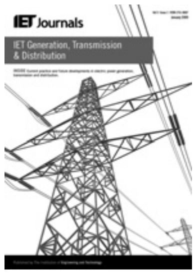
www.ietdl.org

Published in IET Generation, Transmission & Distribution
Received on 31st May 2014
Revised on 6th August 2014
Accepted on 18th August 2014
doi: 10.1049/iet-gtd.2014.0552



ISSN 1751-8687

Evaluation of price-sensitive loads' impacts on transmission network congestion using an analytical approach

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Abstract: An analytical approach is presented to evaluate the impacts of price-sensitive loads on congestion in transmission network. To achieve this aim, the Lagrange multipliers (LMs) of transmission lines' power flow constraints are calculated from independent system operator's social welfare maximisation in which demand-side bidding is considered. It is shown that the LM of each congested line can be decomposed into five components. The first part is a constant value for the specified line, while the next two components are associated to generating units and the last two constitutive parts are related to load service entities (LSEs). The proposed decomposition obtains considerable information regarding the impacts of price-sensitive loads on congestion in transmission network. First, the sensitivity of congestion degree of each congested line to the bidding strategies and maximum price-sensitive demand of LSEs are indicated by weighting coefficients of the last two terms in the decomposed LM. Furthermore, the decomposition of LM to the constitutive components reveals the contribution of each generating unit and LSE to the congestion of corresponding line. The simulation results on a test system confirm the efficiency of the proposed approach.

Nomenclature

N	set of all nodes
N_S	set of all generators
N_D	set of all load service entities (LSEs)
$\overline{N_S}$	set of all bounded generators
$\overline{N_D}$	set of all fully dispatched LSEs
L	set of all transmission lines
L_{cong}	set of all congested transmission lines
a_i, b_i	intercept and slope of bid function for generator $i \in N_S$
c_j, d_j	intercept and slope of the demand function for price-sensitive demand $j \in N_D$
Q_{Si}, $\overline{Q_{Si}}$	power generated and its upper capacity limit for unit i
Q_{Dj}	total load demanded by LSE j
Q_S^n	sum of generated power by GenCos at node $n \in N$
Q_{Dj}^S, $Q_{Dj}^{S,\max}$	price-sensitive load of LSE j and its maximum
Q_{Dj}^F	fixed load demanded by LSE j
Q_D^n	sum of power demanded by LSEs at node $n \in N$
$\underline{\alpha}_l$, $\bar{\alpha}_l$	lower and upper flow limits on line l
$\gamma_{l,n}$	power transmission distribution factor of line l due to node n
λ	Lagrange multiplier (LM) of the equality constraint
μ_i	LM of the maximum generation limit of unit i
ω_j	LM of the maximum demand limit of LSE j
$\Gamma_l^{\min}$, $\Gamma_l^{\max}$	LMs of the lower and upper constraints on the flow of line l
$\Gamma^{\max}$	the vector of $\Gamma_l^{\max}$
$H_{0,l}$, $H_{i,l}$, $H'_{i,l}$	the constant coefficient and the coefficients of a_i and $\overline{Q_{s,i}}$, representing the contribution of GenCo i on LM of line l
$H'_{j,l}$, $H''_{j,l}$	the coefficients of c_j and $Q_{Dj}^{S,\max}$, representing the contribution of LSE j on LM of line l

1 Introduction

In a deregulated electricity industry, open access transmission basis yields in more intense utilisation of transmission network by market participants, which consequently, increases the probability of transmission congestion occurrence. Congestion causes the difference in nodes' price and may lead to price volatility and price spikes [1]. All these effects depend on demand responsiveness [2–4]. Demand response can be defined as 'the changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity' [5]. A wide range of demand response programs exists. Demand bidding can be considered as a subcategory from market-based programs category of demand response [5]. A wide range of literature has focused either on the measurement of load elasticity [6–9] or on the impacts of price responsiveness of loads on electricity prices in power markets [10–14]. In [15], an agent-based simulation is employed to study the effects of demand-side bidding in the exercise of monopoly power of generators. Several congestion management methods have integrated the demand participation [16–18].

Bompard *et al.* [1] investigated the impacts of load elasticity on congestion management and pricing. They deduced that the effects of congestion on the market in terms of price and main economic metrics are alleviated as the demand elasticity increases. The influence of shifting the price responsive demand from periods of high price to periods of low price on congestion and locational marginal price is investigated in [19], although no meter for measuring the impact on congestion is introduced. Reference [20] used electricity demands responsiveness as additional decision variables in the congestion management. Factoring the price elasticity of demand in the optimal power flow (OPF) is presented in [21]. Benefits of demand response for network's operation are studied in [22–24]. In [25], an analytic approach for locational marginal price decomposition was introduced in which transmission network congestion and price-sensitivity of loads were ignored. Despite the presented studies on the demand price responsiveness, no analytical models for measuring the impacts of price-sensitive demand side on transmission congestion based on OPF can be found in the literature.

This paper proposes an analytical approach for investigating the impacts of price-sensitive loads on congestion in transmission network. This aim is accomplished in few steps. In the first step, independent system operator's (ISOs) social welfare maximisation is rephrased considering the supply and demand-side bidding. Then, the optimisation problem is solved using Lagrangian relaxation method, and Lagrange multiplier (LM) of transmission lines' power flow is calculated. Afterwards LM corresponding to each congested line is manipulated and decomposed into five components. We have declared that the first component is a constant value for each bus, which is independent from bidding strategies of GenCos and load service entities (LSEs). The second and third components are associated to generating units that include weighted summation of strategies of marginal units and generated power of bounded generating units, that is, units facing their generation caps. The fourth component of LM is the weighted sum of power demanded by fully dispatched LSEs and fifth part is weighted aggregation of bidding strategies of LSEs, which are not completely dispatched. The presented decomposition produces substantial information about the impacts of price-sensitive loads on congestion in transmission network.

Sensitivity of congestion in transmission lines to the maximum demand and bidding strategies of price-sensitive loads are indicated by weighting coefficients of the fourth and fifth term in the decomposed LM, respectively. Moreover, the decomposition of LM to the constitutive components determines the contribution of each GenCo and LSE to the congestion of corresponding line, which obtains helpful signals for system operators to identify the most influential factors on the congestion in network and consequently make proper decision for mitigating the congestion. The experimental results on a test system confirm the capability of the proposed approach.

The rest of this paper is organised as follows: Problem formulation is presented in Section 2. Sections 3 and 4 include the proposed LM decomposition and assessment of price-sensitive loads' impacts on transmission congestion, respectively. The simulation results for a test system are presented in Section 5. Finally, the paper is summarised and concluded in Section 6.

2 Problem formulation

In this work, a pool-based electricity market is considered in which both GenCos and LSEs submit their hourly bids to an ISO. Another principal trait of the electricity markets is pricing mechanism. In this paper, we focus on the closed auction with non-discriminatory pricing rule, which is the most commonly accepted structure of the spot electricity markets around the world. Assuming the quadratic form for generation cost function of GenCos, the marginal cost function will be in the form of a linear increasing function. GenCos offer a linear supply function to the ISO in which the slope and/or intercept strategically changed with regard to true supply function (marginal cost). In our work, we assume that GenCos only change their strategies by only adjusting the intercept value a_i, which is a rational and common assumption [26, 27]. So, GenCos' supply function is expressed as (1).

$$\begin{aligned} &\mathrm{bid}_i(Q_{Si}) = a_i + b_i Q_{Si} \\ &\text{where:} \quad 0 \le Q_{Si} \le \overline{Q_{Si}} \end{aligned} \tag{1}$$

We assumed that LSE j's demand is composed of a fixed component (Q_{Dj}^F) and a price-sensitive one (Q_{Dj}^S). Therefore, the demand at node j is $Q_{Dj} = Q_{Dj}^F + Q_{Dj}^S$. LSE j offers a linear inverse function for its price-sensitive demand over a known purchase interval

$$\begin{aligned} &\mathrm{bid}_j(Q_{Dj}^S) = c_j - d_j Q_{Dj}^S \\ &\text{where:} \quad 0 \le Q_{Dj}^S \le Q_{Dj}^{S,\max} \end{aligned} \tag{2}$$

Also, we assume that the LSEs adjust their bidding strategies by regulating the intercept of the line (2). Moreover, in order to evaluate the impacts of price-sensitivity of loads on transmission congestion, the ratio R is defined as [28]

$$R = \frac{Q_{Dj}^{S,\max}}{Q_{Dj}^{S,\max} + Q_{Dj}^F} \tag{3}$$

in which the denominator is maximum potential total demand (MPTD). As illustrated in Fig. 1. By increasing R from zero (100% fixed demand case) to the value one (100%

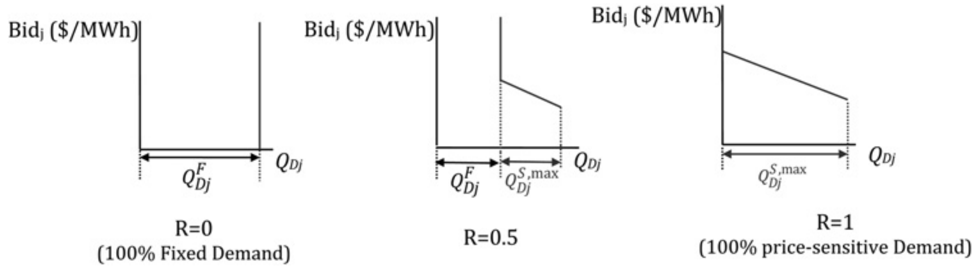

Fig. 1 *Illustration of the R ratio construction for control of relative LSE demand-bid price sensitivity*

IET Gener. Transm. Distrib., 2015, Vol. 9, Iss. 6, pp. 523–530
doi: 10.1049/iet-gtd.2014.0552

price-sensitive case) the impacts of price-sensitivity became clearer.

ISO receives the offers from the GenCos and LSEs, and then settles the market. ISO maximises social welfare while matching supply and demand and satisfying transmission network constraints as expressed in (4).

$$\begin{aligned} \mathrm{Max} J = & \sum_{j \in N_D} \left(c_j Q_{Dj}^S - 0.5 d_j Q_{Dj}^{S2} \right) \\ & - \sum_{i \in N_S} \left(a_i Q_{Si} + 0.5 b_i Q_{Si}^2 \right) \\ \text{s.t.} & \\ & \sum_{j \in N_D} Q_{Dj} - \sum_{i \in N_S} Q_{Si} = 0 \Leftrightarrow (\lambda) \\ & \underline{\alpha}_l \leq \sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n \right) \leq \bar{\alpha}_l \\ & \Leftrightarrow (\Gamma_l^{\min}, \Gamma_l^{\max}) l = 1, \ldots, L \\ & Q_{Si} \leq \overline{Q}_{Si} \Leftrightarrow (\mu_i) \\ & Q_{Dj}^S \leq Q_{Dj}^{S,\max} \Leftrightarrow (\omega_j) \end{aligned} \tag{4}$$

The Lagrangian relaxation method is employed to solve the optimisation problem in (4). The corresponding Lagrangian formulation for the maximisation problem (4) can be stated as

$$\begin{aligned} L = & \sum_{i \in N_S} \left(a_i Q_{Si} + 0.5 b_i Q_{Si}^2 \right) - \sum_{j \in N_D} \left(c_j Q_{Dj}^S - 0.5 d_j Q_{Dj}^{S2} \right) \\ & + \lambda \left(\sum_{j \in N_D} Q_{Dj}^S + \sum_{j \in N_D} Q_{Dj}^F - \sum_{i \in N_S} Q_{Si} \right) \\ & + \sum_{i \in N_S} \left(\mu_i \left(Q_{Si} - \overline{Q}_{Si} \right) \right) + \sum_{j \in N_D} \left(\omega_j \left(Q_{Dj}^S - Q_{Dj}^{S,\max} \right) \right) \\ & + \sum_{l=1}^{L} \left(\Gamma_l^{min} \left(\underline{\alpha}_l - \sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n \right) \right) \right) \\ & + \sum_{l=1}^{L} \left(\Gamma_l^{\max} \left(\sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n \right) - \bar{\alpha}_l \right) \right) \end{aligned} \tag{5}$$

3 Lagrange multiplier (LM) decomposition

Since there is a direct relationship between the transmission congestion and LMs of transmission lines in (4), in this paper the LMs of transmission lines are decomposed and then analysed to evaluate the impacts of price-sensitive loads on transmission congestion.

For simplicity and without loss of generality, it is assumed that at the market equilibrium point the directions of the power flow in transmission lines are already known. Therefore, the lower limits of the lines flow in (4) are relaxed and the lines that belong to L_{cong} bind to their maximum permissible line flows. Moreover, it is assumed that at the market equilibrium point, the power generated by units belong to $\overline{N_s}$ are limited to their upper capacity and LSEs belong to $\overline{N_D}$ are fully dispatched (i.e. power demanded by them are limited to their upper limit).

For the optimisation problem (4) with the Lagrange equation described in (5) and based on the DC power flow, Lemma 1 expresses the decomposition of LM corresponding to congested line l ($\Gamma_l^{\max}$) into five main components, derived from solving the Karush–Kuhn–Tucker (KKT) conditions for the Lagrange (5) at the market equilibrium point.

Lemma 1: For the specified network topology, and based on the DC load flow, the $\Gamma_l^{\max}$ is obtained as follows

$$\begin{aligned} \Gamma_l^{\max} = & H_{0,l} + \sum_{i \in N_S - \overline{N}_S} H_{i,l} a_i + \sum_{i \in \overline{N}_S} H'_{i,l} \overline{Q_{Si}} \\ & + \sum_{j \in \overline{N}_D} H''_{j,l} Q_{Dj}^{S,\max} + \sum_{j \in N_D - \overline{N}_D} H'''_{j,l} c_j \end{aligned} \tag{6}$$

Proof: Lemma 1 is proved in two steps. At the first step, the KKT conditions for the optimisation problem (4) at the market equilibrium point are analysed. At the second step, by manipulating the results of the KKT conditions, the Lemma 1 is proved.

KKT conditions for (4) at the market equilibrium point are presented in Appendix. The quantity of power generated by each unit and quantity of price-sensitive load dispatched at each node are given in (7) and (8), respectively.

$$\begin{cases} Q_{Si} = \left(\lambda - a_i - \sum_{l \in L_{\text{cong}}} \Gamma_1^{\max} \gamma_{l,i} \right) / b_i, & i \in N_S - \overline{N_S} \\ Q_{Si} = \overline{Q}_{Si}, & i \in \overline{N_S} \end{cases} \tag{7}$$

$$\begin{cases} Q_{Dj}^S = \left(-\lambda + c_j + \sum_{l \in L_{\text{cong}}} \Gamma_l^{\max} \gamma_{l,i} \right) / d_j, & j \in N_D - \overline{N_D} \\ Q_{Dj}^S = Q_{Dj}^{S,\max}, & j \in \overline{N_D} \end{cases} \tag{8}$$

By substituting (7) and (8) in equality constraint, λ is given in (9). (see (9))

where

$$C_1 = \sum_{i \in N_S - \overline{N_S}} \frac{1}{b_i} + \sum_{j \in N_D - \overline{N_D}} \frac{1}{d_j}$$

$$\begin{aligned} \lambda = & \frac{\sum_{j \in N_D} Q_{Dj}^F + \sum_{j \in \overline{N_D}} Q_{Dj}^{S,\max} - \sum_{i \in \overline{N_S}} \overline{Q}_{Si} + \sum_{i \in N_S - \overline{N_S}} \left(a_i / b_i \right) + \sum_{j \in N_D - \overline{N_D}} \left(c_j / d_j \right)}{C_1} \\ & + \sum_{l \in L_{\text{cong}}} \left(\Gamma_l^{\max} \frac{\sum_{i \in N_S - \overline{N_S}} \left(\gamma_{l,i} / b_i \right) + \sum_{j \in N_D - \overline{N_D}} \left(\gamma_{l,j} / d_j \right)}{C_1} \right) \end{aligned} \tag{9}$$

If there is no congestion in the network, λ is the market clearing price and equals the first term in the right-hand side of (9). From KKT condition associated to binding inequality constraints and after some manipulation which are given in Appendix, the relationship among $\Gamma_l^{\max}$s, a_is, $\overline{Q_{Si}}$, $Q_{Dj}^{S,\max}$ and c_js is obtained as (10). (see (10))

Equation (11) is the vector form of (10) and shows there is a linear relationship among $\Gamma_l^{\max}$s, GenCos bidding strategies a_is, $\overline{Q_{Si}}$, maximum price-sensitive load and LSEs bidding strategies $c_j s$

$$\alpha_{L_{cong}\times(N_S-\overline{N_S})}a+\beta_{L_{cong}\times L_{cong}}\Gamma^{\max} = C-D_{L_{cong}\times\overline{N_S}}\overline{Q_S}+E_{L_{cong}\times\overline{N_D}}\overline{Q_D^S}+F_{L_{cong}\times(N_D-\overline{N_D})}c \tag{11}$$

in which α, β, C, D, E and F are explained in Appendix in (18). Therefore the relationship between $\Gamma^{\max}$s, a_is, $\overline{Q_S}$, $\overline{Q_D^S}$ and c_js is given by

$$\Gamma^{\max}=\beta^{-1}\times C-\beta^{-1}\times\alpha\times a-\beta^{-1}\times D\times\overline{Q_S}+\beta^{-1}\times E\times\overline{Q_D^S}+\beta^{-1}\times F\times c \tag{12}$$

where a is the vector of strategies of the GenCos, contributing to the price discovery process, c is the vector of strategies of the LSEs which are not fully dispatched, $\overline{Q_S}$ is the vector of maximum generation of the units, bound to their maximum generations and $\overline{Q_D^S}$ is the vector of maximum price-sensitive power demanded by LSEs that completely dispatched.

Therefore for the congested line l, according to (12), the following equation is obtained

$$\Gamma_l^{\max}=H_{0,l}+\sum_{i\in N_S-\overline{N_S}}H_{i,l}a_i+\sum_{i\in\overline{N_S}}H'_{i,l}\overline{Q_{Si}}+\sum_{j\in\overline{N_D}}H''_{j,l}Q_{Dj}^{S,\max}+\sum_{j\in N_D-\overline{N_D}}H'''_{j,l}c_j \tag{13}$$

where

$$\begin{cases}H_{0,l}=\beta^{-1}\times C\\ H_l=-\beta^{-1}\times\alpha\\ H'_l=-\beta^{-1}\times D\\ H''_l=\beta^{-1}\times E\\ H'''_l=\beta^{-1}\times F\end{cases} \tag{14}$$

Thus, the Lemma 1 is proved. □

4 Assessment of price-sensitive loads' impacts on transmission network congestion

The LM decomposition, expressed by Lemma 1, can be effectively employed by system operator for monitoring the impacts of demand responsiveness on the transmission congestion. According to (6) in Lemma 1, the LSEs have been classified into two groups. The first group consists of LSEs that belong to $\overline{N_D}$, which are willing to pay more, so they are fully dispatched in market equilibrium. The second group includes the LSEs that belong to $N_D-\overline{N_D}$, in which the bids were lower so that they did not fully dispatch. Transmission network congestion at operating point is directly influenced by bidding strategies of second group of LSEs. Based on the proposed classifications using a similar approach as in [25], some critical points can be derived from (6):

1. Term $H'''_{j,l}c_j$ represents the contribution of LSE j to the LM associated to congested transmission line l. Furthermore, according to (15), $H'''_{j,l}$ indicates the variation in the congestion of line l according to the variation in the bidding strategy of LSE j.

$$\begin{gathered}\Gamma_l^{\max}=H_{0,l}+\sum_{i\in N_S-\overline{N_S}}H_{i,l}a_i+\sum_{i\in\overline{N_S}}H'_{i,l}\overline{Q_{Si}}+\sum_{j\in\overline{N_D}}H''_{j,l}Q_{Dj}^{S,\max}+\sum_{j\in N_D-\overline{N_D}}H'''_{j,l}c_j\\ \Rightarrow\Delta\Gamma_l^{\max}=H'''_{j,l}\times\Delta c_j\Rightarrow H'''_{j,l}=\frac{\Delta\Gamma_l^{\max}}{\Delta c_j}\end{gathered} \tag{15}$$

$$\begin{aligned}&\sum_{m\in N_S-\overline{N_S}}\left(\frac{\sum_{i\in N_S-\overline{N_S}}(\gamma_{l,i}/b_i)}{C_1b_m}-\frac{\gamma_{l,m}}{b_m}+\frac{\sum_{r\in N_D-\overline{N_D}}(\gamma_{l,r}/d_r)}{C_1b_m}\right)a_m\\
&+\sum_{k\in L_{\text{cong}}}\left[\begin{matrix}\sum_{m\in N_S-\overline{N_S}}\left(\frac{\sum_{i\in N_S-\overline{N_S}}(\gamma_{l,m}\gamma_{k,i}/b_i)}{C_1b_m}\left(-\frac{\gamma_{l,m}\gamma_{k,m}}{b_m}+\frac{\sum_{j\in N_D-\overline{N_D}}(\gamma_{l,m}\gamma_{k,j}/d_j)}{C_1b_m}\right)+\right.\\ \sum_{r\in N_D-\overline{N_D}}\left(\frac{\sum_{i\in N_S-\overline{N_S}}(\gamma_{l,r}\gamma_{k,i}/b_i)}{C_1d_r}-\frac{\gamma_{l,r}\gamma_{k,r}}{d_r}+\frac{\sum_{j\in N_D-\overline{N_D}}(\gamma_{l,r}\gamma_{k,j}/d_j)}{C_1\,d_r}\right)\end{matrix}\right]\Gamma_k^{\max}\\
&=\bar{\alpha}_l+\sum_{j\in N_D}\gamma_{l,j}Q_{Dj}^F+\sum_{j\in\overline{N_D}}\gamma_{l,j}Q_{Dj}^{S,\max}-\sum_{i\in\overline{N_S}}\gamma_{l,i}\overline{Q_{Si}}+\sum_{r\in N_D-\overline{N_D}}\gamma_{l,r}\frac{c_r}{d_r}\\
&-\left(\sum_{m\in N_S-\overline{N_S}}\frac{\gamma_{l,m}}{b_m}+\sum_{r\in N_D-\overline{N_D}}\frac{\gamma_{l,r}}{d_r}\right)\left(\frac{1}{C_1}\right)\left(\sum_{j\in N_D}Q_{Dj}^F+\sum_{j\in\overline{N_D}}Q_{Dj}^{S,\max}-\sum_{i\in\overline{N_S}}\overline{Q_{Si}}+\sum_{j\in N_D-\overline{N_D}}\left(\frac{c_j}{d_j}\right)\right)\end{aligned} \tag{10}$$

doi: 10.1049/iet-gtd.2014.0552

Thus, significant values for coefficient $H'''_{j,l}$ demonstrate high sensitivity of congestion of line l to the bidding strategy of LSE j. Therefore, knowing the values of $H'''_{j,l}$ and $H'''_{j,l}c_j$ at market equilibrium, system operator can easily determine the impact of LSE j on congestion of each line of the power grid. The factors $H'''_{j,l}$ can be either positive or negative. It means increasing in the bid of LSE j does not necessarily result in increasing of $\Gamma_l^{\max}$. Quite contrary, this may lead to congestion decreasing in line l.

2. The value of $H''_{j,l}Q_{Dj}^{S,\max}$ represents the contribution of LSE j which is fully dispatched in congestion of line l. Therefore large value of $H''_{j,l}Q_{Dj}^{S,\max}$ indicates high sensitivity of the congestion of line l to the increasing of demand by LSE j. The coefficients $H''_{j,l}$ can be either positive or negative, which means increasing in the maximum amount of power demanded by LSE j does not necessarily result in increasing of congestion of line l.

3. Term $H_{i,\ l}a_i$ represents the contribution of GenCo i to the LM associated to congested transmission line l. Moreover, in a way similar to (15), $H_{i,\ l}$ indicates the variation in the congestion of line l according to the variation in the bidding strategy of GenCo i.

According to the presented discussions, the coefficients $H''_{j,l}$ and $H'''_{j,l}$ can be employed for effective evaluation of impacts of price-sensitive loads on transmission congestion at market equilibrium. The proposed analytical approach provides a deep insight into the congestion in transmission network and the key factors affecting this phenomenon. Based on the results of this assessment, system operator can diagnose the most influential factors on the congestion in network and consequently make proper decision for mitigating the congestion. This may lead to postponement of the transmission expansion.

5 Case study

The five-bus transmission grid, which is used here for simulation, is taken from ISO-NE/PJM training manuals, where it is used to illustrate the determination of day-ahead market LMP solutions. The topology of the test system is shown in Fig. 2. The details of the line capacities, reactance levels and generators cost data are adopted from [29]. The upper limit for flow of line 5 which is linked to bus 3 and

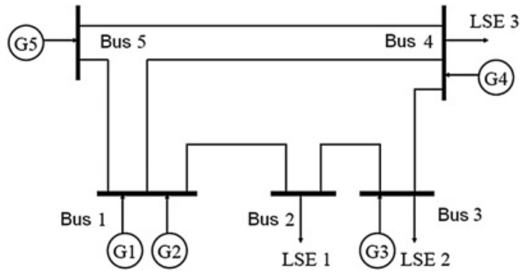

Fig. 2 *Five-bus test system [29]*

bus 4 is decreased from 240 MW to 110 MW, so possibility of congestion is acquired.

The bid and capacity information of generation units and load data are presented in Table 1. It should be noted that for all simulations MPTD is considered constant. In order to evaluate the impacts of price-sensitive loads on transmission congestion, five values for parameter R are considered (0, 0.25, 0.5, 0.75 and 1) and market is dispatched for them separately. Generating unit 4 was limited to its minimum generation, which in this simulation is considered 0. Low-cost small-size unit 1 was bounded to its upper capacity limit in all cases except $R = 1$. Table 2 presents the LM of congested line 5 and contribution of generating units in congestion of this line for different values of R. The LM decomposition results for LSEs and their impact on congestion of line 5 is demonstrated in Table 3. Based on the presented results in Tables 2 and 3, the following comments can be made:

As R was raised and consequently price-sensitivity of loads was increased, congestion of line 5 was decreased. Therefore, in case $R = 1$, there is no congestion.

As mentioned earlier, the coefficients $H''_{j,l}$ are not always positive. This means increasing the maximum demand by LSE with tendency to pay more, does not necessarily result in intensification of congestion in transmission network. For instance, in case $R = 0.5$, if LSE 2 at bus 3 increases its amount of maximum price-sensitive demand, that is, $Q_{Dj}^{S,\max}$ by 1 MW and re-dispatch the network, the congestion of line 5 increases by 0.04, 0.19% variation. In the same case, if LSE 3 at bus 4 increases its maximum price-sensitive

Table 1 Generation and load data for the test system

Unit ID	1	2	3	4	5	LSE ID	1	2	3
at node	1	1	3	4	5	at node	2	3	4
unit size, MW	110	100	520	200	600	maximum potential total demand, MW	201.0	172.3	143.6
a, \$/MWh	14	15	25	30	10	c, \$/MWh	35	40	28
b, \$/MW^{2}h	0.01	0.012	0.02	0.024	0.014	d, \$/MW^{2}h	0.18	0.08	0.12

Table 2 Weighting coefficients and LM components of the GenCos

R	$\Gamma_5^{\max}$	$H_{0,5}$	Q_{si}				$H_{2,5}$	$H_{3,5}$	$H_{5,5}$	$H'_{1,5}$	$H_{2,5}a_2$	$H_{3,5}a_3$	$H_{5,5}a_5$	$H'_{1,5}\overline{Q_{s1}}$
			$i = 1$	$i = 2$	$i = 3$	$i = 5$								
0	22.7824	−5.1684	110	35.5	42.0	331.6	−0.9768	2.1030	−1.1262	0.0117	−14.652	52.575	−11.262	1.287
0.25	22.7824	−7.1120	110	35.5	42.0	331.6	−0.9768	2.1030	−1.1262	0.0117	−14.652	52.575	−11.262	1.287
0.5	21.9141	−8.9392	110	29.7	18.7	328.8	−1.0026	1.9996	−1.1389	0.0120	−15.039	49.99	−11.389	1.320
0.75	10.3615	−84.5190	110	8.5	–	339.0	1.0682	–	−1.7116	−0.0128	16.023	–	−17.116	−1.408
1	0	–	56.7	–	–	326.2	–	–	–	–	–	–	–	–

doi: 10.1049/iet-gtd.2014.0552

Table 3 Weighting coefficients and LM components of the LSEs

R	Q^{S}_{Dj}			$H''_{1,5}$	$H''_{2,5}$	$H''_{3,5}$	$H'''_{1,5}$	$H'''_{3,5}$	$H''_{1,5}Q^{s,\max}_{D1}$	$H''_{2,5}Q^{s,\max}_{D2}$	$H''_{1,5}Q^{s,\max}_{D1}$	$H'''_{1,5}c_1$	$H'''_{3,5}c_3$
	$j=1$	$j=2$	$j=3$										
0	0	0	0	0.0271	0.0421	−0.0345	–	–	0	0	0	–	–
0.25	50.5	43.3	36.0	0.0271	0.0421	−0.0345	–	–	1.3686	1.8229	−1.2420	–	–
0.5	68.9	86.5	72.1	–	0.0400	−0.0341	0. 1419	–	–	3.4600	−2.4586	4.9665	–
0.75	91.5	129.8	106.5	–	0.5019	–	2.0338	−1.3904	–	65.1466	–	71.1830	-38.9312
1	113.5	173.0	96.0	–	–	–	–	–	–	–	–	–	–

demand by 1 MW and re-dispatch the network, the congestion of line 5 decreases by 0.0341, 0.16% variation. It must be noted this LM variation could be anticipated by using the values of $H''_{j,l}$ in Table 3.

The coefficients $H'''_{j,l}$ express the ability of LSE j to manipulate the degree of congestion in line l. For instance, in case $R=0.75$, congestion of line 5 is highly sensitive to the bidding strategies of LSE 1 at bus 2. It is interesting to note that by increase of bid of LSE 1, the LM of line 5 increases while increment in bid of LSE 3 reduces the $\Gamma_5^{\max}$. In the mentioned case, if LSE 1 raises its bidding strategy by 1 \$/MWh and re-dispatch the network, then the LM of line 5 increases by 2.0338, that is, 19.63% variation. On the other hand, under same condition, by augment of 1 \$/MWh in bidding strategy of LSE 3, the $\Gamma_5^{\max}$ decreases by −1.3904, that is, −13.42% variation. In addition, it is important to note that the absolute values of $H'''_{j,l}$ were increased as ratio R was raised.

The variation in the congestion of line l according to the variation in the bidding strategy of GenCo i is indicated by the $H_{i,\ l}$ coefficients. These coefficients can be either positive or negative. As an example in case $R=0.5$, an increment in the bidding strategy of generating unit 2 by 1\$/MWh lead to decrease of $\Gamma_5^{\max}$ by 1.0026 while the same action from generating unit 3 in this case increase the LM of line 5 by 1.9996.

Moreover, Tables 2 and 3 present the decomposed components of the LM based on (6). The influence of each LSE on transmission congestion can be identified by terms $H''_{j,l}Q^{s,\max}_{Dj}$ and $H'''_{j,l}c_j$. In a similar way, the impacts of GenCos on transmission congestion can be determined by terms $H_{i,\ l}a_i$ and $H'_{i,l}\overline{Q_{si}}$. Based on the presented results in Table 2, unit 3 is the most influential unit on the congestion of line 5. In addition, according to the demonstrated results in Table 3, LSE 1 has the largest impact on the congestion of mentioned line.

6 Conclusion

This paper presented a new analytical approach for assessing the impacts of price-sensitive loads on the transmission lines congestion by decomposing LMs associated to congested lines to constitutive components at market equilibrium. The proposed decomposition of the LM indicates the impacts of the bidding strategies and maximum demand of price-sensitive loads on the congestion in transmission lines. This research demonstrated that in the presence of price-sensitive loads, the LM is composed of five constitutive components. The first component is constant, the second component is the weighted sum of strategies of the unbounded generating units (marginal units) and the third component is the weighted sum of power generated by the units bounded by their generation caps. The fourth component is the weighted sum of demanded power of fully dispatched LSEs, and the last component is weighted aggregation of strategies of LSEs, which are not completely dispatched. The weighting coefficients with regard to each LSE and the LM components of the LSEs for each congested line can be employed by the market operator for efficient evaluation of influences of LSEs on mitigation of the congestion in transmission network. The proposed decomposition and evaluation approach was applied and tested on a five-bus test system. The simulation results illustrated the efficiency of the proposed approach.

7 References

1 Bompard, E., Carpaneto, E., Chicco, G., Gross, G.: 'the role of load demand elasticity in congestion management and pricing'. IEEE Power Engineering Society Summer Meeting, July 2000, pp. 2229–2234

2 Rajaraman, R., Sarlashkar, J.V., Alvarado, F.L.: 'The effect of demand elasticity on security prices for the PoolCO and multi-lateral contract model', *IEEE Trans. Power Syst.*, 1997, **12**, (3), pp. 1177–1184

3 Kirschen, D.S.: 'Demand-side view of electricity markets', *IEEE Trans. Power Syst.*, 2003, **18**, (2), pp. 520–527

4 Strbac, G.: 'Demand side management: benefits and challenges', *Energy Policy*, 2008, **36**, (12), pp. 4419–4426

5 Albadi, M.H., El-Saadany, E.F.: 'Demand response in electricity markets: an overview'. IEEE Power Engineering Society General Meeting, June 2007, pp. 1–5

6 Elkhafif, T., Mahmoud, A.: 'Estimating disaggregated price elasticities in industrial energy demand', *Energy J.*, 1992, **13**, (4), pp. 209–217

7 Earle, L.R.: 'Demand elasticity in the California power exchange day-ahead market', *Electr. J.*, 2000, **13**, (8), pp. 59–65

8 Boonekamp, P.G.M.: 'Price elasticities, policy measures and actual developments in household energy consumption—a bottom up analysis for the Netherlands', *Energy Econ.*, 2007, **29**, (2), pp. 133–157

9 Zachariadis, T., Pashourtidou, N.: 'An empirical analysis of electricity consumption in Cyprus', *Energy Econ.*, 2007, **29**, (2), pp. 183–198

10 Hyung Seon, O., Thomas, R.J.: 'Demand-side bidding agents: modeling and simulation', *IEEE Trans. Power Syst.*, 2008, **23**, (3), pp. 1050–1056

11 Wang, B., Li, Y., Jin, W., Wan, Q.: 'Influence of demand side bidding on peak load price based on cost analysis', *Electr. Power*, 2006, **39**, (1), pp. 31–35

12 Rassenti, S.J., Vernon, L.S., Wilson, B.J.: 'Controlling market power and price spikes in electricity networks: demand-side bidding', *Proc. Natl. Acad. Sci.*, 2003, **100**, (5), pp. 2998–3003

13 Song, H., Tang, Y., Zou, Y., Li, Y.: 'Analysis of power generation enterprise bidding strategies considering demand-side bidding', *Electr. Power Autom. Equip.*, 2007, **27**, (6), pp. 43–47

14 Kirschen, D.S., Strbac, G., Cumperayot, P., de Paiva Mendes, D.: 'Factoring the elasticity of demand in electricity prices', *IEEE Trans. Power Syst.*, 2000, **15**, (2), pp. 612–617

15 Tellidou, A.C., Bakirtzis, A.G.: 'Demand response in electricity markets', *Proc. 15th Int. Conf. Intell. Syst. Appl. Power Syst., ISAP 09*, 2009, pp. 1–6

16 Yousefi, A., Nguyen, T.T., Zareipour, H., Malik, O.P.: 'Congestion management using demand response and FACTS devices', *Electr. Power Energy Syst.*, 2012, **37**, (1), pp. 78–85

17 Sun, J., Lo, K.L.: 'A congestion management method with demand elasticity and PTDF approach'. Int. University Power Engineering Conf., 2012, pp. 1–6

18 Tuan, L.A., Bhattacharya, K., Daalder, J.: 'Transmission congestion management in bilateral markets: an interruptible load auction solution', *Electr. Power Syst. Res.*, 2005, **74**, (3), pp. 379–389

19 Singh, K., Padhy, N.P., Sharma, J.: 'Influence of price responsive demand shifting bidding on congestion and LMP in pool-based

doi: 10.1049/iet-gtd.2014.0552

day-ahead electricity markets', *IEEE Trans. Power Syst.*, 2011, **26**, (2), pp. 886–896
20 Chung, T.S., Fang, D.Z., Kong, X.Y.: 'Power market congestion management incorporating demand elasticity effects', *WSEAS Trans, Power Syst.*, 2006, **1**, (7), pp. 1378–1382
21 Jadid, S., Amoli, N.A.: 'Factoring the price elasticity of demand in the optimal power flow'. Power Eng. Conf., IPEC, 2007, pp. 12–17
22 Fahrioglu, M., Alvarado, F.L.: 'Designing incentive compatible contracts for effective demand management', *IEEE Trans. Power Syst.*, 2000, **15**, (4), pp. 1255–1260
23 Affonso, C.M., da Silva, L.C.P.: 'Potential benefits of implementing load management to improve power system security', *Electr. Power Energy Syst.*, 2010, **32**, (6), pp. 704–710
24 Fu, X., Wang, X.: 'Determination of load shedding to provide voltage stability', *Electr. Power Energy Syst.*, 2011, **33**, (3), pp. 515–521
25 Hajiabadi, M.E., RajabiMashhadi, H.: 'LMP decomposition: a novel approach for structural market power monitoring', *Electr. Power Syst. Res.*, 2013, **99**, pp. 30–37
26 Hobbs, B.F., Metzler, C.B., Pang, J.S.: 'Strategic gaming analysis for electric power systems: an MPEC approach', *IEEE Trans. Power Syst.*, 2000, **15**, (2), pp. 638–645
27 Couchman, P., Kouvaritakis, B., Cannon, M., Prashad, F.: 'Gaming strategy for electric power with random demand', *IEEE Trans. Power Syst.*, 2005, **20**, (3), pp. 1283–1292
28 Li, H., Tesfatsion, L.: 'ISO net surplus collection and allocation in wholesale power markets under LMP', *IEEE Trans. Power Syst.*, 2011, **26**, (2), pp. 627–641
29 Sun, J., Tesfatsion, L.: 'Dynamic testing of wholesale power market designs: an open-source agent-based framework', *Comput. Econ.*, 2007, **30**, (3), pp. 291–327

8 Appendix: the KKT conditions for (4) can be expressed as

$$
\begin{aligned}
&\frac{\partial L}{\partial Q_{Si}} = 0 \Rightarrow a_i + b_i Q_{Si} - \lambda + \mu_i \\
&\qquad + \sum_{l=1}^{L} \Gamma_l^{\max} \gamma_{l,i} - \sum_{l=1}^{L} \Gamma_l^{\min} \gamma_{l,i} = 0 \\
&\frac{\partial L}{\partial Q_{Dj}^{s}} = 0 \Rightarrow -c_j + d_j Q_{Dj}^{S} + \lambda + \omega_j \\
&\qquad - \sum_{l=1}^{L} \Gamma_l^{\max} \gamma_{l,j} + \sum_{l=1}^{L} \Gamma_l^{\min} \gamma_{l,j} = 0 \\
&\frac{\partial L}{\partial \lambda} = 0 \Rightarrow \sum_{j \in N_D} Q_{Dj}^{S} \\
&\qquad + \sum_{j \in N_D} Q_{Dj}^{F} - \sum_{i \in N_S} Q_{Si} = 0 \\
&\mu_i \left(Q_{Si} - \overline{Q}_{Si}\right) = 0 \\
&\quad \Rightarrow \begin{cases} \mu_i = 0 & i \in N_S - \overline{N}_S \\ \mu_i > 0 & i \in \overline{N}_S \end{cases} \\
&\omega_j \left(Q_{Dj}^{S} - Q_{Dj}^{S,\max}\right) = 0 \\
&\quad \Rightarrow \begin{cases} \omega_j = 0 & j \in N_D - \overline{N}_D \\ \omega_j > 0 & j \in \overline{N}_D \end{cases} \\
&\Gamma_l^{\max} \left(\sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n\right) - \bar{\alpha}_l \right) = 0 \\
&\quad \Rightarrow \begin{cases} \Gamma_l^{\max} = 0 & l \in L - L_{\text{cong}} \\ \Gamma_l^{\max} > 0 & l \in L_{\text{cong}} \end{cases} \\
&\Gamma_l^{min} \left(\underline{\alpha}_l - \sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n\right) \right) = 0 \\
&\quad \Rightarrow \begin{cases} \Gamma_l^{\min} = 0 & l \in L - L_{\text{cong}} \\ \Gamma_l^{\min} > 0 & l \in L_{\text{cong}} \end{cases}
\end{aligned} \tag{16}
$$

KKT condition for the binding inequality constraints of transmission lines

$$
\sum_{n \in N} \gamma_{l,n} \left(Q_S^n - Q_D^n\right) = \bar{\alpha}_l l \in L_{cong} \tag{17}
$$

By substituting Q_{Si} and Q_{Dj}^{S} from (7) and (8) into (16), we have

$$
\begin{aligned}
&\sum_{m \in N_S - \overline{N}_S} \gamma_{l,m} \left(\frac{\lambda - a_m - \sum_{k \in L_{\text{cong}}} \Gamma_k^{\max} \gamma_{k,m}}{b_m} \right) \\
&- \sum_{r \in N_D - \overline{N}_D} \gamma_{l,r} \left(\frac{-\lambda + c_r + \sum_{k \in L_{\text{cong}}} \Gamma_k^{\max} \gamma_{k,r}}{d_r} \right) \\
&= \bar{\alpha}_l + \sum_{j \in N_D} \gamma_{l,j} Q_{Dj}^{F} + \sum_{j \in \overline{N}_D} \gamma_{l,j} Q_{Dj}^{S,\max} \\
&\quad - \sum_{i \in \overline{N}_S} \gamma_{l,i} \overline{Q}_{Si} l \in L_{cong}
\end{aligned} \tag{18}
$$

After replacing λ from (9) into (17) and some manipulation, (10) can be attained. Matrices α, β, C, D, E and F from (11) are defined as (see (19) at the bottom of next page)

$$
\begin{aligned}
\alpha(l, m) &= \frac{\sum_{i\in N_S-\overline{N}_S}\left(\gamma_{l,i}/b_i\right)}{C_1 b_m} - \frac{\gamma_{l,m}}{b_m} + \frac{\sum_{r\in N_D-\overline{N}_D}\left(\gamma_{l,r}/d_r\right)}{C_1 b_m}\\
\beta(l, k) &= \sum_{m\in N_S-\overline{N}_S}\left(\frac{\sum_{i\in N_S-\overline{N}_S}\left(\gamma_{l,m}\gamma_{k,i}/b_i\right)}{C_1 b_m} - \frac{\gamma_{l,m}\gamma_{k,m}}{b_m} + \frac{\sum_{j\in N_D-\overline{N}_D}\left(\gamma_{l,m}\gamma_{k,j}/d_j\right)}{C_1 b_m}\right)\\
&\quad + \sum_{r\in N_D-\overline{N}_D}\left(\frac{\sum_{i\in N_S-\overline{N}_S}\left(\gamma_{l,r}\gamma_{k,i}/b_i\right)}{C_1 d_r} - \frac{\gamma_{l,r}\gamma_{k,r}}{d_r} + \frac{\sum_{j\in N_D-\overline{N}_D}\left(\gamma_{l,r}\gamma_{k,j}/d_j\right)}{C_1 d_r}\right)\\
C(l) &= \bar{\alpha}_l + \sum_{j\in N_D}\left(\gamma_{l,j} - \left(\sum_{m\in N_S-\overline{N}_S}\frac{\gamma_{l,m}}{b_m} + \sum_{r\in N_D-\overline{N}_D}\frac{\gamma_{l,r}}{d_r}\right)\left(\frac{1}{C_1}\right)\right)Q_{Dj}^{F}\\
D(l, j) &= \gamma_{l,j} - \frac{\sum_{m\in N_S-\overline{N}_S}\left(\gamma_{l,m}/b_m\right) + \sum_{r\in N_D-\overline{N}_D}\left(\gamma_{l,r}/d_r\right)}{C_1}\\
E(l, u) &= \gamma_{l,u} - \frac{\sum_{m\in N_S-\overline{N}_S}\left(\gamma_{l,m}/b_m\right) + \sum_{r\in N_D-\overline{N}_D}\left(\gamma_{l,r}/d_r\right)}{C_1}\\
F(l, z) &= \frac{\gamma_{l,z}}{d_z} - \frac{\sum_{m\in N_S-\overline{N}_S}\left(\gamma_{l,m}/b_m\right) + \sum_{r\in N_D-\overline{N}_D}\left(\gamma_{l,r}/d_r\right)}{C_1 d_z}
\end{aligned}
\tag{19}
$$

IET Gener. Transm. Distrib., 2015, Vol. 9, Iss. 6, pp. 523–530
doi: 10.1049/iet-gtd.2014.0552