July 29th, 2013

Dear Alireza Entezami,

We are pleased to inform you that your paper: "Structural Damage Detection by Sensitivity Analysis of Vibrational Response in Civil and Architectural Buildings " to International Conference on Adaptation and Movement in Architecture (ICAMA2013) has been accepted for **Oral presentation.** All accepted papers are peer reviewed and the authors are expected to register and present their work in person. This letter is to formally invite you to attend ICAMA2013, which will be held on October 11-12, 2013, at Department of Architectural Science, Ryerson University, Toronto, Ontario, Canada.

Please pay careful attention to the reviewers' comments as you revise your paper to make it even better. The deadline for the revised paper submission is 15th of Aug 2013. All accepted papers will be published in the proceeding of the conference, and the BEST papers will be considered for inclusion in a book that the organizers hope to have it published in 2014 (You will be informed after the conference whether your paper will be included in the book).

Please make your own arrangements for accommodation and transportation during the conference. For your convenience we have provided a list of available accommodation in the vicinity of the conference venue in the conference website.

Early Bird registration is now available. Please visit(http://registration.icama2013.com/) for more information. To be entitled to the early registration fee you must have registered and paid by August 4th 2013. Standard registration will be available after August 4th 2013.

Should you have any questions please do not hesitate to contact the conference secretariat at secretariat@icama2013.com.

We are very much looking forward to meeting you in Toronto in October 2013!

Dr. Maziar Asefi
ICAMA2013 Chair

Professor Colin Ripley
ICAMA2013 Honorary chair

Structural Damage Detection by Sensitivity Analysis of Vibrational Response in Civil and Architectural Buildings

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ABSTRACT

This study investigates the use of vibrational response of structure known as modal parameters, for identification of damage in the civil and architectural buildings. The vibrational response is defined as modal parameters that contain of mode shape, natural frequency and modal damping. Damage is also described as adversely performance of vibrational response which is relevant to changes of physical properties of structures. The proposed approach is based on vibration-based method and sensitivity of natural frequency. The Damage estimation algorithms involve two processes. First, the multiple damage location coefficient (MDLC) which uses a sensitivity matrix of natural frequency modification. In this level, the damage location can be detected via difference between natural frequencies of healthy and damaged structures as well as the sensitivity of natural frequency based on damage parameters. Observing the level of correlation between the variations in the modal parameters for healthy and damaged structures enables damage localization. The sensitivity matrix, developed from finite element model, further accommodate multiple damage detection. Next, the least-squares method is used for damage quantification. For verification of proposed damage algorithms, a simple 6-story shear building as discrete dynamic structure is modelled. Then, a 15-bar planner truss is used. Eventually, the numerical results show that the proposed methods are accurate and reliable approaches for damage localization and quantification in the structures.

Key Words: Structural damage detection, modal parameters, natural frequency sensitivity, least squares method.

1. INTRODUCTION

Identifying structural damage by using non-destructive test data has been investigated via many researchers during past decades. The damage identification plays a significant role in better understanding structural behaviour and vibrational responses. Many civil, aerospace and mechanical engineering systems continue to be utilized despite of aging and the associated potential for damage accumulation. Early structural detection is desirable to prevent structural failure and human loss of life. The architectural buildings in all over the world have been constructed and heritage structures with architectural visual are valuable legacy of any country and social communities. The civil engineering and architecture arts are inseparable part of together. Therefore, failure in the structural parameters of building are lead the deficiencies at the architectural aesthetics.

Structural damage is considered to be changes in structural parameters that adversely affect performance. Damage may also be defined as any deviation in the original geometrics of structures or material properties that may cause undesirable stresses, displacements or vibrations in the structure. These weaknesses and deviations may be due to cracks, loose bolts, broken welds, corrosion, fatigue, etc [1]. Damage often causes changes in the vibrational response of structures. Generally the vibrational responses are defined as modal parameters. These data are consisting of natural frequencies, mode shapes and modal damping ratios. The modal data are related to physical properties of structures. The need for additional global damage detection methods that can be applied to complex structures has led to the development of methods that examine changes in the response of the structure. The damage location and severity can be determined through finding difference of structural dynamic characteristics between the intact and damaged structures [2]. Since any change of structural mass or stiffness matrix caused by structural mass or stiffness loss in the given part of the structure will be reflected in the measured natural frequency and mode shape, when the measured data of the natural frequency or mode shape are different from those of the intact system, it indicates the damage emergence in the structural system [3].

Design sensitivity analysis is used to quantify the relationship between parameters used to define an optimum design and calculate outputs used to measure their performance. Design sensitivity analysis of structural and mechanical systems with respect to structural design parameters plays a critical role in inverse and identification problems in engineering applications, as well as structural health monitoring, structural reliability, dynamic model updating, structural design optimization, structural dynamic modification, approximate reanalysis techniques and many other applications. A significant body of research has been devoted to the computation and application of design sensitivity analysis [4, 5].

Structural damage identification through changes in structural characteristics, provide a global approach to evaluate the structural state. Detailed literature review on vibration-based techniques for damage detection has been provided by Doebling et al [6]. Modal frequencies and mode shapes are the most popular parameters used in damage identification. Salawu [7] presents a review on the use of modal frequency changes for damage diagnostics. The damage detection algorithms based on changes at natural frequencies has been evaluated by Su et al. [8]. They proposed a new method for locating of damage in the shear building. Using of sensitivity analysis of natural frequencies was utilized by Esfandiari et al. [9]. They prepared a frequency-based technique is presented to detect any number of localized damages which induce stiffness reduction in a structure. Damage assessment of truss structures were investigated by working of Majumdar et al. [10]. In their method, detect and assess on structural damage from changes in natural frequencies using ant colony optimization (ACO) algorithm were presented.

This paper proposes a method based on numerical sensitivities of natural frequencies to detect and locate damage in the structures that relevant to civil and architectural buildings. The method computes natural frequency sensitivities using the simulated modal data in the healthy state. The simulated modal parameters are normally identified by generalized eigenvalue problem [11, 12]. In this case, only mass and stiffness matrices of healthy structure are used. On the other words, the damping matrix is assumed as proportional. Damage is considered to be directly related to a decrease in stiffness. The approach is verified by a simple 6-story shear building. This structure is modelled as discrete dynamic system. Hence initial physical properties of discrete system is defined based on finite element method [13]. Next, a 15-bar planner truss is utilized. In both cases, results are compared with induced damage based on structural stiffness reduction. Eventually, the numerical results show that the proposed methods can correctly estimate the damage location by multiple damage location coefficient (MDLC) and provide the reliable results of damage severity by sensitivity of vibrational response of structures.

2. DAMAGE LOCALIZATION BASED ON NATURAL FREQUENCIES CHANGES

Determining the location of damage based on correlation between natural frequencies distinct provide a simple tool for locating damage. The damage localization method using of correlation and difference of natural frequencies healthy and damaged structures as $R_\omega=(\omega_h-\omega_d)/\omega_d$ *and* $\Delta\omega=\omega_d-\omega_h$, respectively. Here, ω_h and ω_d denote the natural frequency of the healthy and damaged structures.

$$DL_j = \left| \frac{R_\omega{}^T \Delta\omega_j}{\left|R_\omega\right| \left|\Delta\omega_j\right|} \right| \tag{1}$$

where, the subscripts *j=1,2,…,r* denotes the location of damage and *r* is number of damaged element. Another approach based on correlation matrix known as damage location coefficient (DLC) is expressed in the following form:

$$DLC_j = \left| \frac{\left|R_\omega{}^T . \Delta\omega_j\right|^2}{\left(R_\omega{}^T . R_{\omega j}\right)\left(\Delta\omega_j^T . \Delta\omega_j\right)} \right| \tag{2}$$

As can be seen, Eq. (2) compares two frequency change vectors. Both Eq. (1) and (2) assess the damage location in the damaged structure. All mentioned above methods, capable of detecting a single damage location. In multiple damage situations, if damage locations are directly identified using Eq. (1) and (2), then there is the risk of losing one or more damage sites because of the mutual effect of multiple damages. Therefore, a method that combines the information on the DLC with sensitivity analysis is developed for multiple damage localization in shear buildings. According to fundamental concept of sensitivity analysis, the change of natural frequency based on damage in the structure can be written as follow:

$$\Delta\omega = S.\Delta b \tag{3}$$

where, *S* and *Δb* are the sensitivity matrix and damaged parameter vector based on modification of natural frequencies, respectively. Therefore, the multiple damage location coefficient expression is defined by following form as in

$$MDLC_j = \left| \frac{\left|R_\omega{}^T . \left|S\Delta b_j\right|\right|^2}{\left(R_\omega{}^T . R_{\omega j}\right)\left(\left|S\Delta b_j\right|^T . \left|S\Delta b_j\right|\right)} \right| \tag{4}$$

The only difference between the prior discussed damage localization methods and the MDLC is the presence of the sensitivity matrix. The objective of the proposed damage localization is to find a damaged variable vector Δb_j that makes the MDLC equal to one. On the other words, the variable MDLC larger than of one, shows the damage location.

3. DAMAGE QUANTIFICATION BASED ON NATURAL FREQUENCY SENSITIVITY

Damage is assumed to be directly related to a decrease in stiffness. Therefore, damage can be located using the sensitivities of the natural frequencies with respect to the stiffness parameters. According to changes natural frequencies, the sensitivity matrix are determined as follow [14]:

$$\frac{\partial\omega_i}{\partial b} = \{\varphi\}_i^T \left(\frac{\partial K}{\partial b} - \omega_i{}^2 \frac{\partial M}{\partial b} \right) \{\varphi\}_i \tag{5}$$

where, *M* and *K* are the mass and stiffness of healthy structure, respectively. Also, ω_i and φ_i are natural frequency and mode shape of *i*th mode of healthy structure, respectively. Hence with neglecting of the mass matrix, the Eq. (5) is rewritten to form:

$$\frac{\partial\omega_i}{\partial b} = \{\varphi\}_i^T \frac{\partial K}{\partial b} \{\varphi\}_i \tag{6}$$

In this section, structural damage parameters are estimated by solving of Eq. (3) for natural frequency sensitivity matrix. To attain this aim, sensitivity matrix of natural frequency for healthy and damaged structures must be established firstly. Then, differences of modal parameters as error matrices of natural frequency $\Delta\omega$ of healthy and damaged structures are provided. Consequently, the right side of Eq. (6) indicates the sensitivity matrix of natural frequency for *i*th mode ($S_i=\partial\omega_i/\partial b$).

For damage detection, expressions of (3) must be solved. The least-squares or penalty function method are adequate technique to calculation of damage variables. The changes in the natural frequencies $\Delta\omega=\omega_d-\omega_h$ are expressed as a function of the difference in the damage variable vector $\Delta b=b_d-b_h$. First, the sensitivity S, is developed using the finite-difference method. The differences between natural frequency of healthy and damaged are computed and pre-multiplied by the pseudo-inverse of the sensitivity matrix S^+ yielding the updated damage variable vector Δb.

$$\Delta b = S^{+}.\Delta\omega \tag{7}$$

In these expressions, the subscripts "+" denotes the pseudo-inverse. As the sensitivity matrix S is a non-square matrix, the simplest case of pseudo-inverse of S can be given by:

$$S^{+} = S^{T}\left(SS^{T}\right)^{-1} \tag{8}$$

4. NUMERICAL EVALUATION

4.1. A simple 6-story shear building as discrete structure

The full-scale numerical model is constructed as a 6-story shear building that shown in Fig. 1, which can be modelled as a 6-DOF system with following properties. Consider the beams were confined in the slabs and behave as rigid body; therefore, calculation of stiffness of columns, braces, and concrete shear walls at each story is equalled to corresponding story's stiffness. Also, the mass of each story is accounted according to half weight of above and below walls weight as well as the slab weight of each story, respectively.

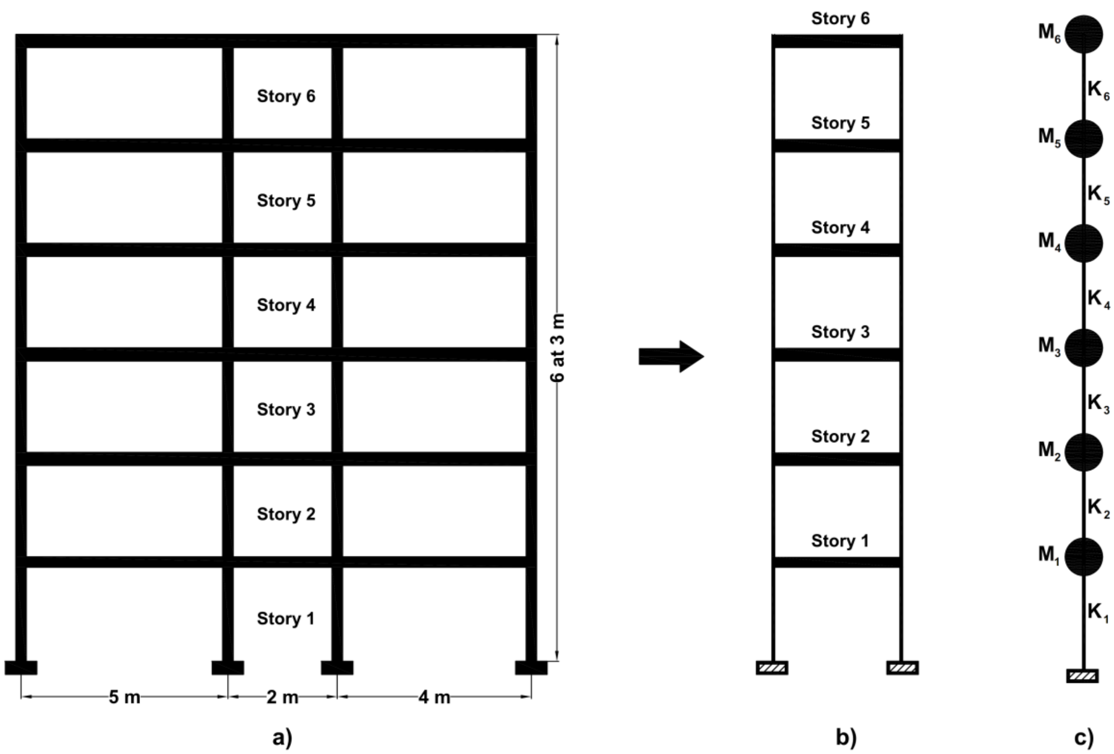

Figure 1: a) Full-scale 6-story shear building, b) Simulated 6-story shear building, c) Discrete system of shear building frame

Table 1: Physical properties of shear building

Physical Properties	Shear Building Stories					
	1th	2nd	3rd	4th	5th	6th
Mass (Ton)	10	10	10	8	8	6
Stiffness (Ton/m)	125	125	111	95	95	83

For damage detection, the initial properties of structures must be determined, hence the mass and stiffness matrices of shear building are formulated by finite element method [13, 15]. In this structure the stiffness reduction is defined as damage index. Four damage cases are considered to investigate the location and number of damaged stories on the results. In the first case, the stiffness of story 1 was decreased by 40%. In the second case, the stiffness of stories 2 and 5 were reduced by 30% and 20% respectively. In the damage case three, the stiffness of story 3 was decreased via 30%. Finally, in the fourth damage case, the stiffness of stories 3 and 4 were reduced by 10% and 20%, respectively. Since the reduction of stiffness, the natural frequencies of structure reduce at the all damage cases. On the other words, the adversely performance of dynamic behaviour due to damages are caused reduction of natural frequencies. Therefore, using of initial physical properties and identification of modal data in the healthy and damaged structures, the damage locations are detected by the proposed multiple damage location coefficient (MDLC).

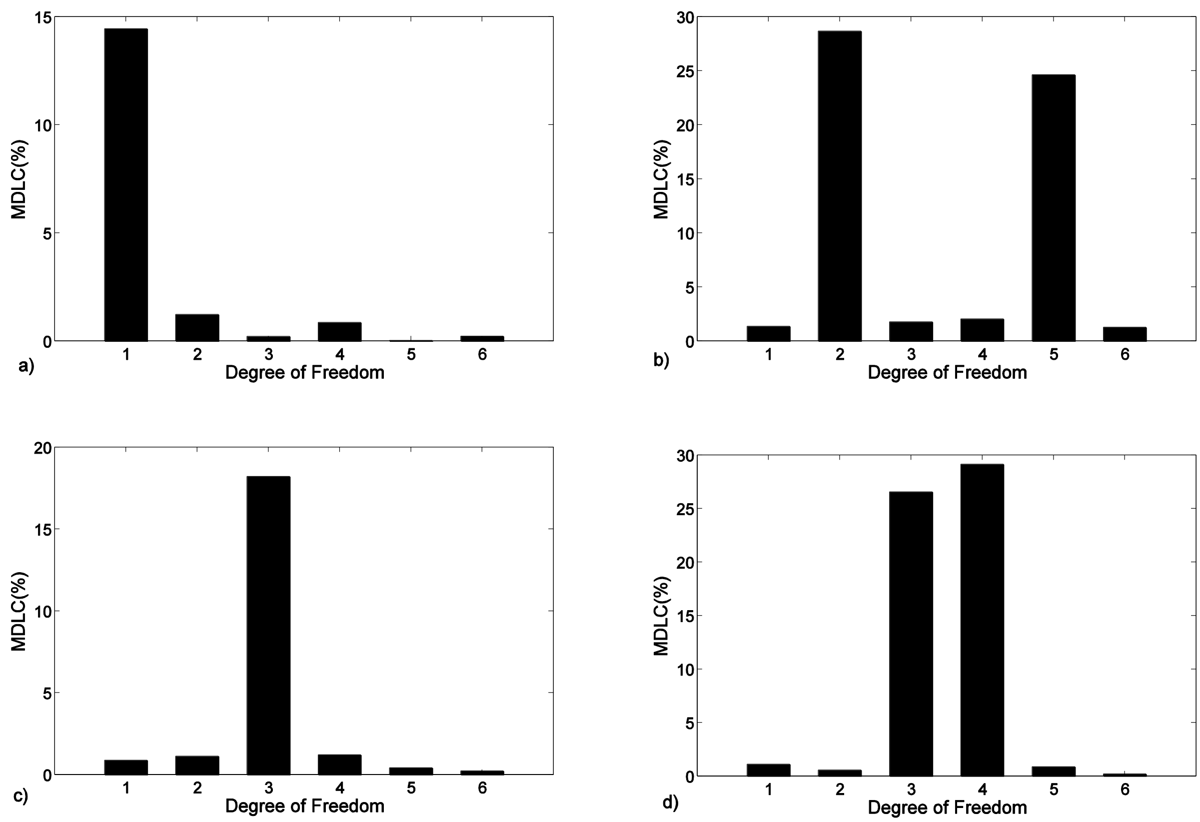

Figure 2: Multiple damage localization based on MDLC (%), a) Damage Case 1, b) Damage Case 2, c) Damage Case 3, d) Damage Case 4

As can be seen, the damage locations at all damage cases have been exactly detected by multiple damage location coefficient (MDLC). Also, at the other degrees of freedom that has healthy (undamaged) situation, the calculation errors are less than 3%. This error ratio is reliable quantities error for damage localization. As a result, the proposed damage localization method (MDLC) method enables to detect any diagnosis in the structures. Furthermore, the damage severity (percent of induced damage) can be estimated via sensitivity of natural frequency. After determination of stiffness matrix of shear building, the ratios of induced damage cases are imposed to identified stiffness matrix and modal parameters of damaged structure are extracted with solving of general eigenvalue problem. Using Eq. (6), sensitivity matrix of natural frequency based on stiffness reduction will be determined. Eventually, with pseudo-inverse of sensitivity matrix the vector of damage parameters are calculated. Table (2) shows the full information of damage quantification and compares with actual quantities of induced damage.

Table 2: Damage quantification of 6-shear building

Damage Cases	Damage Location	Induced damage (%)	Identified damage (%)	Error (%)
Case 1	Story 1	-40	-38.65	3.37
Case 2	Story 2	-30	-29.04	3.19
	Story 5	-20	-18.97	5.12
Case 3	Story 3	-30	-29.09	3.31
Case 4	Story 3	-10	-9.25	7.47
	Story 4	-20	-18.76	6.15

4.2. A 15-bar planner truss as continuous structure

To illustrate characteristics of the proposed damage detection algorithm, a two-dimensional truss structure is presented as shown Fig. 3. The basic parameters of the structure are Young modules E=200 GPa, density ρ=7850 kg/m3. All element of truss are modelled with 100 mm × 100 mm equal double angels and 5 mm thickness.

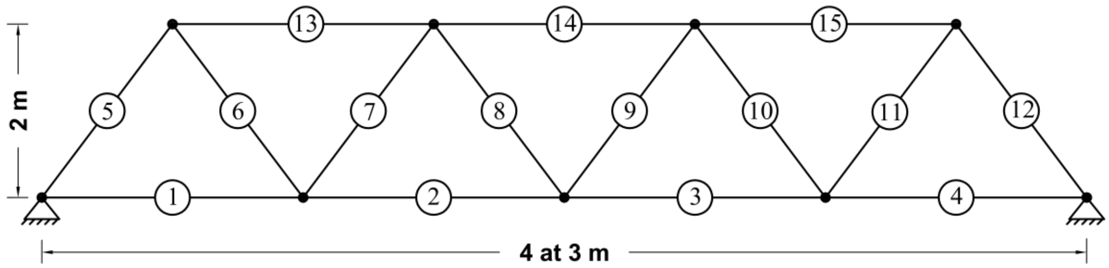

Figure 3: A 15-bar planner truss as continuous structure

Four damage cases are considered to investigate the influence of the location, severity and number of the damaged elements on the results. In the first damage case, the stiffness of elements 2 and 15 were reduced by 40%. In damage case number two, the stiffness of elements 6, 9 and 12 were decreased by 20%, 25% and 30%, respectively. In damage case number three, the stiffness elements 1, 10 and 14 were reduced via 30%, 30% and 20%, respectively. Finally, in damage case number four, the stiffness of elements 7 and 8 were decreased by 20% and 30%, respectively. Changing of stiffness matrix is modified the truss dynamic behaviour. On the other words, reduction of the natural frequency is caused the damage occurrence damage in the truss structure. The location of induced damage cases are detected by multiple damage location coefficient (MDLC) method as follow.

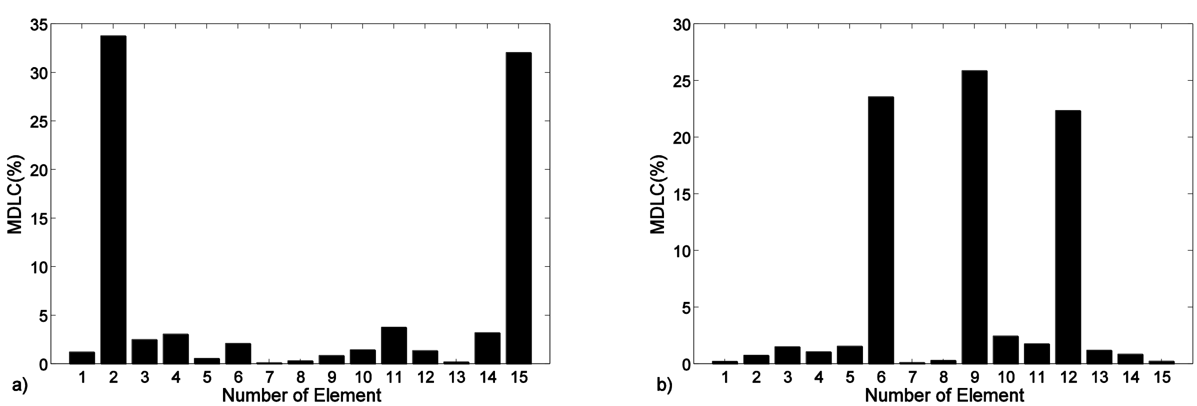

Figure 4: Damage localization by DMLC method, a) Damage Case 1, b) Damage Case 2

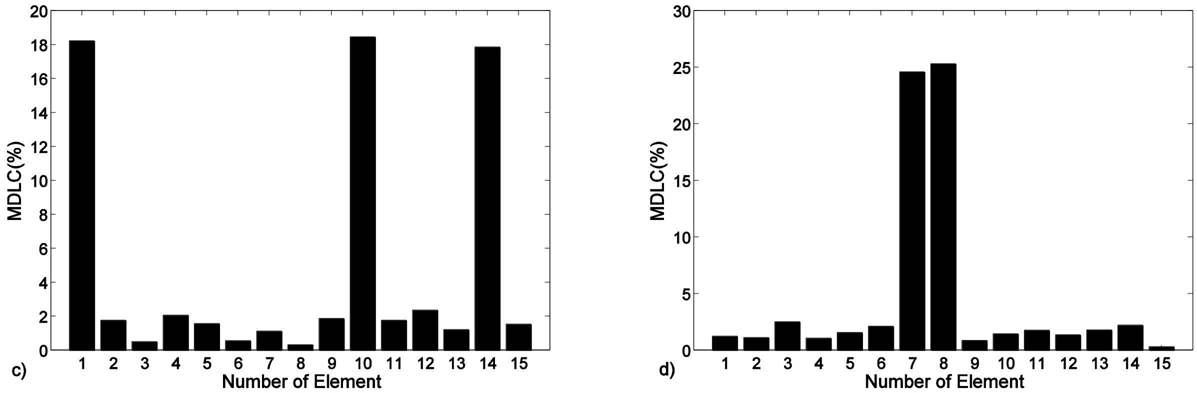

Figure 5: Damage localization by DMLC method, c) Damage Case 3, d) Damage Case 4

It can be observed that the damage location has been also identified by multiple damage location coefficient (MDLC) method. Consequently, the induced damage level was accurately located and identified error is only less than 10%. Furthermore, the damage severity can be also determined by natural sensitivity method, according to expressions (7) and (8). Similar to prior section, Table (3) shows comprehensive and perfect information of identified damage quantifications.

Table 3: Damage quantification in the 15-bar-planner truss

Damage Cases	Damaged Element	Induced damage (%)	Identified damage (%)	Error (%)
Case 1	2	-40	-38.63	3.42
	15	-40	-37.90	5.24
Case 2	6	-20	-18.33	8.31
	9	-25	-23.85	4.58
	12	-30	-27.74	7.52
Case 3	1	-30	-29.31	2.27
	10	-30	-28.81	3.96
	14	-20	-19.03	4.81
Case 4	7	-20	-19.56	2.16
	8	-30	-27.38	8.70

Similar to prior relations, the natural frequency sensitivity method has been also provided the well results in the truss structure. The estimated error in this stage is only less than 10%; therefore, the damage severity has been accurately identified based on proposed approach. As a results, the proposed method had simple formulations, nevertheless, the results of identified damage as well as damage location show the accuracy of them are very high and are reliable tools for damage detection.

5. CONCLUSION

In this study global assessment of structural damage in the building which related to civil engineering and architectural structures has been investigated. The damage detection algorithms were consisted two stages. In the first state, the damage localization based on vibration-based technique and coefficient of multiple damage location is assessed. Therefore, the sensitivity of natural frequency between healthy and damaged structures was formulated and severities of induced damages were determined. The proposed methods were utilized in the simple 6-story shear building and 15-bar planner truss. The numerical results indicate that the damage location and severity were accurately detected by multiple damage location coefficient (MDLC) method and sensitivity of natural frequency approach, respectively.

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