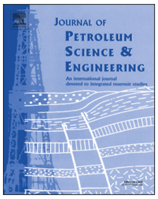
Journal of Petroleum Science and Engineering 126 (2015) 78–82

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol



Empirical relations between strength and static and dynamic elastic properties of Asmari and Sarvak limestones, two main oil reservoirs in Iran

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ARTICLE INFO

Article history:
Received 11 February 2014
Accepted 9 December 2014
Available online 24 December 2014

Keywords:
Limestone
Reservoir
UCS
Young's modulus
Core test

ABSTRACT

Asmari and Sarvak limestone are two main oil producer formations in Iran and the Middle East. Perception and optimal utilization of these reservoirs will have a significant impact on the economy of the petroleum industries. Geomechanical modelling of oil reservoirs are widely used in optimum drilling, production and reservoir compaction. Hence, uniaxial compressive strength (UCS) and static Young' modulus (E_S) are the most essential parameters for any reservoir geomechanical modelling. However, information on the value of UCS and E_S along the well length is often discontinuous and limited to cross well with the core. Therefore, dynamic Young's modulus (E_d) determined from open hole log data such as density (ρ) and compressional and shear wave velocities could results in continuous estimation of elastic properties of the well length. Nevertheless, static parameters are more reliable than the dynamic parameters and they are widely accepted by geomechanics community around the world. Therefore, finding a valid correlation between static and dynamic parameters could result in a continuous and more reliable knowledge on elastic parameters. In this study, the uniaxial compressive strength and ultrasonic tests were carried out on 45 Asmari and Sarvak limestone core specimen. Then, local correlations were established between dynamic and static measurements. Suggested equations were compared with previous relations. Moreover, the sensitivity of the suggested relations to crushed and compacted zones were investigated. These expressions are utilized for future wellbore stability analysis, fracture detections and hydraulic fracturing studies across many oil and gas fields in the country.

1. Introduction

The uniaxial compressive strength (UCS) and elastic properties of rocks such as Young's modulus and Poisson's ratio are widely used to estimate in situ stresses, wellbore stability analysis, reservoir compaction survey, and prediction of optimum drilling mud pressure (Chang et al., 2006; Abdulraheem et al., 2009). Elastic properties of rocks are measured from dynamic and static methods, while UCS is given only from the static method (Al-Shayea, 2004). In the static method, uniaxial or triaxial stresses are gently applied on the core specimens until failure occurred. The stress-deformation curves are traced, and the UCS and static elastic properties of the rock are obtained (Jaeger et al., 2007). In the dynamic method, compressional and shear wave velocities (V_P and V_S, respectively) may be measured in the laboratory or in the field, and the elastic properties are determined accordingly. The elastic parameters obtained from static and dynamic methods are often different (Chang et al., 2006). For example, the static Young's modulus (E_S) is about 3 times smaller than the dynamic Young's modulus (E_d). This is because static measurements are highly affected by pores and cracks (Fjær et al., 2008). Other parameters, such as pore pressure, type of cement, and stress-strain affect the difference between static and dynamic parameters (Lama and Vutukuri, 1978). UCS and static elastic parameters are more realistic than the dynamic parameters and they are widely used in geomechanical modelling (Lacy, 1997). However, measurement of static elastic parameters is more difficult than the corresponding dynamic parameters. This is because the static tests are conducted on the good quality rock core specimens that may not be available in all wells. While, dynamic elastic parameters may be detected using ultrasonic tests on core specimens or acquired from well log data that are available in the oil industry. Therefore, determination of the empirical relationship between dynamic and static parameters is essential for continuous and reliable prediction of mechanical properties of rocks along a wellbore (Chang et al., 2006). Mechanical properties of rocks depend

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http://dx.doi.org/10.1016/j.petrol.2014.12.010
0920-4105/

on a number of variables including pore size and distribution, degree of cementation, and size and shape of aggregates. Therefore, determination of a unique relationship between static and dynamic parameters is very difficult (Mavko et al., 2009). This is the reason why many equations were suggested by previous researchers.

The use of empirical relations for determination of the rock mechanical parameters based on open hole logs is back to 1950. Wyllie et al. (1963) suggested an empirical relationship between porosity and V_P, namely Time-Average equation. Since, the porosity and density have a direct impact on the UCS, Smorodinov et al. (1970) established another empirical relation between UCS and density/porosity. Militzer and Stoll (1973) and Golubev and Robinovich (1976) reported Eqs. (1) and (2) for prediction of UCS from V_P in limestone.

$$\mathrm{UCS} = 2.45V_p^{1.82} \tag{1}$$

$$\log\ \mathrm{UCS} = 0.358V_p + 0.283 \tag{2}$$

where V_P is in km/s and UCS is in MPa. Savich (1984) determined a logarithmic relationship between E_d and E_S (Eq. (3)), and declared that it is more accurate than a linear equation.

$$\log\ E_s = A_0 + A_1\ \log\ E_d \tag{3}$$

where, A_0 and A_1 are constant coefficients. Van Heerden (1978) studied on different rock types that their Young's modulus were in the range of 7–150 GPa and suggested Eq. (4) between E_d and E_S.

$$E_s = aE_d^b \tag{4}$$

where a and b are in the ranges of 0.097–0.152 and 1.388–1.485, respectively. Table 1 lists some empirical relations for prediction of UCS and E_S from dynamic data.

In this study, we carried out uniaxial compressive strength and ultrasonic tests on Asmari and Sarvak limestone core specimens, obtained from an oil well in the southwest of Iran. Then, empirical relationships were derived between UCS and E_S with the E_d and V_P. Using these equations and well log data, the logs of UCS and E_S can be plotted for these carbonate formations.

2. Experimental procedure and results

In this study, 45 limestone core specimens of Asmari and Sarvak formations were obtained. Hydrocarbon content was removed from specimens using soxhlet apparatus. Ultrasonic testing for measurement of V_P and V_S was performed according to ASTM D2845 standard (Fig. 1). Then, E_d was calculated by using Eq. (20) for each specimen (Goodman, 1989).

$$E_d = \rho \times V_s^2 \frac{\left(3V_p^2 - 4V_s^2\right)}{\left(V_p^2 - V_s^2\right)} \tag{20}$$

where ρ is the density (g/cm^3), V_P and V_S are in km/s, and E_d is in GPa. Furthermore, uniaxial compressive strength test was performed according to ASTM D2938 and D3148 standards (Goodman, 1989). In this experiment, LVDTs were used around the sample to measure the axial and lateral strains and the static load was gently applied (Fig. 2). Then, stress-deformation curve was recorded and UCS and E_S was determined. The results of ultrasonic and uniaxial compressive strength tests on the studied specimens are shown in Table 2.

3. Discussion

3.1. Static Young's modulus prediction

A number of investigators suggested empirical relations between E_S and E_d e.g. Eissa and Kazi (1988), Lacy (1997), Ameen et al. (2009) as illustrated in Fig. 3. It is well understood that provided correlations are based on local information, and are not capable of covering a wide range of lithologies. In this regard, development of local correlations for specific lithology is always privileged to general correlations. These

Fig. 1. Ultrasonic testing apparatus for determining of V_P and V_S.

Table 1
Some empirical relations between UCS and E_S with dynamic data.

Eq. nos.	Lithology	Equation	Reference
(5)	Igneous and Metamorphic	$E_S = 1.263E_d - 29.5$	King (1983)
(6)	Igneous and Metamorphic	$\sigma_c = 4.31(E_d/10)^{1.705}$	King (1983)
(7)	Sedimentary	$E_S = 0.74E_d - 0.82$	Eissa and Kazi (1988)
(8)	Sedimentary	$\log_{10}E_S = 0.02 + 0.7\log_{10}\rho E_d$	Eissa and Kazi (1988)
(9)	Soft Rocks	$\sigma_c = 2.28 + 4.0189E_S$	Bradford et al. (1988)
(10)	Sedimentary	$\sigma_c = 0.278E_s^2 + 2.458E_sE_s$	Lacy (1997)
(11)	Sedimentary	$E_s = 0.018E_d^2 + 0.422E_d$	Lacy (1997)
(12)	Hard Rocks (E_S > 15 GPa)	$E_S = 1.153E_d - 15.2$	Nur and Wang (1999)
(13)	Shale	$\sigma_c = 0.77V_P^{2.93}$	Horsrud (2001)
(14)	Shale	$E_S = 0.076V_P^{3.23}$	Horsrud (2001)
(15)	Mudstone	$E_S = 0.103\sigma_c^{1.086}$	Lashkaripour (2002)
(16)	Shale	$E_S = 0.0158E_d^{2.74}$	Ohen (2003)
(17)	Different Rocks	$\sigma_c = 2.304V_P^{2.43}$	Kılıç and Teymen (2008)
(18)	Limestone	$E_S = 0.541E_d + 12.852$*	Ameen et al. (2009)
(19)	Limestone	$\sigma_c = 2.94\left(E_S^{0.83}/n^{0.088}\right)$**	Asef and Farrokhrouz (2010)

* At 27.6 MPa.

** n Is porosity.

Fig. 2. Uniaxial compressive test machine for determination of UCS and E_s.

equations provide more confidence on future geomechanical models for reservoirs. Based on this need, a set of experimental studies carried out on core plugs obtained from Sarvak and Asmari limestone formations (Table 2). Then, new equation developed and compared with other suggested correlations. Fig. 3 clearly illustrates scatter of data points and the trend of some previously suggested equations. It is very clear that the previous suggested equations are unable to predict E_S from E_d for the studied limestone (Fig. 3). Eq. (21) is derived to relate E_S to E_d for Asmari and Sarvak limestone.

$$E_s = 0.014E_d^{1.96}, \quad R^2 = 0.87 \tag{21}$$

where E_d and E_S are in GPa.

Well log data were utilized to acquire V_P and V_S, and E_d is determined from Eq. (20). In case of V_S is not available, E_S could be predicted from V_P directly. Horsrud (2001) reported a nonlinear correlation between E_S and V_P for shale formations. Eq. (22) is an empirical relationship between E_S and V_P for Sarvak and Asmari limestone, as shown in Fig. 4.

$$E_s = 0.169V_p^{3.324}, \quad R^2 = 0.90 \tag{22}$$

3.2. Prediction of UCS

Rock strength is presented hereby as UCS measured from laboratory test on the core specimens. Bradford et al. (1988), Lacy (1997) established a correlation between UCS and E_S. As shown is Fig. 5, the suggested equations could not cover the present data. Hence, Eq. (23) describes empirical relationship between UCS and E_S for the studied limestone, as illustrated in Fig. 5.

$$\text{UCS} = 11.05{E_S}^{0.66}, \quad R^2 = 0.79 \tag{23}$$

where UCS and E_S are in MPa and GPa, respectively. However, UCS could be predicted from dynamic data. King (1983) established a correlation between UCS and E_d. The relationships between UCS

Table 2
Experimental results of uniaxial compressive strength and ultrasonic tests on Asmari and Sarvak limestone.

Sample ID	V_P (km/s)	V_S (K m/s)	Density (g/cm^3)	UCS (MPa)	E_S (GPa)	E_d (GPa)
1	5.381	3.073	2.60	178.2	46.2	61.8
2	4.876	2.712	2.46	122.4	34.3	46.2
3	5.737	3.102	2.60	175.9	48.1	64.7
4	5.951	3.261	2.70	176.9	53.3	73.8
5	4.809	2.797	2.30	81.90	31.0	44.8
6	5.189	2.893	2.60	100.7	36.4	55.5
7	2.690	1.703	2.60	31.90	11.4	17.6
8	4.887	2.809	2.30	68.00	28.8	45.5
9	3.170	1.981	2.40	50.20	13.9	22.2
10	4.831	2.942	2.70	153.8	34.0	56.3
11	4.036	2.691	2.62	105.3	24.9	41.7
12	2.826	1.884	2.43	41.50	10.9	19.0
13	3.924	2.337	2.40	74.00	18.2	32.1
14	3.691	2.043	2.35	34.10	13.3	25.1
15	3.600	2.400	2.61	63.80	17.1	33.1
16	3.229	2.123	2.50	56.50	12.7	25.2
17	3.694	2.44	2.53	58.20	16.7	33.5
18	4.373	2.405	2.41	53.50	17.3	35.8
19	3.454	2.281	2.59	50.80	14.0	30.0
20	3.445	2.261	2.61	59.40	13.3	29.9
21	3.663	2.419	2.44	40.90	12.2	31.8
22	3.696	2.012	2.54	33.60	7.90	26.5
23	3.746	2.475	2.59	48.00	10.0	35.3
24	3.852	2.608	2.63	74.50	12.8	38.5
25	4.221	2.488	2.62	70.80	7.80	40.0
26	3.353	2.357	2.54	75.90	13.4	28.5
27	2.381	1.587	2.47	33.80	7.1	13.7
28	3.855	2.461	2.63	47.50	13.4	36.8
29	3.935	2.644	2.58	78.60	14.5	39.3
30	6.480	3.288	2.70	180.0	90.0	77.4
31	4.063	2.719	2.60	72.60	16.8	42.1
32	4.854	2.880	2.69	118.0	24.4	54.8
33	4.274	2.794	2.65	122.0	22.6	46.6
34	4.854	2.880	2.69	118.0	24.4	54.8
35	5.707	3.149	2.70	157.3	66.0	68.6
36	5.016	3.108	2.70	148.2	51.0	62.0
37	5.744	3.206	2.70	143.7	66.0	70.7
38	4.185	2.690	2.40	91.40	19.5	39.9
39	3.834	2.368	2.10	25.10	17.4	28.1
40	3.064	1.963	2.30	28.15	4.70	20.4
41	3.877	2.418	2.30	46.80	11.5	31.8
42	3.690	2.430	2.70	54.70	13.0	35.6
43	3.77	2.44	2.60	56.50	22.5	35.3
44	4.072	2.618	2.60	75.40	12.0	40.9
45	5.689	3.289	2.70	171.1	50.7	73.0

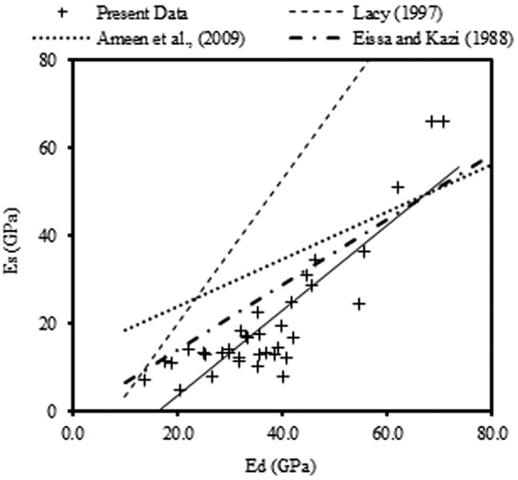

Fig. 3. E_S vs. E_d.

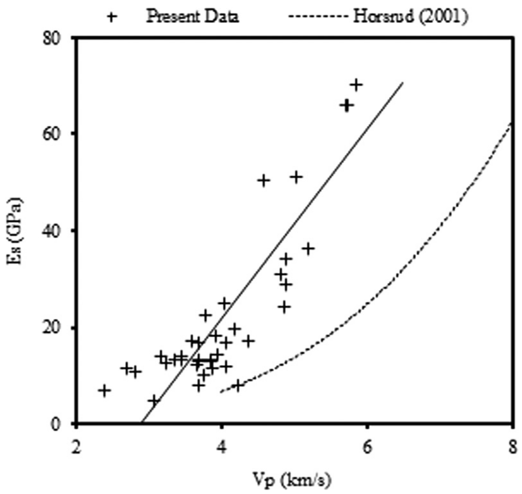

Fig. 4. E_S vs. V_P.

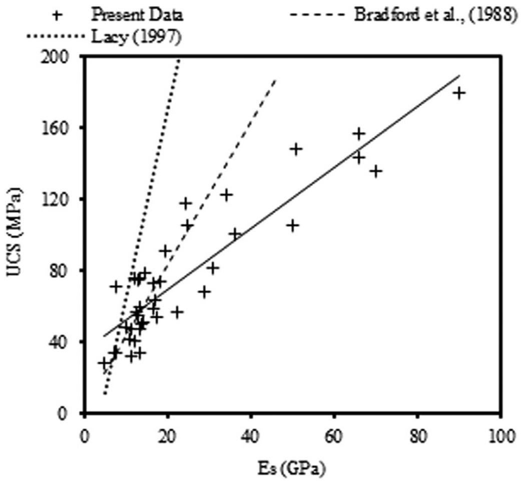

Fig. 5. UCS vs. E_S.

and E_d for the studied limestone are presented in Fig. 6 and Eq. (24).

$$\mathrm{UCS} = 12.8\left(\frac{E_d}{10}\right)^{1.32}, \quad R^2 = 0.88 \tag{24}$$

where UCS and E_d are in MPa and GPa, respectively. Furthermore, in case of V_S and density data are not available for calculation of E_d, UCS could be predicted from V_P directly. Militzer and Stoll (1973) and Horsrud (2001) reported nonlinear relationships between UCS and V_P. Eq. (25) is suggested to predict UCS from V_P for Asmari and Sarvak limestone. The scatter of data points for Eq. (25) are shown in Fig. 7.

$$\mathrm{UCS} = 3.67V_P^{2.14}, \quad R^2 = 0.81 \tag{25}$$

where UCS and V_P are in MPa and km/s, respectively.

The accuracy of the predicted UCS based on aforementioned methods (Eqs. (23)–(25)) were further examined and compared by statistical analysis.

- In Eq. (23) UCS is predicted based on E_S, where E_S is obtained from V_P or E_d.
- In Eq. (24) UCS is predicted from E_d.
- And in Eq. (25) UCS is predicted from V_P.

The value of predicted UCS for 45 Asmari and Sarvak limestone specimens (based on Eqs. (23)–(25)) were compared with the corresponding measured data. The following table shows the root mean square error (RMSE) for each method.

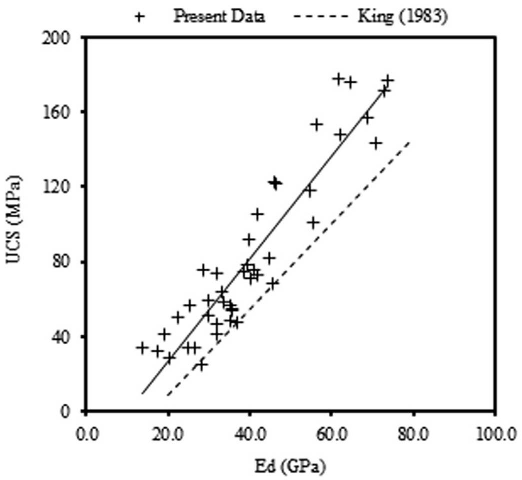

Fig. 6. UCS vs. E_d.

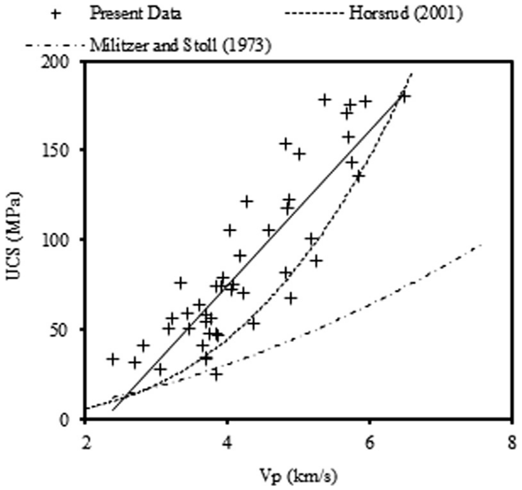

Fig. 7. UCS vs. V_P.

Table 3
The RMSE of Eqs. (23)–(25) in prediction of UCS.

RMSE (MPa)	Method
UCS predicted from E_d	16.1
UCS predicted from V_P	20.9
UCS predicted from E_S	21.9

As shown in Table 3, the predicted UCS from E_d is the best fitted method and RMSE is 16.1 MPa for each specimen. Also, the accuracy of UCS predicted from V_P is better than UCS predicated from E_S. This is because in this method, E_S was predicted previously from E_d or V_P using Eqs. (21) or (22), respectively. Therefore, prediction of UCS from E_d is the best method and prediction of UCS from V_P must be limited to conditions where E_d is not available.

Utilizing the sonic and density logs of the studied oil well, the uniaxial compressive strength log was determined based on Eqs. (23)–(25) as shown in Fig. 8.

As it can be seen in Fig. 8, the predicted UCS from E_S is often smaller than the predicted UCS from E_d (except over the crushed zones). In compacted zones, estimated UCS from E_d is the highest value and it has the least value in crushed intervals. In crushed, porous and saturated zones, the V_S is more strongly decreased and most affect compared to V_P. Therefore, estimated UCS from E_d may better distinguish crushed and compacted zones. Predicted UCS from

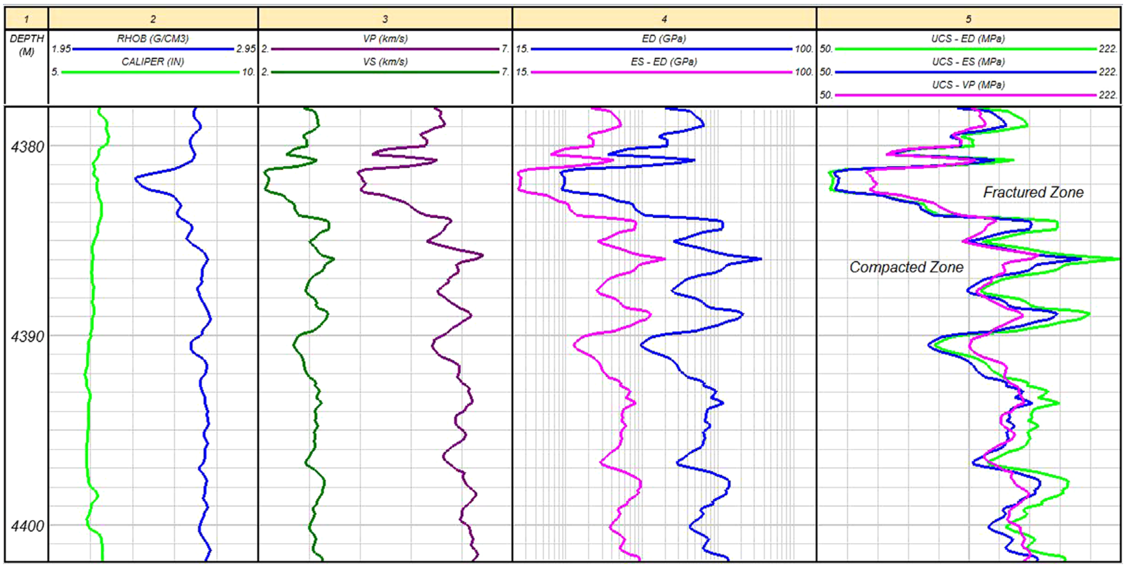

Fig. 8. Predicted UCS logs from Eqs. (23)–(24) in the studied well.

V_P is less sensitive to crushed and compacted intervals. Therefore, in crushed zones UCS from V_P is higher than the UCS predicted from E_d. Also, in compacted zones, the UCS predicted from V_P is less than UCS predicted from E_d and E_S. Therefore, it is suggested that the UCS should predicted from E_d, and in case of V_S and density are not available, it is better to be predicted from V_P, directly.

4. Conclusions

Uniaxial compressive strength and ultrasonic tests were performed on Sarvak and Asmari limestone core specimens and the values of UCS, E_S, V_P, and V_S were measured. Empirical relations between UCS and E_S with E_d and V_P that reported by previous litterateurs, were compared to authors' data. The locally validated equations could not adequately cover our data. Due to the importance of UCS and E_S in studies of petroleum reservoir geomechanics, it is always worth to predict these parameters from empirical relations that suggested for other formations with different lithology. By performing this study, locally validated empirical relationships between UCS and E_S with E_d and V_P for Asmari and Sarvak limestone were established. Besides, an equation to predict UCS from E_S was established. The accuracy of predicted UCS based on E_S, E_d, and V_P were compared using statistical analysis. Accordingly, UCS predicted based on E_d has higher accuracy. Hence, this method is recommended for practical applications. Evidently, estimation of UCS from E_S has less accuracy over other methods, because E_S was predicted from E_d or V_P. In summary, prediction of UCS from V_P is recommended for conditions where V_S and density data are not available and V_P is only dynamic data from open hole logs. Furthermore, it was demonstrated that UCS predicted from V_P is more sensitive to the crushed zones.

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