

Damage localization and quantification by direct structural dynamic parameters updating method

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Abstract

The objective of this paper is detecting the location and extent of structural damage from measured vibration test data based on direct structural finite element model updating. The method is based upon a mathematical model representing the undamaged vibrating structure and a local description of the damage and introduces a new finite element model updating approach for damage detection. The problem of modeling errors and their influence to damage localization accuracy is discussed and an approach to obtain reliable results in this case is presented. The concept of direct updating of individual dynamic parameters is used and according to that algorithms the mathematical function for damage assessment are defined. Then, error matrix of dynamic properties of healthy and damaged structures is established to detect the damage location and severity. For validation of damage detection approaches, two numerical examples are utilized. At the all examples, consider the modal data are incomplete and inverse of rectangular matrices is accomplished by Moore–Penrose inverse matrix without using any multipliers. It will be shown that the proposed procedure is simple to implement and may be useful for structural damage identification.

Key words: Damage localization and quantification, Direct optimization functions, Updating of mass and stiffness, Moore-Penrose inverse method

1. Introduction

Structural damage detection using measured or simulated dynamic data has emerged as a new research area in civil, mechanical and aerospace engineering communities in recent years. Recently, many methods have been presented to identify structural damage using the changes in modal parameters such as mode shapes and natural frequencies and dynamic properties of structures including mass and stiffness changes. In engineering practice, only a few of eigenvalues and partial mode shapes can be obtained by a modal survey for large flexible structures. Then, these existing damage identification methods can be categorized by the type of solving the incomplete measurement problem. One group includes these methods in which mode shape expansion or model reduction cannot be avoided in damage identification. Many

usual methods belong to this group, such as the finite element model (FEM) update techniques based on the residual force vector [1].

Finite Element (FE) model updating is the process by which an initial model of a dynamical system is modified (updated) in order to minimize a metric of the difference between system response measurements and model predictions [2]. Also, in the modern analysis of structural dynamics, much effort is devoted to the derivation of accurate models. These accurate models are used in many applications of civil engineering structures like damage detection, health monitoring, structural control, structural evaluation and assessment. The Finite element (FE) model of a structure is constructed on the basis of highly idealised engineering and designs that may not truly represent all the physical aspects of an actual structure. When field dynamic tests are performed to validate the analytical model, inevitably their results, commonly natural frequencies and mode shapes, do not coincide well with the expected results from the analytical model [3]. Many modal-based damage detection methods attempt to detect changes in the natural frequencies of a structure. In an earlier work by Cawley and Adams [4], it was shown that the ratio of frequency changes in different modes is only a function of damage location and not the magnitude of damage. Salawu [5] reviewed the different methods of structural damage detection through changes in natural frequencies. Analysis of changes in mode shapes due to damage represents another subgroup of modal-based methods. Usually, changes in a mode shape's curvature are more sensitive to damage. Pandey et al. [6] introduced the use of mode shape curvatures. Another class of damage identification methods uses the updating of individual dynamic properties of structures. Yang and Chen [7] proposed a new approach for estimated of mass and stiffness matrices by direct finite element model updating method. The main afford of their research was to determine the error mass and stiffness matrices by initial physical properties and modal data. Lee and Eun [8] also provided a new method for estimation of dynamic parameters by error matrix and model updating method. At the other research of them [9] the mass and stiffness error matrices determined that can be utilized of them for damage detection. The applicable approach for damage detection in the shear building frame presented by Shiradhonkar and Shrikhande [10]. They proposed a new method for detecting and locating in the beams with the aid of vibration based system identification and finite element model updating method, using limited number of responses recorded during strong earthquakes.

In this paper, procedure for detecting the damage quantities in the dynamical structures according to finite element model updating is presented. The method is based upon a mathematical model representing the undamaged vibrating structure and a local description of the damage and introduces a new finite element model updating approach for damage detection. The concept of direct updating of individual dynamic parameters is used and according to that algorithms the mathematical function for damage assessment are defined. Then, error matrix of dynamic properties of healthy and damaged structures is established to detect the damage location and severity. In the damage detection process, two objective

functions are introduced to identify damage localization and quantification. For validation of damage detection approaches, two numerical examples are utilized. At the all examples, consider the modal data are incomplete and inverse of rectangular matrices is accomplished by Moore–Penrose inverse matrix without using any multipliers. It will be shown that the proposed procedure is simple to implement and may be useful for structural damage identification.

2. Theory

Measured and analytical data are unlikely to be equal due to measurement noise, model inadequacies, and some damage, etc. The mode shapes and natural frequencies obtained from incomplete modal tests do not usually satisfy the eigenvalue equation and orthogonality requirements. Thus, the mass and stiffness matrices should be modified for simulation and design studies [11]. The direct model updating method for damage detection process uses analytical undamaged mass and stiffness matrices M_u and K_u from the analysis assumed to be incorrect and test data such as natural frequencies and their corresponding mode shapes assumed to be correct. Here M_u and K_u are the analytical mass and stiffness matrices to be corrected, respectively. Damage occurrence is caused to change these individual properties of structures that can be denote as damaged mass and stiffness of structures. In the model updating method, relations between dynamic parameters of damaged and healthy structures are usually defined as error matrices of mass and stiffness properties which can be described as follow

$$\Delta M = M_u - M_d \quad (1)$$

$$\Delta K = K_u - K_d \quad (2)$$

At all damage cases, effect of mass changes in the adversely performance of dynamic behaviour that is introduced a basic damage definition is very less than stiffness influences. Therefore, in this study; changes of stiffness matrix are described as damage state.

The mode shapes and natural frequencies obtained from vibration tests or simulated modal analysis methods are often incomplete and usually do not satisfy the dynamic equation and orthogonality requirements [9]. Therefore, the desired mass and stiffness matrices must be modified to fulfill the eigenvalue equation and the orthogonality constraints using modal test data. The constraint equations for damage detection process are the basic orthogonality requirement and eigenvalue equation as

$$\varphi^T K \varphi = \Lambda \quad (3)$$

$$K \varphi = M \varphi \Lambda \quad (4)$$

where, M and K are individual mass and stiffness matrices of structure, respectively. Φ and Λ are the vibrational modal data that can be defined as eigenvector and eigenvalue, respectively. For damage detection process, Eq. (4) is used to estimate the of damage quantities. This process will be implemented by a new proposed method according to finite element model

updating for individual dynamic properties. Damage is considered to be directly related to a decrease in stiffness; hence the dynamic orthogonality conditions in the damage state can be rewritten as follow

$$\varphi_d^T K_d \varphi_d = \Lambda_d \quad (5)$$

$$K_d \varphi_d = M_u \varphi_d \Lambda_d \quad (6)$$

In these expressions subscript d denotes as damage state of structure. It is assumed that if damage changes the structural stiffness ΔK matrix, a corresponding change in eigenvalues and eigenvectors of the original structure will result. On the other hand, stiffness changes are caused to adversely performance of vibrational response of structures. In the following equations, we derive the change of the stiffness matrix ΔK under the assumption of correct modal data and mass matrix in the satisfaction of the constraint of Eq. (4).

$$(K_u + \Delta K) \varphi_d = M_u \varphi_d \Lambda_d \quad (7)$$

or

$$\Delta K \varphi_d = M_u \varphi_d \Lambda_d - K_u \varphi_d \quad (8)$$

To estimation of error matrix as stiffness difference matrix, the objective function of Berman and Nagy [12] will be used as follow

$$J = \|M_u^{-1/2} (K_d - K_u) M_u^{-1/2}\| = \|M_u^{-1/2} \Delta K M_u^{-1/2}\| \quad (9)$$

Modifying the constraint equation of Eq. (9), it follows that

$$M_u^{-1/2} (\Delta K) M_u^{-1/2} M_u^{-1/2} \varphi_d = M_u^{-1/2} M_u \varphi_d \Lambda_d - M_u^{-1/2} K_u \varphi_d \quad (10)$$

Since the mode shape matrix φ is also rectangular, an infinite number of solutions of ΔK exist. This derivation selects the matrix ΔK to minimize the objective function. Solving Eq. (10) with respect to $M_u^{-1/2} (\Delta K) M_u^{-1/2}$ yields

$$M_u^{-1/2} (\Delta K) M_u^{-1/2} = (M_u^{-1/2} M_u \varphi_d \Lambda_d - M_u^{-1/2} K_u \varphi_d) (M_u^{1/2} \varphi_d)^+ + y_1 \left[I - (M_u^{1/2} \varphi_d) (M_u^{1/2} \varphi_d)^+ \right] \quad (11)$$

where, y_1 denotes an $(N \times N)$ arbitrary matrix and I is an $(N \times N)$ identity matrix and “+” denotes the inverting of rectangular matrices by Moore-Penrose method. Utilizing the condition to minimize the objective function of Eq. (9) into Eq. (11) and solving the result with respect to the arbitrary matrix yields

$$y_1 = (-M_u^{-1/2} M_u \varphi_d \Lambda_d + M_u^{-1/2} K_u \varphi_d) (M_u^{1/2} \varphi_d)^+ \left[I - (M_u^{1/2} \varphi_d) (M_u^{1/2} \varphi_d)^+ \right] + y_2 (M_u^{1/2} \varphi_d) (M_u^{1/2} \varphi_d)^+ \quad (12)$$

With expanding and compacting of Eq. (12), the arbitrary matrix y_1 will be solved to form

$$y_1 = y_2 (M_u^{1/2} \varphi_d) (M_u^{1/2} \varphi_d)^+ \quad (13)$$

where y_2 is an $(N \times N)$ arbitrary matrix. Utilizing Eq. (13) in Eq. (11) and arranging the result yields

$$M_u^{-1/2} (\Delta K) M_u^{-1/2} = (M_u^{-1/2} M_u \varphi_d \Lambda_d - M_u^{-1/2} K_u \varphi_d) (M_u^{1/2} \varphi_d)^+ \quad (14)$$

Pre-multiplying and post-multiplying both sides of Eq. (14) by $M_u^{-1/2}$, the change of the stiffness matrix is derived by

$$\Delta K = (M_u \varphi_d \Lambda_d - K_u \varphi_d) (M_u^{1/2} \varphi_d)^+ M_u^{1/2} \quad (15)$$

As can be seen, the damage index based on error stiffness matrix ΔK is only formulated by initial physical properties of intact structures as well as the modal parameters in the undamaged and damaged states. Indeed, Eq. (15) obtains the comprehensive information for damage localization according to changing of the stiffness matrix. However, damage localization index ΔK depends on geometrical condition and combining of degree of freedoms in the construction of discrete matrices. In the present work, the diagonal values of stiffness error matrix are used instead of ΔK . Therefore the damage localization index is reproduced as diagonal values of stiffness error matrix.

$$dR(\%) = \left(\frac{\Delta K}{K_u} \right)_j . 100 \quad (16)$$

In this expression, dR , ΔK_j and K_{uj} are relative indicator of damage index based on stiffness reduction, diagonal values of ΔK and diagonal values of undamaged stiffness matrix, respectively. This relative indicator is advantageous in some situations to give an indicator of relative damage levels. It will be observed that the peaks for curve charts or tallest columns of the column charts are located of damage and their maximum values give an indication on the relative damage levels.

To attain the damage severity, a proportional damage is utilized. In a proportional damage model ΔK are expressed as a function of the undamaged element stiffness and mass matrices by the equations

$$\Delta K = \sum_{j=1}^{N_d} K_{uj} \delta k_j \quad (17)$$

Where, K_{uj} is stiffness matrix of the healthy structure, δk is proportional stiffness damage modification factor for element j and N_d is number of damaged element, respectively. The summation sign in Eq. (17) signify matrix assembly. By expressing the damage in terms of the damage modification factors, the number of unknowns in ΔK is reduced to δk per damaged element. The problem is therefore rendered much less prohibitive. Upon substitution of Eq. (17) into Eq. (8), can be rewritten to form

$$\sum_{j=1}^{N_d} \varphi_i^T K_j^e \varphi_i \delta k_j = M_u \varphi_d \Lambda_d - K_u \varphi_d \quad (18)$$

It should be noted that φ_d , Λ_d , M_u and K_u can be obtained from a finite element modeling of the structure in its undamaged state. Suppose N_d damaged eigenvalues are available, application of the Eq. (18) to each measured eigenvalue leads to a matrix equation of the form

$$S.\delta k = P \quad (19)$$

where, S is a $(N_d \times N_d)$ matrix and δk is $(N_d \times 1)$ vector, given by

$$S_{ij} = \varphi_i^T K_j^e \varphi_i \quad (20)$$

$$P_{ij} = (M_u)_j \varphi_{id} \lambda_{id}^2 - (K_u)_j \varphi_{id} \quad (21)$$

For incomplete modal data, solution of Eq. (19) will be possible by definition of linear objective function that consists of initial properties of undamaged structures as well as modal data of damage structures. This objective function can be determined as follow

$$J = \|(S.\delta k - P)^T \delta k (S.\delta k - P)\| \quad (22)$$

Since the mode shape matrix φ is also rectangular, an infinite number of solutions of ΔK exist. This derivation selects the matrix δk to minimize the objective function.

$$\delta k = S^+ P + z_1 (S^+ S - I) \quad (23)$$

where, z_1 denotes an $(N_d \times N_d)$ arbitrary matrix and I is an $(N_d \times N_d)$ identity matrix. Utilizing the condition to minimize the objective function of Eq. (22) and solving the result with respect to the arbitrary matrix yields

$$z_1 = S^+ P (I - S S^+)^+ + z_2 (S^+ P - I) \quad (24)$$

where z_2 is an $(N_d \times N_d)$ arbitrary matrix. Utilizing Eq. (24) in Eq. (23) and arranging the damage severity based on stiffness changes can be rewritten to form

$$\delta k = S^+ P + (S^+ S - P) \quad (25)$$

The Eq. (25) is a unique solution of sensitivity analysis of stiffness matrix which introduce as damage index. Also, Moore-Penrose inversing method provides the easily procedure to use the incomplete modal data and correct results of damage detection method.

3. Numerical investigation

3-1- A 15-bar planner truss

To illustrate characteristics of the proposed damage detection algorithm, a two-dimensional truss structure is presented as shown Fig. 1. The basic parameters of the structure are Young modulus $E=200$ GPa, density $\rho=7850$ kg/m³. All element of truss are modeled with 100 mm \times 100 mm equal double angels and 5 mm thickness. Each nodal of truss have two degrees freedom (DOF). In this example, the first 5 vibrating modes are used for identifying the damage.

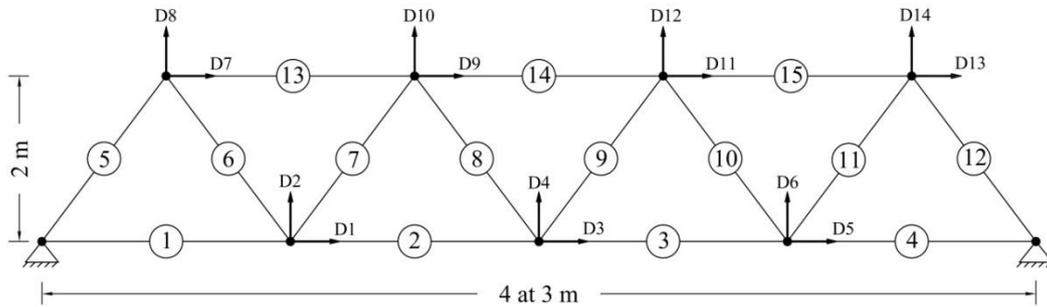


Figure 1. The 15-bar planner truss

The above structure is a continuous dynamic system and the mass and stiffness matrices can be determined by basic concept of finite element method [13]. After determination of physical parameters of intact truss structure, the generalized eigenvalue problem is used and the modal parameters including natural frequencies and mode shapes are calculated. Assume that the proportional damping is dominated in the structure behaviour and consequently the modal parameters are extracted as real data.

Four damage cases are considered to investigate the influence of the location, severity and number of the damaged elements on the results. In the first damage case, the stiffness of elements 2 and 14 were reduced by 40%. In damage case number two, the stiffness of elements 6, 9 and 12 were decreased by 20%, 25% and 30%, respectively. In damage case number three, the stiffness elements 3, 8 and 15 were reduced via 30%, 30% and 20%, respectively. Finally, in damage case number four, the stiffness of elements 6 and 13 were decreased by 20% and 30%, respectively. Changing of stiffness matrix is modified the truss dynamic behaviour. In the other words, reduction of the natural frequencies is caused the damage occurrence damage in the truss structure. The location and severity of induced damage cases are detected by proposed finite element model updating methods as follow.

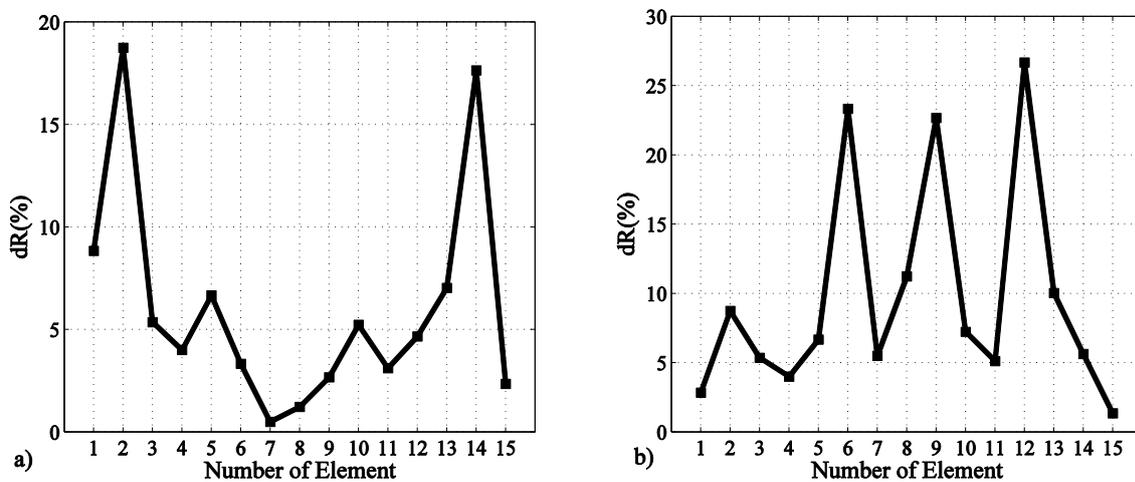


Figure 2. Damage localization of planner truss, a) Damage scenario 1, b) Damage scenario 2

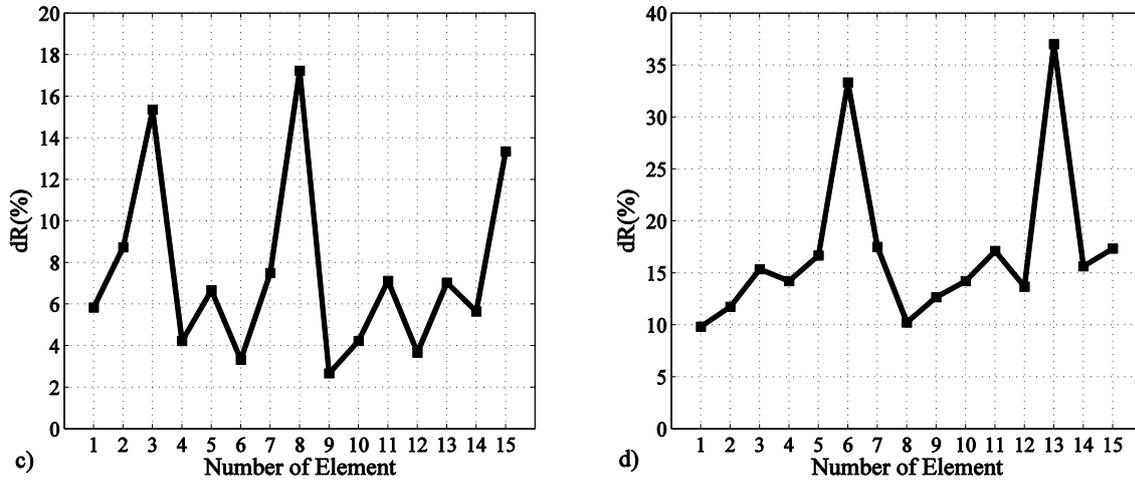


Figure 3. Damage localization of planner truss, c) Damage scenario 3, d) Damage scenario 4

As can be seen, the peaks of figures illustrate the damage location. In the other words, each damage state is precisely detected by proposed model updating method when incomplete modes are present. Using of incomplete identified modes and prediction of initial properties of structures are advantage of damage localization method. Also, objective function for this aim is also converges to direct identification of dynamic parameters of structures. In the other words, we used the updating method of mass and stiffness matrices for damage detection.

After determination of location of damage, based on proposed objective; the damage extent as stiffness modification factor ∂k are estimated. Figures 4a-b and 5c-d show the damage severity.

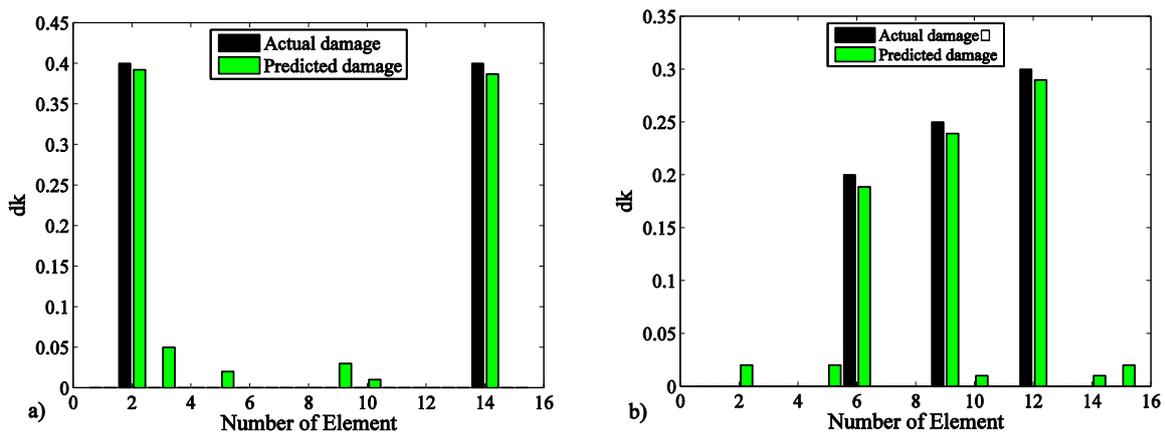


Figure 4. Predicted damage in the planner truss, a) Damage scenario 1, b) Damage scenario 2

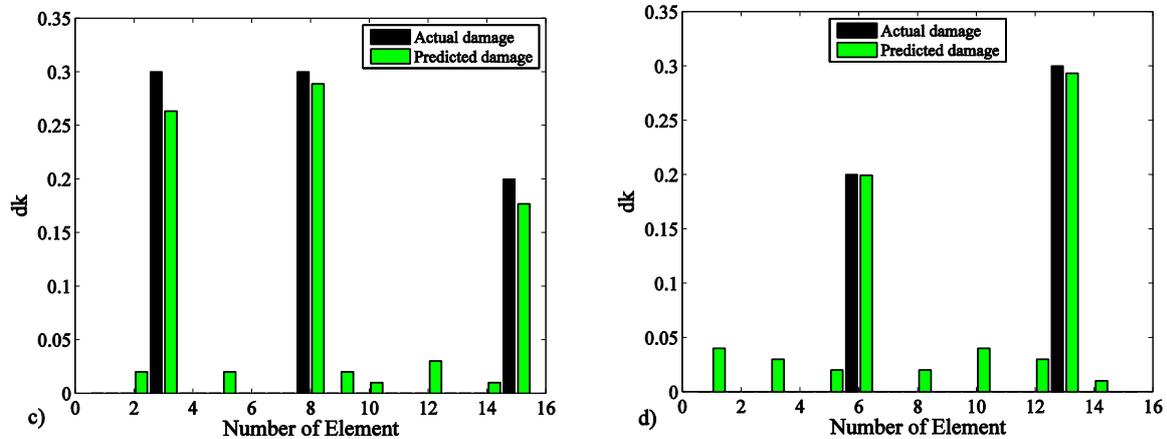


Figure 5. Predicted damage in the planner truss, c) Damage scenario 3, d) Damage scenario 4

As can be indicated, the induced damage were accurately estimated and error function based on updating process is approximately less than 7% for each damage cases. Therefore, the proposed objective function for damage quantification and Moore-Penrose can provide reliable results for damage detection process.

3-2- A cantilever beam

In this section, the damage detection and damage severity models describes in the preceding section used to identify the location and determine the magnitude of reduction of stiffness on a cantilever beam. The beam has been shown in Fig. 6. The length, thickness and width of the beam are 1.20, 0.05 and 0.1 m, respectively. The mass density is 7850 kg/m³ and the elasticity modulus is 210 GPa. In this example, the first 5 vibrating modes are used for identifying the damage. Therefore, consider the incomplete modal data are available.

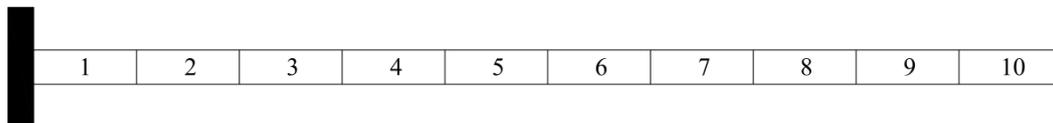


Figure 6. A cantilever beam

The finite element analysis is carried out to simulate the modal data, using two-node beam elements [14]. Here, four damage cases are assumed to investigate the capabilities of the proposed methods in detection of the occurred damage of a flexural structure. In the first damage case, the stiffness of element 2 was decreased by 30%. In damage case number two, the stiffness of elements 5 reduced by 40%. In the third damage case, the stiffness of element 2 and 5 decreased via 20% and 30%, respectively. Finally, in the damage case number four, the stiffness of element 8 reduced by 25%. Based on the proposed damage assessment algorithms, location of induced damages is detected similar to prior section. Figures 7a-b and 8c-d indicate the location of induced damages.

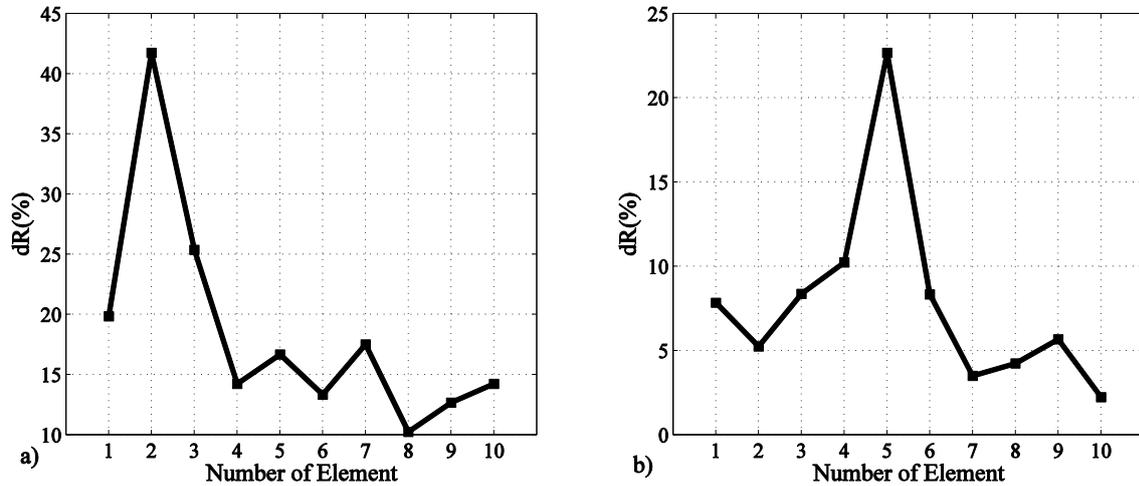


Figure 7. Damage localization of planner truss, a) Damage scenario 1, b) Damage scenario 2

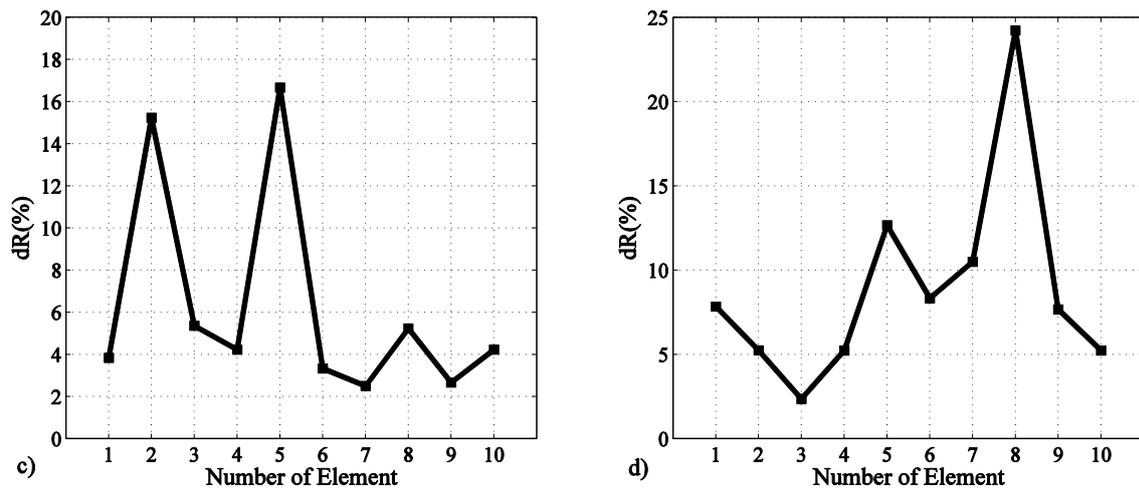


Figure 8. Damage localization of cantilever beam, c) Damage scenario 3, d) Damage scenario 4

As can be seen, similar to prior section, the location of damage is also detected as maximum peak of each figure according to damage cases. Therefore, dR factor can exactly predicted the damage location only utilizing diagonal values of stiffness error matrix. Also, these figures show that the incomplete modal data cannot cause the limitation to damage detection. In the other words, the finite element model updating can solve each problem especially incomplete data and based on updating algorithm the best result of damage assessment are achieved.

Corresponding to damage quantification, objective function of Eq. (22) is firstly minimized to specify the changes of stiffness matrix. Subsequently, Moore–Penrose inverse of stiffness sensitivity matrix S is computed. Eventually, using of Eq. (25) the vectors of damage parameters as stiffness modification factor will be estimated.

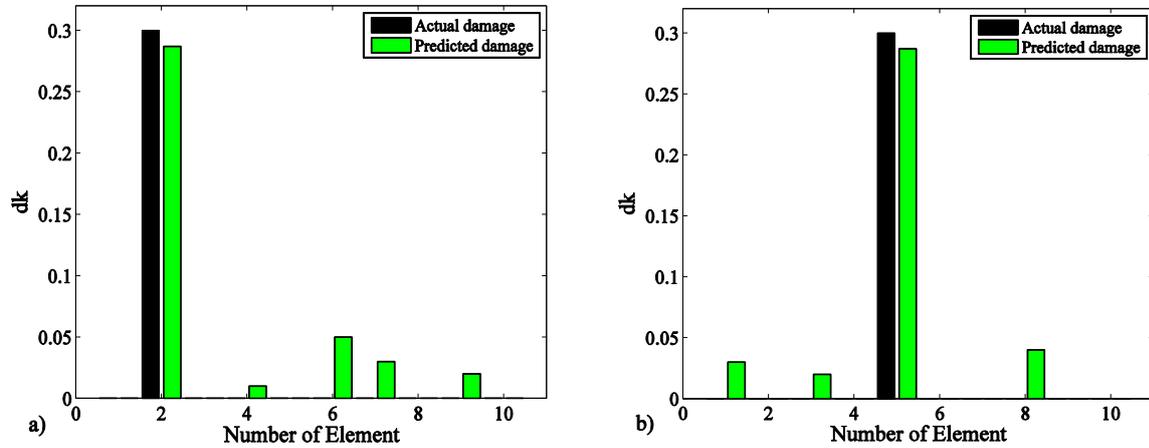


Figure 9. Predicted damage in cantilever beam, a) Damage scenario 1, b) Damage scenario 2

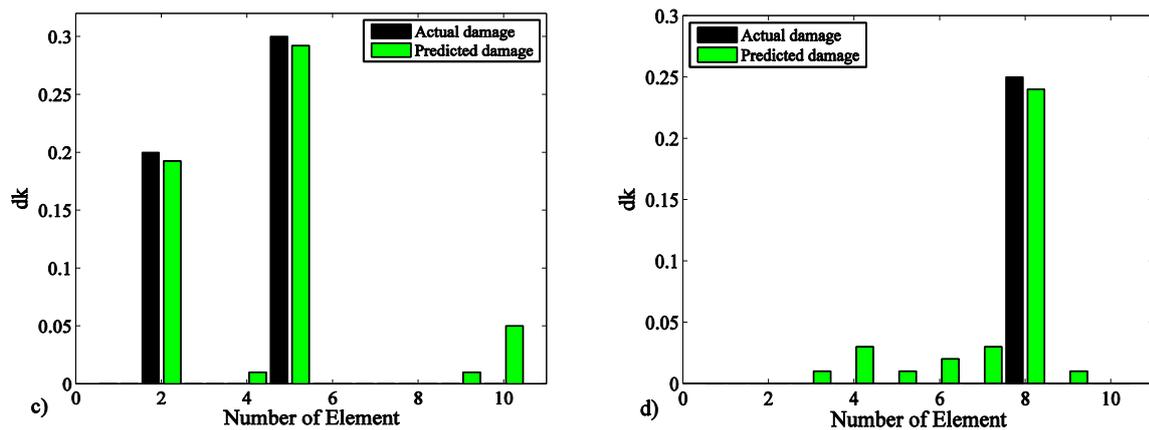


Figure 10. Predicted damage in cantilever beam, c) Damage scenario 3, d) Damage scenario 4

As mentioned to prior section, for cantilever beam the extent of damage as stiffness modification factor are also estimated corresponding to damage cases. The error function in the damage detection algorithm is approximately less than 5%. Typically, this error function pertains to prediction of initial dynamic properties of healthy structures and earlier assumption of proportional damping. Nonetheless, the identified damage is closed to actual damage states.

4. Conclusion

In this paper, the direct finite element model updating technique is presented to identification of location and extent of structural damage. At the each damage stage, a separation objective functions that related to updating of structural dynamic parameters are utilized. To overcome the difficulty of identification of incomplete modal data, the Moore-Penrose inversing method is used for the rectangular matrix of vibrational modes. The damage detection process is commenced to detect the location of damage by determination of stiffness error matrix that

can be determined as difference stiffness matrices of healthy and damage structures. After prediction of damage location, a new objective function based on sensitivity analysis of stiffness matrix is introduced to estimate of damage severity. Expansion of orthogonality conditions and eigenvalue problem provide a correct solution of damage detection method when incomplete modal data are present. Therefore, stiffness modification factor are presented to predict the damage extent. Also, the diagonal values of error stiffness matrix have accurate results for damage localization. For validation of proposed methods two numerical examples are accomplished. First, a 15-bar planer truss with four damage cases is induced to truss element. Also, a cantilever beam with 10 elements is modelled. At the all examples, generalized eigenvalue problem is utilized to extract the modal parameters. Eventually, numerical results show the proposed methods can provide reliable, simple and correct results to damage detection process, when incomplete modal data are present.

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