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Assessing manufacturing process capability with imprecise data based on fuzzy Cpi

Samaneh Asghari*[1],Bahram Sadeghpour Gileh[2]

[1] Ph. D. student, Department of Statistics, Ferdowsi University of Mashhad,
Mashhad, Iran, asghari.samaneh@stu-mail.um.ac.ir

[1]Academic member, Department of Statistics, Ferdowsi University of Mashhad,
Mashhad, Iran, Sadeghpour@um.ac.ir

Abstract: In the present study, we develop a realistic approach that allows the consideration of imprecise output data resulting from the measurements of the products quality. A general method combining the vector of fuzzy numbers to produce the membership function of fuzzy estimator of inertial capability index is introduced for further testing process capability. Two useful fuzzy inference criteria, the critical value and the fuzzy P-value, are proposed to assess the manufacturing process capability based on C_{pi}.

Keywords: Inertial capabilty index; Fuzzy hypothesis testing; Critical value; Fuzzy P-value.

1. Introduction

Process capability indices are indirect measures of potential ability to meet the required quality criteria. This indices are very usable statistics to summarize process performance and have been a focus of research in quality assurance for the last years. The most commonly indices introduced in the published literature and used in manufacturing industries are C_p, C_{pk}, C_{pm} and C_{mpk}. Montgomery [10] , Kotz and Johnson [9] dicussed such indices.

More recently, inertial capability index C_{pi} defined by Pillet [11] that has many properties, in particularly in the case of the mixed batches. There are many different situations in which the measurements of product quality are insufficiently precise. These situations with fuzzy set theory developed by Zadeh [13]. B.S. Gildeh and S. Asghari [6] obtain a confidence interval for C_{pi} based on fuzzy data and propose a membership function for it. They apply the Dp,q-distance between fuzzy numbers as a criteria to choose the preferable suppliers [7]. Buckley [1, 2] and Buckley and Eslami [4] introduced a method of estimation of mean and variance in fuzzy statistics which uses a set of confidence intervals to produce a triangular number as the estimator. Buckley [3] further developed a decision rule for hypothesis testing, which is basically determined by the percentage of the area of the fuzzy test statistic that results in a crisp rejection. Filzmoser and Viertl [5] presented an approach for testing hypotheses on the basis of fuzzy data, by introducing the fuzzy p-value. Hsu, Shu [8] and Wu [12] use fuzzy inference for testing process performance with fuzzy data based on process capability indices C_{pm} and C_{pk}, respectively. In this paper we develop this method for C_{pi}.

The organization of this paper is as follows. In section 2 by consider fuzzy data, the membership function for fuzzy estimator for C_{pi}, based on buckly's approach, is presented. Section 3 contains testing manufacturing process capability based on fuzzy estimation. At last a numerical example is given to illustrate the efficiency our approach.

2. Membership function for fuzzy estimator C_{pi}

Inertial capablity index (C_{pi}), presented by pillet [11], introduce such as

$$C_{pi} = \frac{I_{\max}}{I_{Batch}} = \frac{I_{\max}}{\sqrt{\sigma_X^2 + \delta_X^2}}, \tag{1}$$

where $I_{\max}$ indicate maximom inertial and I_{Batch} indicate root of mean square error of target $\left(I_{Batch}^2 = E(X-T)^2 = \sigma^2 + \delta^2 \right)$. Fuzzy set theory was applied for PCIs to obtain more sensitive results. Let us review some preliminaries.

Definition 1:
Let $\Re$ denote the set of real numbers. Then a fuzzy subset $\tilde{A}$ of $\Re$ is characterized by its membership function $\eta_{\tilde{A}} : \Re \to [0,1]$, which assigns to each element $x \in \Re$ a real number $\eta_{\tilde{A}}(x)$ in the interval [0, 1], where the value of $\eta_{\tilde{A}}(x)$ reflects the membership function of x in $\tilde{A}$. Thus, the value of $\eta_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$.

Definition 2:
The set of elements that belong to the fuzzy set $\tilde{A}$ at least to the degree of membership β is called the β-cuts,denoted by $\tilde{A}[\beta] = \left\{ x \,\middle|\, \eta_{\tilde{A}}(x) \geq \beta,\ x \in \Re \right\}$ where $\beta \in [0,1)$.

In this section, for obtian a fuzzy estimator for C_{pi}, we apply Buckly's approach [1, 2] . Therefor, first we construct fuzzy estimators $\tilde{\bar{X}}$ and $\tilde{S}_n^2$ from a set of confidence interval $100\,(1-\beta)\%$ with using set β-cuts fuzzy estimator $\tilde{\bar{X}}$ and $\tilde{S}_n^2$ as follows:

$$\tilde{\bar{X}}[\beta] = \left[l_{\tilde{\bar{X}}}(\beta), \psi_{\tilde{\bar{X}}}(\beta) \right] = \left[\bar{X} - t_{1-\beta/2,n-1} \frac{S_n}{\sqrt{n-1}}, \bar{X} + t_{1-\beta/2,n-1} \frac{S_n}{\sqrt{n-1}} \right], \qquad for \quad \forall \beta \in (0,1] \tag{2}$$

$$\tilde{S}_n^2[\beta] = \left[l_{\tilde{S}_n^2}(\beta), \psi_{\tilde{S}_n^2}(\beta) \right] = \left[\frac{nS_n^2}{\chi^2_{1-\beta/2,n-1}}, \frac{nS_n^2}{\chi^2_{\beta/2,n-1}} \right], \qquad for \quad \forall \beta \in (0,1] \tag{3}$$

where $t_{(1-\beta/2),n-1}$ is the $\beta/2$ percentile of the t distribution with $n-1$ degrees of freedom and $\chi^2_{1-\beta/2,n-1}$ is the $\beta/2$ percentile of the ordinary central χ^2 with $n-1$ degrees of freedom. Then place these β-cut intervals, one on top up of the other, to produce triangular shaped fuzzy numbers $\tilde{\bar{X}}$ and $\tilde{S}_n^2$, respectively. In this way we use more information than just a point estimate, or just a single interval estimation.
Figure 1 shows the membership function of the fuzzy estimators $\tilde{\bar{X}}$ and $\tilde{S}_n^2$ for the proccess with $\bar{X} = 5.187$, $S_n^2 = 1.281$ and $n = 50$.
In order to obtain membership function for $\tilde{\hat{C}}_{pi}$ choosing $g \in \tilde{\bar{X}}[\beta]$ and $h \in \tilde{S}_n^2[\beta]$,

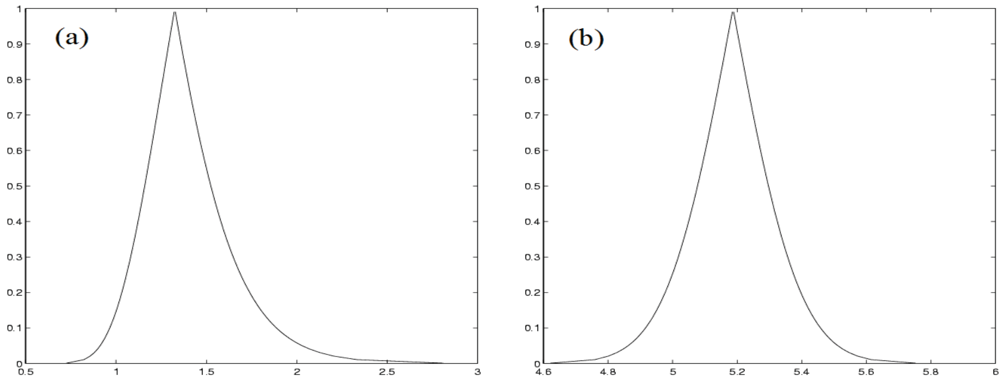

Figure 1: (a)Membership function for S_n^2 and (b) Membership function for $\overline{X}$

we obtain $\tilde{\hat{C}}_{pi}(g,h) = \dfrac{I_{\max}}{\sqrt{h+(g-h)^2}}$. The β -cut of the fuzzy estimator $\tilde{\hat{C}}_{pi}$, is defined as

$$\tilde{\hat{C}}_{pi}[\beta] = \left[l_{\tilde{\hat{C}}_{pi}}, \psi_{\tilde{\hat{C}}_{pi}} \right].$$

$\tilde{\hat{C}}_{pi}(g,h)$ is increasing function of g and decreasing function of h, when $\overline{X} \le T$ and When $\overline{X} \ge T$, is decreasing function of g and h. Therefore we can formulate the β -cut of the fuzzy estimator $\tilde{\hat{C}}_{pi}$ as follows:

If $\overline{X} \le T$, Then
$$\begin{cases} l_{\tilde{\hat{c}}_{pi}} = \dfrac{I_{\max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{\beta/2,n-1}} + \left(\overline{X} - t_{1-\beta/2,n-1}\dfrac{S_n}{\sqrt{n-1}}\right)^2}}, & (4) \\ \psi_{\tilde{\hat{c}}_{pi}} = \dfrac{I_{\max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{1-\beta/2,n-1}} + \left(\overline{X} + t_{1-\beta/2,n-1}\dfrac{S_n}{\sqrt{n-1}}\right)^2}}, & (5) \end{cases}$$

If $\overline{X} \ge T$, Then
$$\begin{cases} l_{\tilde{\hat{c}}_{pi}} = \dfrac{I_{\max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{\beta/2,n-1}} + \left(\overline{X} + t_{1-\beta/2,n-1}\dfrac{S_n}{\sqrt{n-1}}\right)^2}}, & (6) \\ \psi_{\tilde{\hat{c}}_{pi}} = \dfrac{I_{\max}}{\sqrt{\dfrac{nS_n^2}{\chi^2_{1-\beta/2,n-1}} + \left(\overline{X} - t_{1-\beta/2,n-1}\dfrac{S_n}{\sqrt{n-1}}\right)^2}}, & (7) \end{cases}$$

Figure 2 shows the membership function for fuzzy estimator $\tilde{\hat{C}}_{pi}$ **for (a)** $\tilde{\hat{C}}_{pi} = 1.221$, $I_{\max} = 0.06$, $T = 3.5$ **and** $\overline{X} = 3.510$ $(\overline{X} > T)$, $S_n = 0.0537$ **and** $n = 30, 75, 150$ **(b)**

$\tilde{\hat{C}}_{pi} = 1.143$, $I_{max} = 0.17$, $T = 2.9$ **and** $\overline{X} = 2.825$ $(\overline{X} < T)$, $S_n = 0.0537$ **and** $n = 30, 75, 150$.

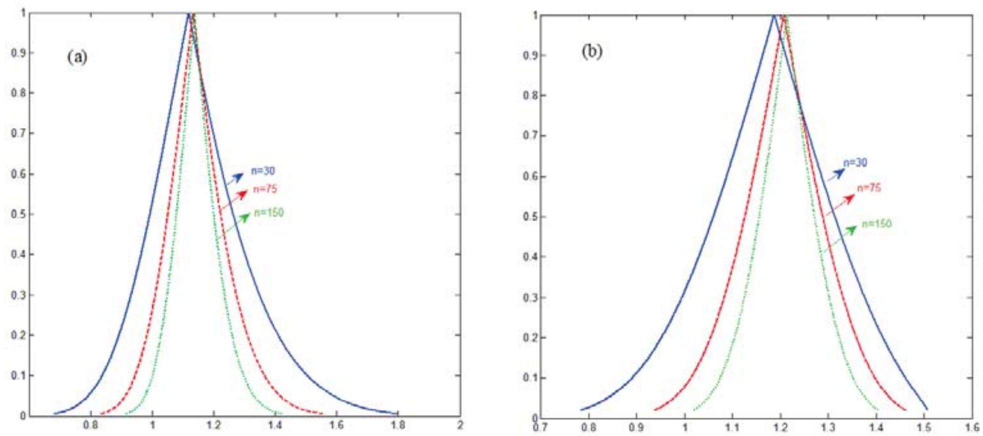

Figure 2 : (a) Membership function for fuzzy estimator $\tilde{\hat{C}}_{pi}$ when $\overline{X} < T$ and (b) when $\overline{X} > T$

3. Testing manufacturing process capability based on fuzzy estimation

For assessing process capability, we using following hypothesis test :

$$\begin{aligned} &H_0 : C_{pi} \le C \quad process \quad isnot \quad capable \ , \\ &H_1 : C_{pi} > C \quad process \quad is \quad capable \ , \end{aligned} \tag{8}$$

by using decision rules critical value and p-value, we test H_0 vs H_1 hypothesis and make decision.

3-1- Make decision based on critical value rule

In level of type one error α , the risk of misjudging an incapable process, critial value c_0 can be obtained by solving the following equation

$$P\left(\hat{C}_{pi} \ge c_0 \middle| C_{pi} = C\right) = \int_0^{\frac{C\sqrt{n(1+\hat{\xi}^2)}}{c_0}} G\left(\frac{C^2 n(1+\hat{\xi}^2)}{c_0^2} - t^2\right)\left(\phi\left(t+\xi\sqrt{n}\right) + \phi\left(t-\xi\sqrt{n}\right)\right) dt = \alpha . \tag{9}$$

In order to assessing process capability by critical value apply the following steps:
step 1: Determine value of C and α risk.
step 2: Find membership function for $\eta_{\tilde{\hat{C}}_{pi}}$.
step 3: With using values of C, α and n, find value of c_0 .
step 4: Determine β, α a certain degree of imprecision on sample data.
step 5: Conclude as the following:

(a) if $l_{\tilde{\hat{C}}_{pi}}(\beta) > c_0$, reject H_0 and accept H_1 .

(b) if $\psi_{\tilde{\hat{C}}_{pi}}(\beta) < c_0$, accept H_0 and reject H_1.

(c) if $l_{\tilde{\hat{C}}_{pi}}(\beta) < c_0 < \psi_{\tilde{\hat{C}}_{pi}}(\beta)$, cannot conclude and further study is needed.

3-2- Make decision based on p-value rule

P - value which presents the actual risk of misjudging an incapable process as a capable one also widely used for making decisions in process capability testing. Corresponding to given c^*, P - value can be obtained by solving the following equation

$$\text{P - value} = P\left(\hat{C}_{pi} \geq c^* \middle| C_{pi} = C\right) = \int_0^{\frac{C\sqrt{n(1+\hat{\xi}^2)}}{c^*}} G\left(\frac{C^2 n(1+\hat{\xi}^2)}{c^{*2}} - t^2\right)\left(\phi\left(t+\hat{\xi}\sqrt{n}\right) + \phi\left(t-\hat{\xi}\sqrt{n}\right)\right)dt \qquad (10)$$

Thus, if P - value $< \alpha$ then one rejects the null hypothesis, and concludes that the process is capable.
β -cuts fuzzy P - value can be obtained by using β -cuts $\eta_{\tilde{\hat{C}}_{pi}}$ as the following,

$$\tilde{P}[\beta] = \left[l_{\tilde{P}}(\beta), \psi_{\tilde{P}}(\beta)\right] = \left[P\left(\tilde{\hat{C}}_{pi} \geq \psi_{\tilde{\hat{C}}_{pi}}(\beta) \middle| C_{pi} = C\right), P\left(\tilde{\hat{C}}_{pi} \geq l_{\tilde{\hat{C}}_{pi}}(\beta) \middle| C_{pi} = C\right)\right]. \qquad (11)$$

Testing procedure for fuzzy P - value to assess process performance is given in the following

step 1: Determine value of C and α risk.

step 2: Obtiane membership function for $\eta_{\tilde{\hat{C}}_{pi}}$.

step 3: Obtianed membership function fuzzy p-value with using values of C, α and n.

step 4: Determine β, α certain degree of imprecision on sample data.

step 5: Conclude as the following:

(a) if $l_{\tilde{P}}(\beta) > \alpha$, accept H_0 and reject H_1.

(b) if $\psi_{\tilde{P}}(\beta) < \alpha$, reject H_0 and accept H_1.

(c) if $l_{\tilde{P}}(\beta) < c_0 < \psi_{\tilde{P}}(\beta)$, cannot conclude and further study is needed.

4. Numerical example

For process with $I_{max} = 8.33$, $T = 65$, $S_n^2 = 24.56$, $\overline{X} = 68.27$ and $n = 50$. The capability requirement in the company was set to $C = 1.10$ (capable). We must determine whether the manufacturing process meets $C_{pi} > 1.1$. We need to testing the following hypotesis

$H_0: \ C_{pi} \leq 1.1,$

$H_1: \ C_{pi} > 1.1.$

By using critical value rule for $\alpha = 0.025, 0.05, 0.1$ we obtain critical value C_0 such as ($C_0 = 1.3539$, $\alpha = 0.025$), ($C_0 = 1.3080$, $\alpha = 0.05$), ($C_0 = 1.2583$, $\alpha = 0.1$). with choice $\beta = 0.7$, obtain $l_{\tilde{\hat{C}}_{pi}}(0.7) = 1.3165$ l and $\psi_{\tilde{\hat{C}}_{pi}}(0.7) = 1.4620$. Thus for $\alpha = 0.05, 0.1$ critical value rule cause the reject H_0 ($l_{\tilde{\hat{C}}_{pi}}(0.7) > c_0$) and accept capability for process. For $\alpha = 0.025$ cannot conclude and further study is needed. By using P - value rule, first obtain membership function for p-value. α -cuts for determined $\beta = 0.7$ is $\tilde{P}(0.7) = [0.0042 \ , \ 0.0441 \ ]$. For $\alpha = 0.05, 0.1$ p-value rule cause the reject H_0 ($\psi_{\tilde{P}}(0.7) < \alpha$) and for $\alpha = 0.025$ cannot conclude and further study is needed, $l_{\tilde{P}}(0.7) < \alpha < \psi_{\tilde{P}}(0.7)$. Figures 3 and 4, illustrate this example and analysis it.

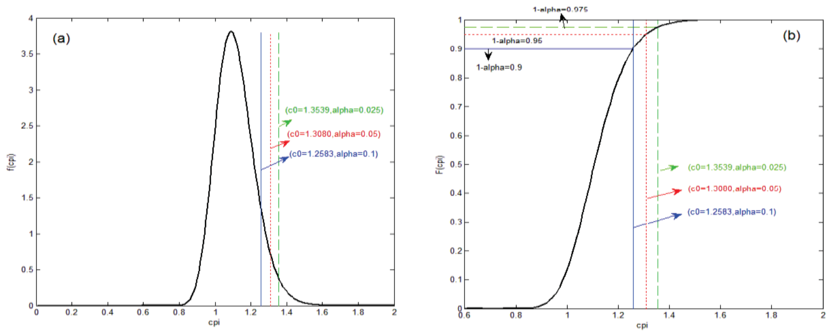

Figure 3: (a) pdf of $\tilde{\hat{C}}_{pi}$ (b) cdf of $\tilde{\hat{C}}_{pi}$

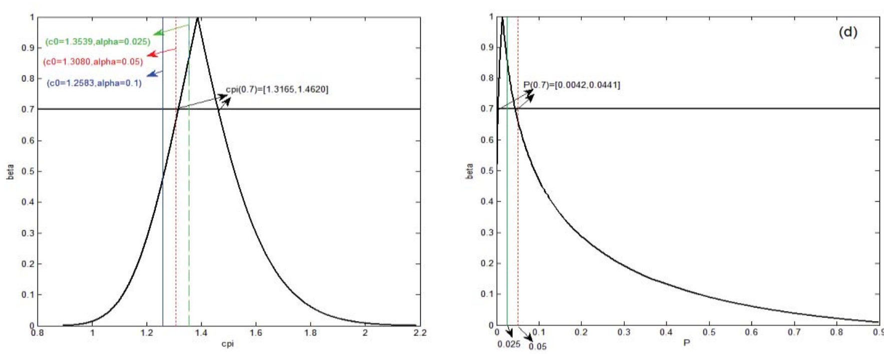

Figure 4: (c) Membership fuction of $\tilde{\hat{C}}_{pi}$ (d) Membership fuction fuzzy p-value

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