ISAV2014-4133

Size-Dependent Flexural Wave Propagation and Free Vibration of Functionally Graded Rayleigh's Micro-Beams Based on the Modified Couple Stress Theory

Amir R. Askari[1], Masoud Tahani [2], Mahmoud Shariati [3]

[1]PhD Student, Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran, a.r.askari@stu-mail.um.ac.ir and amaskari@gmail.com.

[2]Professor, Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran, mtahani@um.ac.ir.

[3] Professor, Department of Mechanical Engineering, Ferdowsi University of Mashhad, Mashhad, Iran, mshariati44@gmail.com.

Abstract

The objective of present paper is to provide closed-form solutions for the size-dependent flexural wave propagation speed and natural frequencies of micro-beams made of functionally graded materials. For this aim, the modified couple stress theory (MCST) together with the Rayleigh's beam model is considered and the size-dependent equation of motion which accounts for the effects of rotary inertia, axial residual and couple stress components as well as through-thickness variation of the material properties is derived using the Hamilton's principle. Utilizing the derived equation of motion, closed-form exact expressions for flexural wave propagation speed of the system as well as its natural frequencies are presented. A detailed parametric study is also conducted to emphasis on the effects of couple stress components, rotary inertia and through-thickness variation of the material properties on both flexural wave propagation speed and natural frequencies of the system. The results show that accounting for couple stress components results in increasing both natural frequencies and flexural wave propagation speed, while the rotary inertia reduces both of them. It is found that the effects of couple stress components and rotary inertia on the wave propagation speed increases with an increase of the wave source frequency. Furthermore, it is observed that higher natural frequencies of the system will be affected more than the lower ones if the couple stress components and rotary inertia effects are taken into account.

Iranian Society of Acoustics and Vibration
4th ISAV2014 International Conference on Acoustics and Vibration
10-11 Dec 2014 Tehran - Iran
Iran University of Science & Technology

Size-Dependent Flexural Wave Propagation and Free Vibration of Functionally Graded Rayleigh's Micro-Beams Based on the Modified Couple Stress Theory

Amir R. Askari [a*], Masoud Tahani [b] and Mahmoud Shariati [b]

[a] *PhD Student, Department of Mechanical Engineering, Ferdowsi University of Mashhad, 91775-1111, Mashhad, Iran.*

[b] *Professor, Department of Mechanical Engineering, Ferdowsi University of Mashhad, 91775-1111, Mashhad, Iran.*

* *Corresponding author e-mail: a.r.askari@stu-mail.um.ac.ir and amaskari @ gmail.com*

Abstract

The objective of present paper is to provide closed-form solutions for the size-dependent flexural wave propagation speed and natural frequencies of micro-beams made of functionally graded materials. For this aim, the modified couple stress theory (MCST) together with the Rayleigh's beam model is considered and the size-dependent equation of motion which accounts for the effects of rotary inertia, axial residual and couple stress components as well as through-thickness variation of the material properties is derived using the Hamilton's principle. Utilizing the derived equation of motion, closed-form exact expressions for flexural wave propagation speed of the system as well as its natural frequencies are presented. A detailed parametric study is also conducted to emphasis on the effects of couple stress components, rotary inertia and through-thickness variation of the material properties on both flexural wave propagation speed and natural frequencies of the system. The results show that accounting for couple stress components results in increasing both natural frequencies and flexural wave propagation speed, while the rotary inertia reduces both of them. It is found that the effects of couple stress components and rotary inertia on the wave propagation speed increases with an increase of the wave source frequency. Furthermore, it is observed that higher natural frequencies of the system will be affected more than the lower ones if the couple stress components and rotary inertia effects are taken into account.

Keywords: Modified couple stress theory; Rayleigh's beam model; Functionally graded materials; Vibrational analysis.

1. Introduction

Structures at micron and sub-micron scales are frequently used nowadays. Resonant micro-sensors as one of the largest category of micro-systems are extensively utilized in different applications such as signal filtering, and chemical and mass sensing [1]. The building blocks of these sensors are mechanical micro-structures which can be modelled as micro-beams whose vibrational characteristics are employed as the sensing method [2]. Therefore, extracting the vibrational characteristics of micro-beams is essential and quiet useful in designing such sensors.

Recently, variety of experiments showed that the material mechanical behavior in small scales is size-dependent [3, 4]. Size-dependent behavior is an intrinsic property of certain materials, which emerges when the characteristic size, e.g. the diameter or the thickness is comparable to the material length scale parameter which can be determined experimentally [3-5].

The classical continuum mechanics cannot predict the size-dependent behavior of materials which occurs in micron and sub-micron scale structures. In 1960s Toupin [6], Koiter [7] and Mindlin [8] proposed the classical couple stress elasticity theory based on the Cosserat continuum mechanics [9]. Based on this theory, beside the classical stress tensor, the couple stress components should be included to describe the manner of media. In comparison to the classical continuum mechanics, the couple stress theory has two additional parameters (high-order material length scales) other than two classical Lame's constants in constitutive equations for isotropic elastic materials. Recently, a modified version of this theory has been elaborated by Yang et al., in which constitutive equations involve only one additional internal material length scale parameter besides two classical material constants [10].

The modified couple stress theory (MCST) has been successfully utilized to predict mechanical behavior of micro-structures in recent years. Park and Gao [11] showed that the bending rigidity predicted by the MCST is larger than that calculated by the classical theory (CT) and the difference between the deflections predicted by these two models is significant when the beam thickness is small. Kong et al. [12] investigated the size effect on natural frequencies of the Euler-Bernoulli micro-beams. Ke et al. [13] studied the thermal effect on the free vibration and buckling of micro-beams using the MCST and Timoshenko beam theory through the differential quadrature method (DQM).

To achieve all material and economical requirements for micro-structures, functionally graded materials (FGMs) are extensively employed in the past years [14, 15]. Therefore, many researchers are motivated to develop size-dependent mechanical models for functionally graded (FG) structures at micron and sub-micron scales using the MCST. Asghari et al. [16] presented some closed-form solutions for static bending and free vibration analysis of size-dependent FG micro-beams using the Euler-Bernoulli beam model. Reddy [17] developed size-dependent non-linear Euler-Bernoulli and Timoshenko beam theories for micro-beams made of FGM with two material phases based on the MCST. He also presented some analytic solutions as well as finite element models for investigating free vibration, bending, buckling and post-buckling responses in such systems. Ke et al. [18] investigated non-linear free vibration of FG Timoshenko micro-beams using the MCST through iterative DQM.

Although many researchers have dealt with the mechanical behavior of micro-beams, the research effort devoted to wave propagation analysis of these structures are very limited. The objective of present work is to extract the size-dependent flexural wave propagation speed as well as the natural frequencies of FG micro-beams on the basis of the MCST. To do so, size-dependent FG Rayleigh's beam model is developed using the Hamilton's principle and flexural wave propagation speed of the system as well as its natural frequencies are extracted through closed-form exact expressions. A detailed parametric study is also conducted to show the effects of couple stress components, rotary inertia and through-thickness variation of the material properties on both wave propagation speed and natural frequencies of the system.

2. The MCST formulation for FG Rayleigh's micro-beams

According to the MCST presented by Yang et al. [10] in 2002, both strain tensor (conjugated with stress tensor) and curvature tensor (conjugated with couple stress tensor) are included in the strain energy density. Based on this theory, the strain energy U in a deformed isotropic linear elastic material occupying region Π is given by

$$U = \frac{1}{2}\int_{\Pi}\left(\vec{\vec{\boldsymbol{\sigma}}} : \vec{\vec{\boldsymbol{\varepsilon}}} + \vec{\vec{\mathbf{m}}} : \vec{\vec{\boldsymbol{\chi}}}\right) \mathrm{d}\Pi \tag{1}$$

where $\vec{\vec{\boldsymbol{\sigma}}}$, $\vec{\vec{\boldsymbol{\varepsilon}}}$, $\vec{\vec{\mathbf{m}}}$ and $\vec{\vec{\boldsymbol{\chi}}}$ are the Cauchy stress, strain, deviatoric part of couple stress and symmetric curvature tensors, respectively. These tensors for cases with small slopes and deflections can be written as

$$\vec{\vec{\boldsymbol{\sigma}}} = \lambda \operatorname{tr}\left(\vec{\vec{\boldsymbol{\varepsilon}}}\right)\vec{\vec{\mathbf{I}}} + 2\mu\vec{\vec{\boldsymbol{\varepsilon}}}\,,\ \vec{\vec{\boldsymbol{\varepsilon}}} = \frac{1}{2}\left[\nabla\mathbf{u} + \left(\nabla\mathbf{u}\right)^{\mathrm{T}}\right],\ \vec{\vec{\mathbf{m}}} = 2l^2\mu\,\vec{\vec{\boldsymbol{\chi}}}\,,\ \vec{\vec{\boldsymbol{\chi}}} = \frac{1}{2}\left[\nabla\boldsymbol{\theta} + \left(\nabla\boldsymbol{\theta}\right)^{\mathrm{T}}\right] \tag{2}$$

where $\nabla = \mathbf{e}_x\partial/\partial x + \mathbf{e}_y\partial/\partial y + \mathbf{e}_z\partial/\partial z$, $\mathbf{u}$ is the displacement vector, λ and μ are Lame's constants (μ is also known as shear modulus), l is the internal material length scale parameter, $\vec{\vec{\mathbf{I}}}$ is the identity tensor and $\boldsymbol{\theta}$ is the rotation vector obtained as $\boldsymbol{\theta} = 1/2\left(\operatorname{curl}\mathbf{u}\right)$.

According to the basic hypothesis of the Rayleigh's beam model which accounts for the effect of inertia due to both axial and transverse displacements of the beam [19], the displacement field $(\tilde{u},\tilde{w})$ of an arbitrary point on the micro-beam can be expressed as

$$\tilde{u} = -z\,\partial w/\partial x\,,\ \tilde{w} = w\left(x,t\right) \tag{3}$$

where w is the transverse displacement of a point on the mid-plane of the micro-beam (i.e. $z = 0$).

Figure 1 shows a schematic of a micro-beam made of a non-homogenous FGM with two distinct material phases. The thickness, length and width of the micro-beam are h, L and b, respectively. It is assumed that the properties of the beam vary continuously through its thickness according to the power-law [17] as

$$\Theta\left(z\right) = \Theta_1 + \left[\mathrm{VF}_1\left(z\right)\right]^m\left(\Theta_2 - \Theta_1\right),\ \mathrm{VF}_1\left(z\right) = \left(\frac{2z+h}{2h}\right)^m \tag{4}$$

where $\mathrm{VF}_1\left(z\right)$ is the volume fraction of the material phase 1, m is the power-law index and $\Theta\left(z\right)$ can be considered as any property of the beam such as its Young modulus ($E\left(z\right)$), shear modulus ($\mu\left(z\right)$), density ($\rho\left(z\right)$) and material length scale parameter ($l\left(z\right)$). Also, parameters with sub-scripts 1 and 2 are referred to the properties of the material used in the bottom and top surfaces of the beam, respectively. It is noteworthy that, by setting $m = 0$, the present formulation would be simplified for homogeneous micro-beams made of material phase 2. Also the volume fraction of material phase 2 can be written as $\mathrm{VF}_2\left(z\right) = 1 - \mathrm{VF}_1\left(z\right)$ [17].

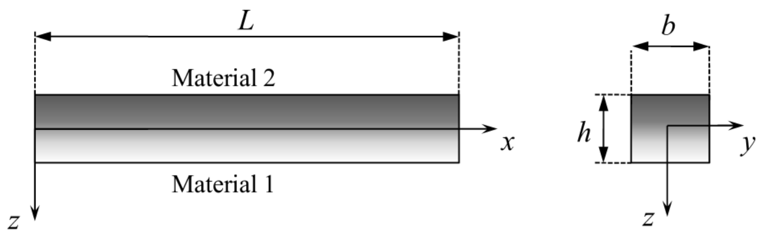

Figure 1. Schematic of a FG micro-beam.

The strain energy expression for the displacement field introduced in Eq. (3) takes the form

$$U = \frac{1}{2}\int_V \left(\sigma_x \varepsilon_x + 2m_{xy}\chi_{xy}\right)dV = \frac{1}{2}\left[\left(EI\right)_{\text{eq}} + \left(\mu A l^2\right)_{\text{eq}}\right]\int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{5}$$

where $\left(EI\right)_{\text{eq}}$ and $\left(\mu A l^2\right)_{\text{eq}}$ are

$$\left(EI\right)_{\text{eq}} = \int_A z^2 E\left(z\right)dA,\ \left(\mu A l^2\right)_{\text{eq}} = \frac{1}{2\left(1+\nu\right)}\int_A l^2\left(z\right)E\left(z\right)dA \tag{6}$$

in which ν is the Poisson's ratio of the beam assumed to be constant for the FGM [17]. Also, A refers to the cross sectional area of the micro-beam.

The kinetic energy of the micro-beam takes the form

$$T = \frac{1}{2}\int_V \left\{\rho\left(z\right)\left[\left(\frac{\partial \tilde{u}}{\partial t}\right)^2 + \left(\frac{\partial \tilde{w}}{\partial t}\right)^2\right]\right\}dV = \frac{1}{2}\left(\rho I\right)_{\text{eq}}\int_0^L \left(\frac{\partial^2 w}{\partial x\,\partial t}\right)^2 dx + \frac{1}{2}\left(\rho A\right)_{\text{eq}}\int_0^L \left(\frac{\partial w}{\partial t}\right)^2 dx \tag{7}$$

where $\left(\rho A\right)_{\text{eq}}$ and $\left(\rho I\right)_{\text{eq}}$ are

$$\left(\rho A\right)_{\text{eq}} = \int_A \rho\left(z\right)dA,\ \left(\rho I\right)_{\text{eq}} = \int_A z^2\rho\left(z\right)dA \tag{8}$$

Due to the mismatch of both thermal expansion coefficient and crystal lattice period between substrate and micro-beam film which is un-avoidable in surface micro-machining techniques, a resultant axial force $N = \sigma_r bh$ is applied to the micro-beam [20], where σ_r represents the axial residual stress. The external virtual work done by the axial residual force N can be expressed as [19]

$$\delta W_{\text{ext}} = -\int_0^L N\,\frac{\partial w}{\partial x}\,\delta\frac{\partial w}{\partial x}\,dx \tag{9}$$

The Hamilton principle for an elastic body states $\int_{t_i}^{t_f}\left(\delta T - \delta U + \delta W_{\text{ext}}\right)dt = 0$ [21]. By substitution of Eqs. (5), (7) and (9) into the Hamilton's principle, the equation of motion is obtained as

$$\left(\rho A\right)_{\text{eq}}\frac{\partial^2 w}{\partial t^2} - \left(\rho I\right)_{\text{eq}}\frac{\partial^4 w}{\partial x^2\partial t^2} + \left[\left(EI\right)_{\text{eq}} + \left(\mu A l^2\right)_{\text{eq}}\right]\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} = 0 \tag{10}$$

The corresponding boundary conditions are also determined as

$$\delta w = 0 \ \text{ or } \ \left[\left(EI\right)_{\text{eq}} + \left(\mu A l^2\right)_{\text{eq}}\right]\left(\frac{\partial^3 w}{\partial x^3}\right) - \left(\rho I\right)_{\text{eq}}\frac{\partial^3 w}{\partial x\,\partial t^2} = 0 \qquad \text{at} \quad x = 0, L \tag{11a}$$

$$\delta\frac{\partial w}{\partial x} = 0 \ \text{ or } \ \left[\left(EI\right)_{\text{eq}} + \left(\mu A l^2\right)_{\text{eq}}\right]\left(\frac{\partial^2 w}{\partial x^2}\right) = 0 \qquad \text{at} \quad x = 0, L \tag{11b}$$

It is to be noted that, for extracting the wave propagation speed and natural frequencies of the system, no initial conditions are required.

Eq. (10) and its corresponding boundary conditions represent the size-dependent governing equation of motion for a FG Rayleigh's micro-beam based on the MCST. This equation will be utilized in the next two sections for extracting the size-depended flexural wave propagation speed and natural frequencies in FG Rayleigh's micro-beams.

3. Extracting the flexural wave propagation speed

For extracting the flexural wave propagation characteristics, the harmonic wave function for transverse deflection $w(x,t)$ can be expressed as [22]

$$w(x,t)=W\ \exp\left[-2\pi i\left(\frac{x}{\lambda}-\frac{t}{T}\right)\right] \tag{12}$$

where T denotes the period of the wave which is equal to that of the wave source and λ is the wavelength. Also $i=\sqrt{-1}$. Upon substitution from Eq. (12) into Eq. (10), the wavelength can be obtained as

$$\lambda=2\pi\sqrt{\frac{2\left[(EI)_{\text{eq}}+\left(\mu A l^2\right)_{\text{eq}}\right]}{\left((\rho I)_{\text{eq}}(2\pi f)^2-N\right)+\sqrt{\left((\rho I)_{\text{eq}}(2\pi f)^2-N\right)^2+4(\rho A)_{\text{eq}}(2\pi f)^2\left[(EI)_{\text{eq}}+\left(\mu A l^2\right)_{\text{eq}}\right]}}} \tag{13}$$

where f is the frequency of the wave source measured in Hz. The flexural wave propagation speed can also be extracted as $C_p=f\lambda$. According to Eq. (13), both wavelength and wave propagation speed depend to the frequency of the wave source. It is noteworthy that by neglecting the effects of size, through-thickness variation of material properties and axial residual force, Eq. (13) would be simplified to that of simple classic Rayleigh's beam model [22].

4. Obtaining the natural frequencies

By assuming $w(x,t)=\varphi_n(x)\exp(i\omega_n t)$, the eigenvalue problem associated with Eq. (10) will be obtained as

$$\left[(EI)_{\text{eq}}+\left(\mu A l^2\right)_{\text{eq}}\right]\frac{\partial^4\varphi_n}{\partial x^4}+(\rho I)_{\text{eq}}\,\omega_n^2\frac{\partial^2\varphi_n}{\partial x^2}-\omega_n^2(\rho A)_{\text{eq}}\,\varphi_n-N\frac{\partial^2\varphi_n}{\partial x^2}=0 \tag{14}$$

where ω_n and φ_n denote the nth natural frequency and its associated mode-shape of the micro-beam, respectively. The solution of Eq. (14), can be expressed as

$$\varphi_n(x)=\sum_{i=1}^{4}C_i\ \exp(s_i x) \tag{15}$$

where s_1, s_2, s_3 and s_4 are the roots of the characteristic equation associated with Eq. (14). Also the constants C_1, C_2, C_3 and C_4 as well as the natural frequencies of the system can be determined from the boundary conditions. Herein for the sake of brevity, we only investigate the simply supported boundary conditions. For this case, the nth natural frequency of the system can be easily obtained as

$$\omega_n=\sqrt{\frac{(n\pi/L)^4\left[(EI)_{\text{eq}}+\left(\mu A l^2\right)_{\text{eq}}\right]+N\,(n\pi/L)^2}{(\rho A)_{\text{eq}}+(\rho I)_{\text{eq}}(n\pi/L)^2}} \tag{16}$$

As it is seen from Eq. (16), by neglecting the effects of size, through-thickness variation of material properties and axial residual force, this expression would be simplified to that of simple classic Rayleigh's beam model [19].

5. Results and discussion

Herein, using the derived equations in the previous sections, some numerical case studies are presented to delineate the effects of size, rotary inertia and through-thickness variation of the material properties on both flexural wave propagation speed and natural frequencies of the system. It is assumed that the micro-beam is made of FG poly-SiAg structure where silver (Ag) and silicon (Si) are used as material phase 1 and material phase 2, respectively. The material properties of both silicon and silver are listed in Table 1. Also, it is assumed that there exists no axial residual stress in the micro-beam and the geometric properties of the system are set to: $L = 100\ \mu\text{m}$, $b = 10\ \mu\text{m}$ and $h = 1\ \mu\text{m}$.

Table 1. Material properties of silver and silicon.

Material	E (GPa)	ν	ρ (kg/m^3)	l (μm)
1. Silver (Ag)	83	0.37	10490	6.233 [23]
2. Silicon (Si)	169	0.33	2331	0.592 [24]

5.1 Flexural wave propagation speed

Figure 2 shows the effects of rotary inertia and couple stress components on flexural wave propagation speed in a micro-beam made of silicon. According to Figure 2(a), in which the couple stress components are neglected, ignoring the rotary inertia effect increases the values of the wave propagation speed and produces very in-accurate results especially for cases with high-frequency of the wave source. It is to be noted that, if the effect of rotary inertia has been taken into account, by increasing the frequency of the wave source, the values of the wave propagation speed (i.e. C_p) may lead to the speed of the sound in the medium (i.e. $C_0 = \sqrt{E/\rho}$). However, this value leads to infinity if the rotary inertia effect is neglected; which is physically un-acceptable [22]. Therefore, it is essential to account for the effect of rotary inertia especially for high-frequency excited systems.

The effect of couple stress components is also investigated in Figure 2(b). Based on the results of this figure, neglecting the effect of couple stress components decreases the wave propagation speed. The error of ignoring this effect for wave source frequencies less than 10^9 Hz is about 21%, while this error may increase up to the values about 37% for high-frequency excited systems. It is to be mentioned here that, due to the wide range of wave propagation speed and frequencies of the wave source, we have chosen to use the logarithmic scales for Figures 2(a) and 2(b) just for the sake of improving the quality of presentation.

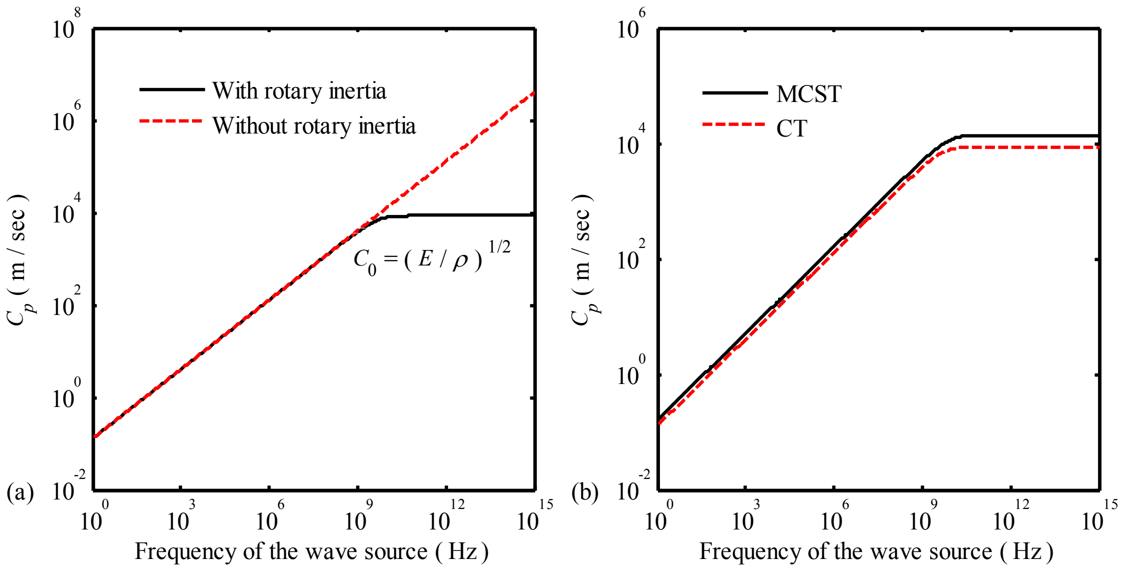

Figure 2. Effects of rotary inertia and couple stress components on the flexural wave propagation speed.

Table 2 investigates the effect of through-thickness variation of the material properties on the wave propagation speed using both CT and MCST. It is to be noted that the value of the wave source frequency is also set to 1 MHz for producing the results of this table. As it is seen from this table, by increasing the values of the power-law index, the CT results are decreased, while the MCST findings are increased. This is due to the fact that, by increasing the value of the power-law index, the properties of the FGM become more and more similar to those of silver which will be stiffer than silicon in such dimensions. Because the large material length scale parameter of the silver plays a more crucial role than its Young modulus in the dimensions of present micro-beam. This is the concept of size-dependent behavior at micron and sub-micron scales which must be taken into account for designing micro-structures.

Table 2. Flexural wave propagation speed (m/sec) obtained by both CT and MCST versus power-law index.

Theory	silicon ($m = 0$)	$m = 0.1$	$m = 1$	$m = 10$	silver ($m = \infty$)
CT	124.2542	113.5363	89.6626	76.1817	71.4168
MCST	157.1371	194.4342	243.9851	257.8768	258.3494

5.2 Natural frequencies

Figure 3 displays the effects of rotary inertia and couple stress components on the first hundred natural frequencies of present silicon micro-beam. According to the results of this figure, these effects play more crucial roles on higher natural frequencies than the lower ones. Therefore, it is more essential to account for these effects for high-frequency excited systems.

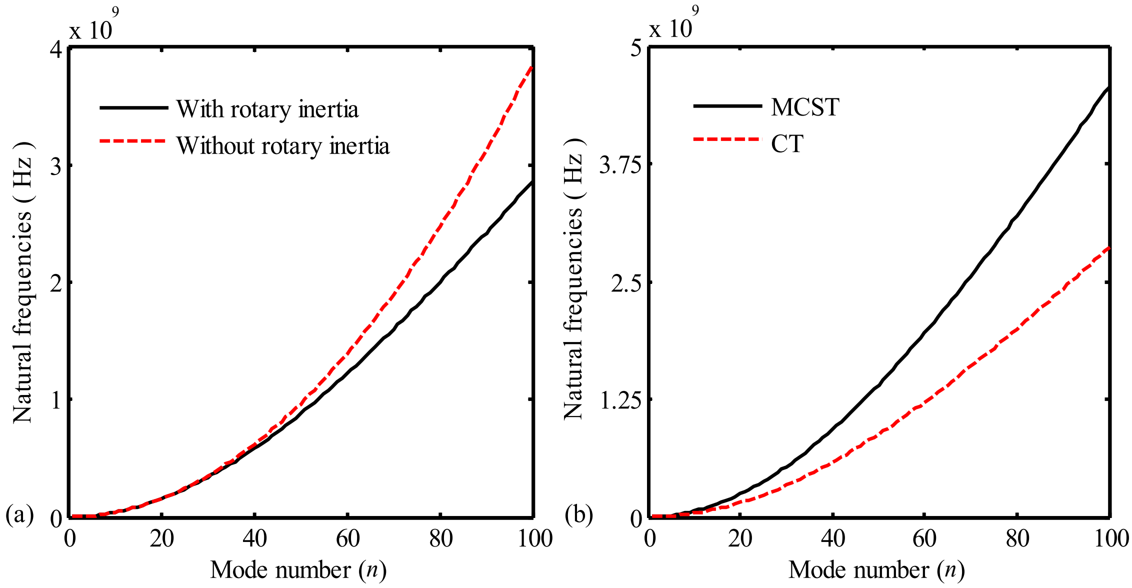

Figure 2. Effects of rotary inertia and couple stress components on the first hundred natural frequencies of present simply supported silicon micro-beam.

Table 3 represents the effect of through-thickness variation of the material properties on the fundamental frequency of preset simply supported micro-beam using both CT and MCST. As it is seen from this table, by increasing the values of the power-law index, the results of FG micro-beam become more and more similar to those of micro-beam made of silver. Furthermore, the difference between CT and MCST results is also increased by increasing the values of the power-law index. This is due to the fact that, the effect of couple stress components increases with an increase of the value of the internal material length scale parameter [11, 12].

Table 3. Fundamental natural frequency (kHz) obtained by both CT and MCST versus power-law index.

Theory	silicon ($m = 0$)	$m = 0.1$	$m = 1$	$m = 10$	silver ($m = \infty$)
CT	386.0028	322.2947	201.0175	145.1232	127.5448
MCST	617.3171	945.1193	1488.1986	1662.4864	1668.5824

6. Conclusions

Extracting the size-dependent flexural wave propagation speed and natural frequencies of FG Rayleigh's micro-beams was the main goal of the present paper. For this object, the MCST together with the Rayleigh's beam model were employed and the governing equation of motion was derived through the Hamilton's principle. Using the derived equation of motion, the flexural wave propagation speed as well as natural frequencies of the system was determined through closed-form exact expressions. A detailed parametric study was also conducted to show the effects of couple stress components, rotary inertia and through-thickness variation of the material properties on both wave propagation speed and natural frequencies of the system. The results showed that neglecting the effects of couple stress components and rotary inertia decreases the bending rigidity and inertia of the system, respectively. Also, it was found that accounting for these effects at micron and sub-micron scales would be more essential for the case of high-frequency excited systems.

REFERENCES

[1] G. M. Rebeiz, *RF MEMS: theory, design, and technology,* John Wiley & Sons, NewYork, 2003.

[2] R. C. Batra, M. Porfiri, and D. Spinello, "Review of modeling electrostatically actuated microelectromechanical systems", *Smart Mater. Struct.* 16, R23-31 (2007).

[3] N. A. Fleck, G. M. Muller, M. F. Ashby, and J. W. Hutchinson, "Strain gradient plasticity: theory and experiment", *Acta Metall. Mater.* 42, 475–487 (1994).

[4] J. S. Stolken, and A. G. Evans, "Microbend test method for measuring the plasticity length scale", *Acta Mater.* 46, 5109-5115 (1998).

[5] M. R. Begley, and J. W. Hutchinson, "The mechanics of size-dependent indentation", *J. Mech. Phys. Solids* 46, 2049-2068 (1998).

[6] R. A. Toupin, "Elastic materials with couple-stresses", *Arch. Ration. Mech. Anal.* 11, 385-414 (1962).

[7] W. T. Koiter, "Couple-stresses in the theory of elasticity: I and II", *Proc, K. Ned. Akad. Wet. B.* 67(1), 17-44 (1964).

[8] R. D. Mindlin, "Micro-structure in linear elasticity", *Arch. Ration. Mech. An.* 16(1), 51-78 (1964).

[9] E. Cosserat, and F. Cosserat, *Theorie des Corps Deformables,* Hermann et Fils, Paris, 1909.

[10] F. Yang, A. C. M. Chong, D. C. C. Lam, and P. Tong, "Couple stress based strain gradient theory for elasticity", *Int. J. Solids Struct.* 39, 2731-2743 (2002).

[11] S. K. Park, and X. L. Gao, "Bernoulli-Euler beam model based on a modified couple stress theory", *J. Micromech. Microeng.* 16, 2355-2359 (2006).

[12] S. Kong, S. Zhou, Z. Nie, and K. Wang, "The size-dependent natural frequency of Bernoulli-Euler micro-beams", *Int. J. Eng. Sci.* 46, 427-437 (2008).

[13] L. L. Ke, Y. S. Wang, and Z. D. Wang, "Thermal effect on free vibration and buckling of size-dependent microbeams", *Physica E* 43, 1387-1393 (2011).

[14] A. Witvrouw, and A. Mehta, "The use of functionally graded poly-SiGe layers for MEMS applications", *Mater. Sci. Forum* 492-493, 255-260 (2005).

[15] M. Gromova, A. Mehta, K. Baert, and A. Witvrouw, "Characterization and strain gradient optimization of PECVD poly-SiGe layers for MEMS applications", *Sensor Actuat. A-Phys.* 130–131, 403-410 (2006).

[16] M. Asghari, M. T. Ahmadian, M. H. Kahrobaiyan, and M. Rahaeifard, "On the size-dependent behavior of functionally graded micro-beams", *Mater. Design* 31(5), 2324-2329 (2010).

[17] J. N. Reddy, "Microstructure-dependent couple stress theories of functionally graded beams", *J. Mech. Phys. Solids* 59, 2382-2399 (2011).

[18] L. L. Ke, Y. S. Wang, J. Yang, and S. Kitipornchai, "Nonlinear free vibration of size-dependent functionally graded microbeams", *Int. J. Eng. Sci.* 50, 256-267 (2012).

[19] S. S. Rao, *Vibration of Continuous Systems,* John Wiley & Sons, USA, 2007.

[20] J. Qian, C. Liu, D. C. Zhang, and Y. P. Zhao, "Residual Stresses in Microelectromechanical System", *Journal of Mechanical Strength* 23, 393-401 (2001).

[21] J. N. Reddy, *Energy principles and variational methods in applied mechanics,* John Wiley & Sons, New York, 2002.

[22] W. Johnson, *Impact Strength of Materials,* Edward Arnold, London, 1972.

[23] M. Zhao, W. S. Slaughter, M. Li, and S. X. Mao, "Material-length-scale-controlled nanoindentation size effects due to strain gradient plasticity", *Acta Mater.* 51, 4461-4469 (2003).

[24] M. Rahaeifard, M. H. Kahrobaiyan, M. Asghari, and M. T. Ahmadian, "Static pull-in analysis of microcantilevers based on the modified couple stress theory", *Sensor Actuat. A-Phys.* 171, 370-374 (2011).