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Vertically thickness of a magnetized advection dominated accretion flow

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Abstract. We examine the effect of magnetic field in the dynamical structure of advection dominated accretion flow. We have presented self similar solution for MHD equations in the spherical framework. Our results show that the vertical thickness of the disk was modified by the magnetic field strength.

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1. Introduction

Accretion disks an important ingredient in our current understanding of many astrophysical systems on all scales. Accretion of the matter onto a compact object powers many energetic astrophysical systems, such as cataclysmic variables, X-ray binaries, and active galactic nuclei. The birth of modern accretion disk theory is traditionally attributed to the original model presented by Shakura and Sanyev [1]. This standard geometrically thin, optically thick accretion disk model (SSD) can successfully explain most of observational features in active galactic nuclei (AGNs) and X-ray binaries. In the standard thin disk model, the motion of matter in the accretion disk is nearly Keplerian, and gravitational energy released in the disk is radiated away locally. An alternative accretion disk model, namely, the advection-dominated accretion flows (ADAFs), was suggested for the black holes accreting at very low rates [2, 3]. In the ADAF model, only a small fraction of gravitational energy released in the accretion flow is radiated away due it's inefficient cooling, and most of the energy is stored in the accretion flow and will advected to central accretor. So ADAFs are optically thin and very hot (compare with standard Shakura & Sanyev disks), which radiated mostly in the X-ray band. This model can successfully explain X-ray binaries and low luminosity AGNs.

Although great progress has been made in recent years in increasingly sophisticated numerical accretion disk simulations, simple analytic disk models still are the only accessible way of making direct link between the theory and observations. The theoretical treatment can estimate the spectra and other observational features of the accretion powered objects. This fact justified the continues effort to improve the theoretical models for understanding the underlying physics of accretion disks.

One of the most important assumptions in the SSD is that they are geometrically thin, every where on the disks, the half-thickness $H(r)$ is much smaller than cylindrical radius, $H/r \ll 1$. So the average motion of the flow in the vertical direction must be negligible compare with the radial motion. This assumption would for the inner regions of the disks in some specific situations. For example when the accretion rate suppresses as its critical value, Eddington Luminosity, or when the disk is in the radiation inefficient regime. In these cases, the inner region of the thick will get geometrically thick, $H/r \sim 1$. Based on these understandings and the concept of advection dominance, two new types of model was introduced namely the optically thick, radiation pressure supported slim disk [4] and the optically thin advection dominated accretion flow [3, 4]. ADAFs and Slim disk were supposed to be a geometrically slim, neither thin nor thick. The reasons for this restriction comes from the definition of advection parameter, $f = Q_{adv}/Q_{vis}$, which is argued by Abramowitcz et al. [4]. Here Q_{adv} and Q_{vis} are the advective cooling and viscose heating rate per unit area, respectively. Advective parameter should satisfied in this relation [4]

$$f \geq (\frac{H}{R})^2.$$

Since in advection paradigm $f \leq 1$, ADAFs can be a valid model for the disks that are not thin, but the disk can not be thick either, because the value of f can not exceed 1.

Recently, Gu & Lu [5] (hereafter GL07) and Gu et al [6] (hereafter GU09) addressed a problem on the vertical thickness of ADAF types disks. They have shown that the vertical component of gravitational potential was introduced by Hoshi (1997) which was widly used in slim disk model is valid only for geometrically thin disk with $H/R \preceq 0.2$. For a larger thickness it would greatly magnify the gravitational force in the vertical direction. GL07 have shown that when the vertical gravitational force is correctly calculated in a cylenderival coordinate with the explicit potential, $\psi(R, z)$, slim this are much thicker than previously thought. GU09 revisit the problem of the vertical structure of black hole accretion disks in a spherical coordinates. By comparing the advective cooling with the viscous heating, they have shown that the ADAFs are geometrically thick, with half-opening angle $\Delta\theta > 2\pi/5$.

2. The basic equations

We consider a steady state axi-symmetric ($\frac{\partial}{\partial\varphi} = \frac{\partial}{\partial t} = 0$) accretion disk in spherical coordinate (r, θ, φ). For simplicity the self-gravity and general relativistic effects have been neglected and use the Newtonian potential for gravity, $\psi = -\frac{GM_*}{r}$, where M_* is the black hole mass. also we consider a toroidal configuration for magnetic field. The basic equation of continuity and momenta are

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\rho v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\rho v_\theta) = 0, \quad (1)$$

$$v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}(\frac{\partial v_r}{\partial\theta} - v_\theta) - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{GM_*}{r^2} + \frac{1}{4\pi\rho}(J_\theta B_\varphi - J_\varphi B_\theta), \quad (2)$$

$$v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}(\frac{\partial v_\theta}{\partial\theta} + v_r) - \frac{v_\varphi^2}{r}\cot\theta = -\frac{1}{\rho r}\frac{\partial p}{\partial\theta} + \frac{1}{4\pi\rho}(J_\varphi B_r - J_r B_\varphi), \quad (3)$$

$$v_r\frac{\partial v_\varphi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\varphi}{\partial\theta} + \frac{v_\varphi}{r}(v_r + v_\theta\cot\theta) = \frac{1}{4\pi\rho}(J_r B_\theta - J_\theta B_r) + \frac{1}{\rho r^3}\frac{\partial}{\partial r}(r^3 T_{r\varphi}), \quad (4)$$

where v_r, v_θ and v_φ are the velocity components in a spherical coordinate. Also the components of the current density, $\mathbf{J}$, are

$$J_r = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(B_\varphi\sin\theta), \quad J_\theta = -\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi), \quad J_\varphi = \frac{1}{r}[\frac{\partial}{\partial r}(rB_\theta) - \frac{\partial B_r}{\partial\theta}]. \quad (5)$$

In the viscous prescription, we assume that only $r\varphi$-component of the viscous stress tensor is important, which is

$$T_{r\varphi} = \nu\rho r\frac{\partial}{\partial r}(\frac{v_\varphi}{r}), \quad (6)$$

where $\nu = \frac{\alpha c_s^2 r}{v_k}$ is the kinematic viscosity coefficient, α is the constant viscosity parameter, c_s is the speed of sound which is defined as $c_s^2 = p/\rho$ and also $v_k = (\frac{GM_*}{r})^{1/2}$

is the Keplerian velocity. we do not have any outflow or wind production from the surface of the disk, so $v_\theta = 0$ is a reasonable approximation for the disks with any thickness [7]. And also by considering only φ-component of magnetic field, $B_\varphi \neq 0$, the third component of current density become zero, $J_\varphi = 0$. With the above assumptions, our main equations are reduced to

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \rho v_r) = 0, \tag{7}$$

$$v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi^2}{r} = -\frac{1}{\rho}\frac{\partial P_g}{\partial r} - \frac{GM_*}{r^2} - \frac{B_\varphi}{4\pi\rho r}\frac{\partial}{\partial r}(rB_\varphi), \tag{8}$$

$$v_\varphi^2 \cot\theta = \frac{1}{\rho}\frac{\partial P_g}{\partial \theta} + \frac{B_\varphi}{4\pi\rho\sin\theta}\frac{\partial}{\partial\theta}(B_\varphi \sin\theta), \tag{9}$$

$$v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_r v_\varphi}{r} = \frac{1}{\rho r^3}\frac{\partial}{\partial r}(\nu\rho r^4 \frac{\partial}{\partial r}(\frac{v_\varphi}{r})). \tag{10}$$

3. Self-similar solutions

The self Similar Solutions can not able to describe the global behavior of the accretion flows, because in this method there are not any boundary conditions. however as long as we are interested the solutions near the boundaries, it would benefit to have overall dynamical behavior of the solutions. Following [3], we will adopt the self-similarity in the radial direction as

$$v_r(r,\theta) = r\Omega_K(r)V(\theta) \tag{11}$$

$$v_\varphi(r,\theta) = r\Omega_K(r)\Omega(\theta) \tag{12}$$

$$c_s(r,\theta) = r\Omega_K(r)C(\theta) \tag{13}$$

$$\rho(r,\theta) = \rho_0\rho(\theta)(r/r_0)^{-3/2} \tag{14}$$

Where $\Omega_K(r) = \sqrt{GM_*/r^3}$ is Keplerian angular velocity, ρ_0 and r_0 provide convenient units with which the equations can be written in non-dimensional forms. Also we consider that the radio of magnetic pressure to the thermal pressure, β, is spatially constant [8, 9]

$$\beta = \frac{p_m}{p_g} = \frac{B_\varphi^2}{8\pi p} \tag{15}$$

The above similarity solution will satisfy in the main MHD equations. They will satisfy in continuity equation automatically and the components of momentum equation are reduced to be

$$\frac{1}{2}V^2 + \frac{5}{2}(1+\beta)C^2 + \Omega^2 - 2\beta C^2 - 1 = 0 \tag{16}$$

$$(1+\beta)\frac{C^2}{p}\frac{\partial p}{\partial\theta} = (\Omega^2 - 2\beta C^2)\cot\theta \tag{17}$$

$$V = -\frac{3}{2}\alpha C^2 \tag{18}$$

We have four unknown quantities, namely V, Ω, C and p which they are appeared in the three above equations. This is becauce we do not use the energy equation. we will assume that the energy equation form in radial direction is

$$q_{vis} = q_{adv} + q_{rad} \tag{19}$$

Where q_{vis} is viscous heating rate per volume, q_{rad} is the radiative cooling rate per volume and q_{adv} is the advection energy by accretion materials per unit volume and they are expressed as

$$q_{adv} = -\frac{5-3\gamma}{2(\gamma-1)}\frac{pv_r}{r} \tag{20}$$

$$q_{vis} = \frac{9}{4}\frac{\alpha p v_\varphi^2}{r v_K} \tag{21}$$

And also Q_{vis}, Q_{rad} and Q_{adv} represent local values of energy transport by viscous heating, radiation cooling and advection respectively. Then Q_{adv} and Q_{vis} are given by the vertical integration

$$Q_{adv} = \int_{\frac{\pi}{2}-\Delta\theta}^{\frac{\pi}{2}+\Delta\theta} q_{adv} r \sin\theta d\theta \tag{22}$$

$$Q_{vis} = \int_{\frac{\pi}{2}-\Delta\theta}^{\frac{\pi}{2}+\Delta\theta} q_{vis} r \sin\theta d\theta \tag{23}$$

Where $\Delta\theta$ is the half-opening angle of the disk. Due to complication in calculating the radiation process in a global ADAF solutions [3], $Q_{adv} = fQ_{vis}$ was used as an energy equation, where f was given as a constant. One of our goal is that how f varies with the thickness of the disk and how this parameter is effected by magnetic field. To do this, we further assume a polytropic relation, $p = K\rho^\gamma$, in the vertical direction, which is often adopted in the vertically integrated models of geometrically slim disks [10]. We admit that the polytropic assumption is a simple way to close the system, and then enable us to calculate the dynamical quantities.

By combination of the polytropic relation and the definition of the sound speed $c_s^2 = p/\rho$, the polar component of Euler equation, (Eq.17), becomes

$$\frac{dC^2}{d\theta} = (\frac{\gamma-1}{\gamma})(\frac{1}{1+\beta})(\Omega^2 - 2\beta C^2) \tag{24}$$

This equation with Eq.16 and Eq.18 can be solved for V, Ω and C. A boundary condition is required for solving the differential equation 24, which is set to be $c_s = 0$ at the surface of the disk, because both density and pressure are zero at the surface of the disk,($\rho = p = 0$).

Then Q_{vis} and Q_{adv} is obtained from Eq.23 and Eq.22 respectively. So we can calculate the advection parameter, $f_{adv} = \frac{Q_{adv}}{Q_{vis}}$.

4. Discussion and conclusions

In this paper first, we have obtained numerical solutions of equation (16-18) and (24) by considering viscosity parameter (α) and magnetic field parameter (β). The behavior of the solutions shows as the four panels in figure.1 . The top left panel displays the dimensionless redial velocity $V(\theta)$ as a fixed disk's half-opening angle $\Delta\theta$, for different values of magnetic field parameter, β. it is obvious that $V(\theta)$ is zero at surface of the disk(this is a boundary condition) and reach to the maximum value at the equatorial plane ($\theta = \pi/2$). It is seen that the profiles of $V(\theta)$ decrease with increasing of magnetic field parameter β and achieve in maximum values at the equatorial plane.

In the right top panel of figure.1, we plot the angular velocity, $\Omega(\theta)$. According to this figure, $\Omega(\theta)$ decreases in all profiles , when it near to the equatorial region from the surface of the disk, It means that $\Omega(\theta)$ has a maximum in the surface of the disk. Moreover, the same as redial velocity, as β increase, the profiles of angular velocity decrease toward the fixed half-opening angular, $\Delta\theta$. It is clear from equation(18),the variation of sound speed is similar to variation of redial velocity and we show the sound speed similarity function, $C(\theta)$ in the bottom left panel of figure 1. As it is expected, the velocity similarity functions $V(\theta)$, $\Omega(\theta)$ and $C(\theta)$ are sub-Keplerian.

And the Bottom right panel of figure.1 displays the density similarity function, $\rho(\theta)$. we show that the density decrease with increasing the different values of magnetic parameter, β, and it is zero at the surface of the disks and reach to the maximum value at the equatorial plane.

The main aim of this investigation was to show the variation of the advection factor, f_{adv} with the disk's half-opening $\Delta\theta$ for different values of magnetic parameter, β and the fixed value of radio of specific heats $\gamma = 4/3$ and viscosity parameter α. In our opinion, the difference results from different assumption, i.e, Narayan & Yi [11] assumed an anergy advection factor, f'_{adv} in advance, whereas we solve for energy advection factor, f_{adv} self-consistently based on polytropic relation in the vertical direction. In addition we assumed that the radio of magnetic pressure to gas pressure, $\beta = p_m/p_g$ has a fixed value. so we make improvement over GL07 and GX09 .Our results are shown in figure.2. It is seen that advection dominated ($f_{adv} > 0.5$) is possible, but only for $\Delta\theta > 2\pi/5$. Therefore advection-dominated disks must be geometrically thick rather than slim as previously supposed.

The key concepts of slim and ADAF disk models is advection dominance. This concept was introduced rather as an assumption, whether and under what physical conditions can it be realized have not been clarified. Here the main results of our work was to have shown that in order for advection to be dominated, the disk must be geometrically thick. And also by increasing the magnetic parameter β, the half-opening angle $\Delta\theta$ become smaller than before.

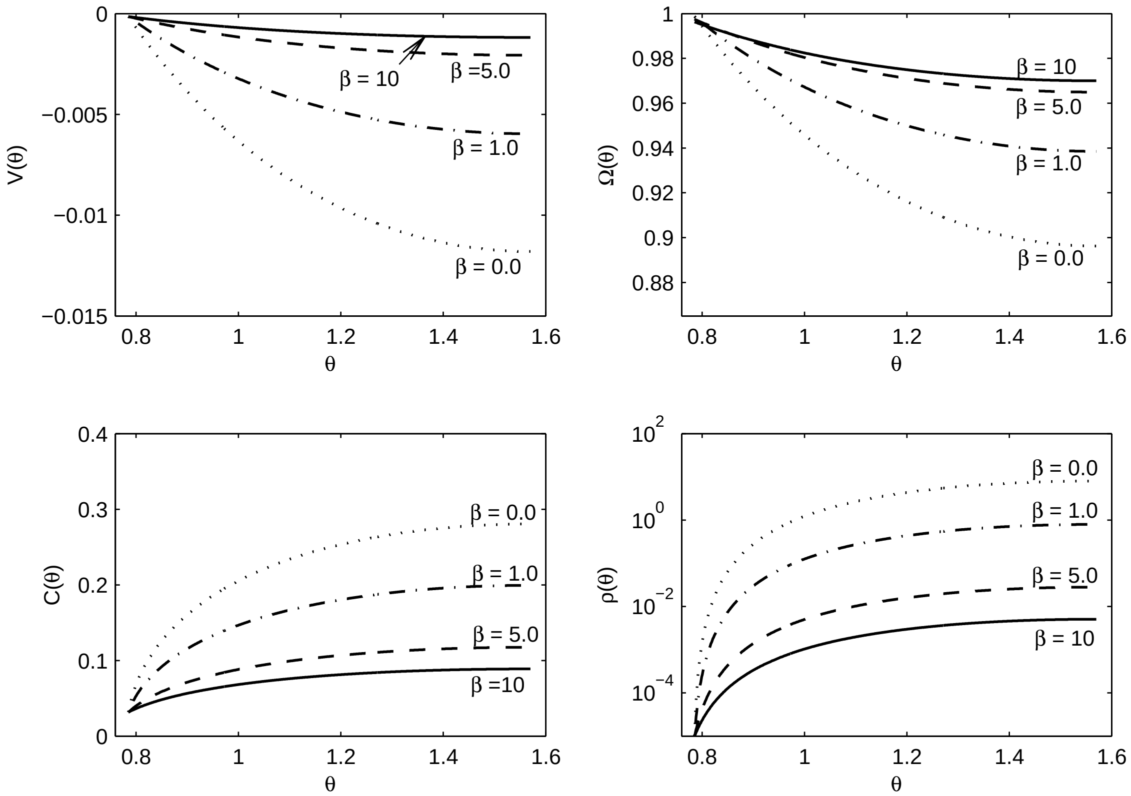

Figure 1. The self-similar solutions of $V(\theta)$, $\Omega(\theta)$, $C(\theta)$ and $\rho(\theta)$ as a function of polar angel θ corresponding to $\alpha = 0.1$, $\gamma = 4/3$, $\Delta\theta = 0.25\pi$ and $\beta = 0, 1, 5, 10$

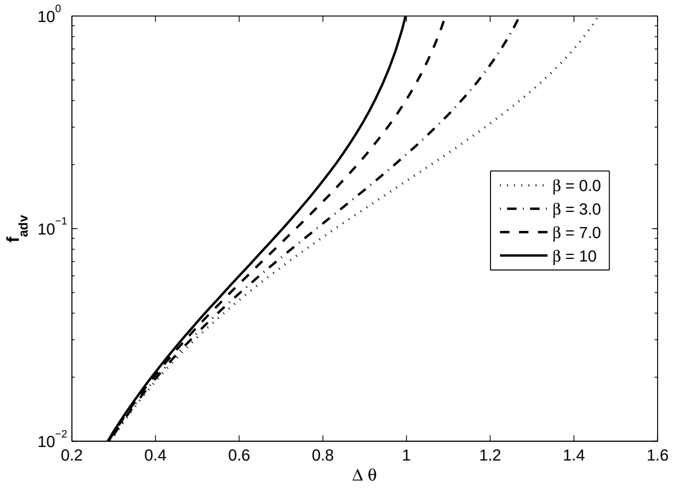

Figure 2. Variation of the advection factor f_{adv} with the disk's half-opening angle $\Delta\theta$ for diffrent values of magnetic feild, $\beta = 0, 3, 7, 10$ corresponding to $\alpha = 0.1$, $\gamma = 4/3$

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