,351	Left side	Lenjan	Rice	1	28.97	27.52	1,277	1,213
1,251	Left side	Nadjafabad	Rice	1	0.00	0.00	0	0
1,671	Left side	Borkhar	Alfalfa	1	4.58	4.81	166	175
1,672	Left side	Borkhar	Alfalfa	2	4.58	4.81	166	175
1,673	Left side	Borkhar	Alfalfa	3	4.58	4.81	166	175
2,812	Right side	Isfahan	Wheat	2	0.16	0.17	12	12

Table 5. Optimal Annual and Total Gross Margin for Nominaland Robust Problems

Year	Nominal	Robust
2012, $t = 1$	133.8	125
2013, $t = 2$	90.1	77.1
2014, $t = 3$	114.6	100.1
Total	338.5	302.2

Note: Optimum values are in units of U.S. dollars (1 U.S. dollar is approximately equal to 10,000 Iranian Rials).

which protected the Nekooabad irrigation network against uncertainty with an increase in the degree of conservatism, should be used by the owner of an irrigation network.

The writers devote the remainder of the analysis for different irrigation water prices, irrigation efficiency, and degree of conservatism.

Sensitivity Analysis of the Total Gross Margin

The effectiveness of the optimization model that the writers proposed in this paper can be tested with a sensitivity analysis. As the importance of the total gross margin variations over the planning horizon, sensitivity analysis can be applied to analyze these variations in different scenarios, such as changes in the irrigation water price or efficiency, changes in the degree of conservatism, and other factors. Thus, a sensitivity analysis that is based on the parameters model can be determined with various combinations of optimal allocation net flows and areas, in addition to their effects on the optimal gross margin.

The writers obtained total gross margin values by solving the modified robust model for the Nekooabad irrigation network with different levels of given uncertainty (ε) and probability of constraint violation (p) with other conditions as constants ($SR = 0.35$ and existing values for irrigation water price), presented in Fig. 3. The optimal total gross margin depends on both levels of given uncertainty and probability of constraint violation. Fig. 3 clearly indicates that the optimal total gross margin values deteriorate as the degree of conservatism (Γ) increases (decrease in the probability of constraint violation), at a constant level of given uncertainty. Fig. 3 also indicates that the optimal total gross margin increases as the level of given uncertainty decreases (at a constant level of p). As indicated by Fig. 3, at less than the constraint violation probability level of 0.5, the values of total gross margin is relatively flat. Furthermore, the total gross margin from $p = 0.5$ to $p = 0.8$ sharply increases, and is then relatively flat again.

Fig. 4 presents the total gross margin values for robust problem of the Nekooabad irrigation network with different levels of the constraint violation probability (p) and irrigation efficiency (SR) with other conditions as constants ($\varepsilon = 0.5$ and existing values for irrigation water price). Fig. 4 demonstrates that as irrigation efficiency is increased, the total gross margin increases with a constant of p. Offsetting the loss of total gross margin values can be determined with the robust problem, and can be used to increase the irrigation efficiency in the Nekooabad irrigation network. Because of old infrastructure such as irrigation water canals, water loss can be increased through canal modification. The different distance between the lines (with the probability of constraint violation as constants) is attributable to the nonlinear relationship between the gross margin and irrigation efficiency.

Fig. 5 presents the total gross margin values for robust problem with different levels of the constraint violation probability (p) and irrigation water price with other conditions as constants ($\varepsilon = 0.05$ and $SR = 0.35$). The optimal total gross margin decreases as the irrigation water price increases with a constant of p. This decrease is attributable to the negative sign for irrigation water price in the problem.

Evaluating the Model with Monte Carlo Simulation

To examine the quality of the proposed model, the writers ran 1,000 simulations of random uncertain data with the parameters of gross margin (N_{hrjt}), quantity of available water (Q_t), maximum irrigation water demand ($D_{\max - hrjt}$), and minimum irrigation water demand ($D_{\min - hrjt}$). The writers compared nominal (i.e., all $\Gamma = 0$) and robust ($\varepsilon = 0.05$ and $p = 0.1$) problem solutions. The writers implemented Monte Carlo simulations for their analysis. The writers recorded the percentage of scenarios in which the solution was determined to be unviable. A normal distribution was assumed within the interval, with a 95% coverage (i.e., 1.96× the standard deviation of the distribution) for

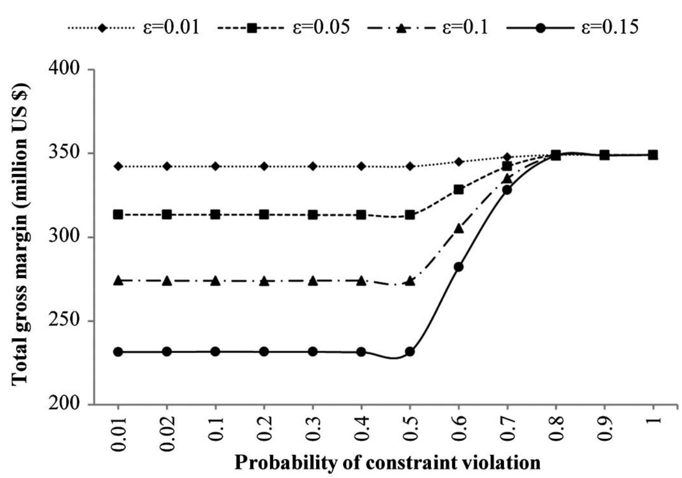

Fig. 3. Total gross margin values at different levels of robustness

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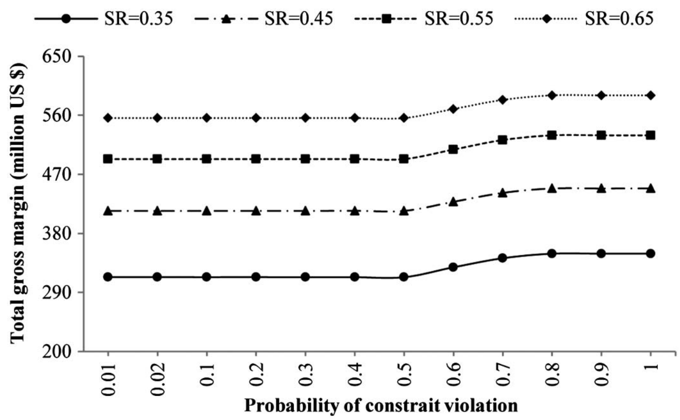

Fig. 4. Total gross margin values at different levels of water irrigation efficiency

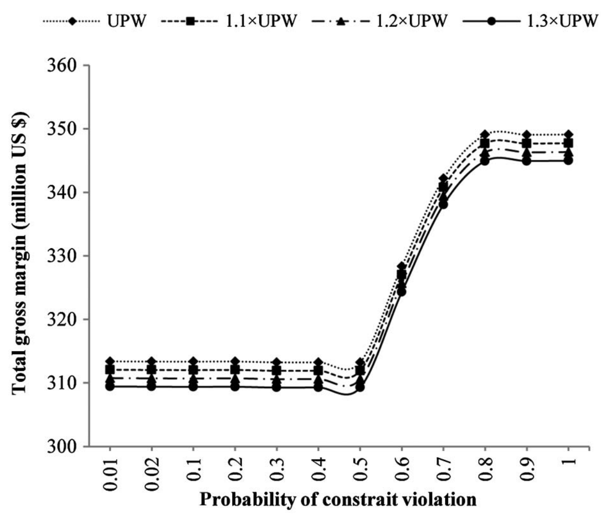

Fig. 5. Total gross margin values at different levels of water irrigation price

simulation runs [see Freund (1985)]. The Monte Carlo simulation determined probabilities for viability in nominal and robust problems as percentages of 53.7 and 10.6%, respectively, assuming a 95% coverage normal distribution.

Conclusion

Linear programming models can be used as an effective tool for determining an optimal allocation of irrigation water and areas in an irrigation water network. With this tool, administrative operational manager, decision-makers, and water authorities can make better analysis of their study areas for determining the best alternative allocation of agricultural irrigation water and areas in an irrigation water network. In this paper, the writers applied a linear programming model to the Nekooabad irrigation network to maximize the total gross margin of farmers for delivering water to agricultural land that covers an irrigation network over a 3-year planning horizon. The writers also presented a robust optimization approach to support the allocation of agricultural irrigation water to areas under uncertainty. The writers applied this approach to their linear model without introduction of additional complexity into the original problem. The primary uncertainty parameters of the model were gross margin per unit of allocated water, quantity of available water, and maximum and minimum demands for irrigation water.

In the application, the writers solved the model for sensitivity analysis of the total gross margin as the input parameters that are associated with various irrigation water prices, irrigation efficiency levels, and levels of robustness using *GAMS* software. The results indicate that as the degree of conservatism (Γ) increases (a decrease in the probability of constraint violation), the optimal solution structure changes. The optimal total gross margin in the nominal and robust problems indicates that the optimal total gross margin decreases with higher robustness levels, which is in agreement with the conclusions of Bohle et al. (2010) and Chung et al. (2009). The writers also observed that as the irrigation efficiency decreases and irrigation water price increases, the optimal total gross margin decreases. The writers implemented Monte Carlo simulation to analyze the probabilities of viability in the nominal and robust problems. The writers determined the probabilities of viability in the nominal and robust problems as 53.7 and 10.6%, respectively, assuming a 95% coverage under normal distribution. This indicates that the writers' proposed model is both reliable and flexible.

Recommendations for future research include better representations of the proposed model and its decisions. For example, this paper is based on a linearity assumption of the objective function and constraint. However, if water irrigation requirements of the Nekooabad irrigation network are completely fulfilled, then there is nonlinearity in the relationship between crop yield and irrigation water that correlates with a logarithmic relationship for greater volumes of irrigation water.

Notation

The following symbols are used in this paper:

$A_{\max\text{-}hrjt}$ = maximum of total areas of crop j in region r at planning horizon year t (ha);
B_{hrjt} = capacity of branch canal in the region r for total areas of crop j at a planning horizon year t;
C_{ht} = capacity of the primary canal h at the planning horizon year t;
$\bar{D}_{\max\text{-}hrjt}$ = nominal value of maximum irrigation water demands;
$\tilde{D}_{\max\text{-}hrjt}$ = random form of $\bar{D}_{\max\text{-}hrjt}$;
$\bar{D}_{\min\text{-}hrjt}$ = nominal value of minimum irrigation water demands;
$\tilde{D}_{\min\text{-}hrjt}$ = random form of $\bar{D}_{\min\text{-}hrjt}$;
f = objective function value, which represents the total gross margin of farmers for delivering water to agricultural areas that cover an irrigation network over a planning horizon (US$);
GM_t = annual gross margin at year t (US$/year);
h = primary canal ($h = 1, \ldots, H$);
j = crop ($j = 1, \ldots, J$);
je = subset of J for perennial plants;
L_{hr} = distance between the starting point of branch canal h for region r and the diversion dam (km);
N_{hrjt} = gross margin of crop j per unit of allocated water to the primary canal h for region r at planning horizon year t (US$/m^3);
$\bar{N}_{hrjt}$ = nominal value of gross margin per unit of allocated water (US$/m^3);
$\tilde{N}_{hrjt}$ = random form of $\bar{N}_{hrjt}$;
NI_{hrj} = net irrigation need for crop j in region r for the primary canal h (m^3/ha);

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PW_{hrj} = water price of crop j per unit of allocated water to the primary canal h for region r (US\$/m^3);
p_{hrjt} = additional variables for robust constraint a1 [Eq. (22)];
$p_{\max -hrjt}$ = additional variables for robust constraint a3 [Eq. (24)];
$p_{\min -hrjt}$ = additional variables for robust constraint a4 [Eq. (25)];
p_t = additional variables for robust constraint a2 [Eq. (23)];
Q_t = quantity of available water for each planning horizon year t (m^3 × 10^6);
$\bar{Q}_t$ = nominal value of quantity of available water (m^3×10^6/year);
$\tilde{Q}_t$ = random form of $\bar{Q}_t$;
$q_{hr(je)t}$ = allocated agriculture irrigation water and area for perennial plants (m^3×10^6/year);
q_{hrjt} = net flow that is allocated to the primary canal h for total areas of crop j in region r at a particular planning horizon year t (m^3×10^6/year);
$q_{hrjt-\text{opt}}$ = optimal net flow that is allocated to the primary canal h for total areas of crop j in region r at planning horizon year t (m^3×10^6/year);
r = region ($r = 1, \ldots, R$);
S = loss rate (%/km);
SR = indicator for technological improvements to an irrigation system (irrigation efficiency);
t = planning horizon ($t = 1, \ldots, T$);
UN_{hrjt} = gross margin of crop j per unit of agricultural area (hectare) to the primary canal h in region r at planning horizon year t (US\$/ha);
UPW_{hrj} = irrigation water price of crop j per unit of agricultural area to the primary canal h in region r (US\$/ha);
WQ_{hrjt} = water quota for total areas of crop j in region r at planning horizon year t (for primary canal h);
w_{hrjt} = unitary irrigation demand of crop j in region r for the primary canal h (m^3/ha);
x_{hrjt} = total area of crop j in region r at planning horizon year t for the primary canal h (ha/year);
z_1 = additional variables for robust constraint a1 [Eq. (22)];
z_2 = additional variables for robust constraint a2 [Eq. (23)];
z_3 = additional variables for robust constraint a3 [Eq. (24)];
z_4 = additional variables for robust constraint a4 [Eq. (25)];
δ_{hr} = quantity of water lost in the primary and branch canals;
ε = given uncertainty level;
η_1 = random variable [Eq. (18)];
η_2 = random variable [Eq. (19)];
η_3 = random variable [Eq. (20)];
η_4 = random variable [Eq. (21)];
ξ_{hrj} = loss rate that is associated with branch canal at region r (primary canal h) for total areas of crop j;
Γ_1 = degree of conservatism for robust constraint a1 [Eq. (22)];
Γ_2 = degree of conservatism for robust constraint a2 [Eq. (23)];
Γ_3 = degree of conservatism for robust constraint a3 [Eq. (24)]; and
Γ_4 = degree of conservatism for robust constraint a4 [Eq. (25)].

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