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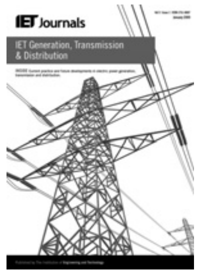
Published in IET Generation, Transmission & Distribution
Received on 30th August 2011
Revised on 24th January 2013
Accepted on 15th April 2013
doi: 10.1049/iet-gtd.2012.0370



ISSN 1751-8687

Cobweb theory-based generation maintenance coordination in restructured power systems

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Abstract: A cobweb theory-based maintenance coordination algorithm is proposed in this study. It is an iterative process in which, initially, the independent market operator (IMO) declares the interval electricity price for the period of concern. The generation companies (GenCos) would then provide the independent system operator (ISO) and the IMO with their own maintenance proposals; based on their own objectives and constraints. The ISO would evaluate the impact of the maintenance proposals on the reliability indices and assign some penalties/rewards to the GenCos; in proportion to their contributions in reliability index violation from a desirable level. On the other hand, the IMO would declare the new electricity prices, considering the new maintenance proposals. These two signals would be used by the GenCos to review and modify their maintenance proposals. The procedure is repeated until an equilibrium point is reached. For convergence assurance, a memory rate is introduced by which the GenCos earlier experiences in providing the ISO with the maintenance proposals are, somehow, taken into account. The capabilities of the proposed algorithm are assessed and evaluated on IEEE reliability test system.

Nomenclature

Sets and indices[1]

i	index for a GenCo
k	index for a generating unit
t	index for the time interval
r	index for an iteration of cobweb theory based process
z	index for the convergence criterion
Δ	set of GenCos
Λ_i	set of generating units of i

Constants and parameters

a	slope of the supply curve (MW^{-1})
$cmax_{k,i}$	maximum generation capability of ki (MW)
$cmin_{k,i}$	minimum generation capability of ki (MW)
CUE	cost of unserved energy (\$/MWh)
DP_t	duration of t (in hours)
$EENS^r_{t,max}$	standard maximum expected energy not supplied at tr (MWh)
EIR_{ref}	standard energy index of reliability (%)
EIR^r_t	energy index of reliability at tr (%)
$FC_{k,i}$	fixed cost of ki (\$/h)
$FOR_{k,i}$	forced outage rate of ki
L_t	load at t (MW)
$MC_{k,i,t}$	maintenance cost of kit (\$)
$MD_{k,i}$	maintenance duration of ki
$MN_{i,t}$	maximum number of generating units of it allowed to go on maintenance
N	number of GenCos
N_i	number of generating units belonging to i
NG	number of generating units
$PC_{k,i}$	production cost of ki (\$/MWh)
T	total number of maintenance coordination intervals
TE_t	total energy required to be supplied at t (MWh)
$u_{k,i,t}$	utilisation factor of kit (%)
μ_t	expected supply shift index at t:
γ_i	memory rate of i ($0 \leq \gamma_i \leq 1$)

[1](*Note:* For conciseness in nomenclature, combinations of indices are also used. For instance, *kit*, means generating unit k of GenCo i at interval t.)

Variables

B^r_i	profit function of ir (\$)
b^r_t	supply shift index at tr
C^r_t	total capacity at tr (MW)
$Cmax_{k,i}$	Bernoulli random variable for the capacity of ki (MW)
$Cont^r_{k,i,t}$	contribution of ki in $EENS^r_t$ (MWh)
$EENS^r_t$	expected energy not supplied at tr (MWh)
$g^r_{k,i,t}$	generation quantity of $kitr$ (MWh)
$Pen^r_{k,i,t}$	penalty assigned to $kitr$ (\$)
$Rew^r_{k,i,t}$ $\sum'_1$	reward assigned to $kitr$ (\$)
$Tpen^r_{k,i,t}$	total cumulative penalty of $kitr$ (\$)
$Trew^r_{k,i,t}$	total cumulative reward of $kitr$ (\$)
$WEWAP^r$	weekly energy weighted average price in r
$\boldsymbol{X}^r_i$	maintenance outage index matrix of ir

$x^r_{k,i,t}$ maintenance outage index of $kitr$ (= 1 if out, = 0 if in service)
ρ^r_t electricity market price at tr (\$/MWh)

1 Introduction

Generation maintenance scheduling is an issue of concern in mid-term power system planning. In a restructured environment, this scheduling is replaced by maintenance coordination through independent system operator (ISO). The generation companies (GenCos) propose their optimal maintenance schedules to the ISO, aiming at maximisation of their own profits. These proposals are typically carried out in a non-coordinated decentralised manner. The ISO would then run an algorithm to coordinate the proposals, whereas observing the technical and reliability constraints [1–6].

The coordination process may be simulated in either a static or a dynamic way. The former is mainly based on game theory [2–4] in which some simplifying assumptions (e.g. considering uniform behaviours in playing game and deterministic criteria in reliability evaluation) result in lack of practical capability. The dynamic approaches are proposed for practical applications. In [5, 6] are two typical research reported in which an iterative procedure is proposed. Rational GenCos, without observing the other GenCos actions and their private information, provide the ISO with their proposals on maintenances. To preserve the reliability, the ISO generates some incentives/disincentives or maintenance capacity limitation, and communicates with the GenCos in an iterative procedure to reach an agreement on maintenance schedules.

The present paper proposes a procedure, aiming at resolving the drawbacks of existing dynamic approaches as follows:

- The usefulness may be seriously affected by considering the probabilistic criteria of reliability.
- The proposed procedures are based on fixed market prices, insensitive to maintenance schedules. However, the price variation can be used as extra information, in addition to incentives/disincentives; to help the GenCos and the ISO to adjust the maintenance schedules in a weaker conflicting way [7].

On the one hand, ‘the conflict in the maintenance coordination is the result of the lack of liquidity in the electricity markets. By designing instruments to send the future price signals, responding to maintenance schedules one year ahead, the conflict would be removed and a market-based maintenance scheduling would be emerged’ [7, 8]. On the other hand, if the mid-term prices are not transparent enough, economically efficient maintenance schedules could not be achieved [9]. Therefore if a GenCo is provided with the future price variation in response to maintenance schedules, it can obtain some information of the rivals’ maintenance proposals (without the need to have the complete information of the rivals) and as a result, the maintenances could be scheduled in a more efficient and a less conflicting way.

To provide the GenCos with the future price variation, either an extra supplementary forward electricity market should be established, or a proxy of its behaviour should be simulated, in which, the resulting prices are publicly made available. Such a market design carries out the features that are similar to those of the capacity markets, designed to solve the adequacy problems of energy-only markets [10]. Especially, the participation of the GenCos (to submit the offers and to schedule the maintenances) and load serving entities, or at least the ISO as a substitute (to forecast the interval-based loads) in the proposed forward market, must be mandatory. This market can simulate the future interval-based prices so that the maintenance impacts are accounted for. Designing such a market can be a subject of another research and is out of the scope of this paper. Here, we assume that a proxy of the market behaviour is available, provided by the independent market operator (IMO) that would operate the mentioned forward market, if it is established. In this case, the IMO is not responsible for price forecasting, and, only provides the GenCos with the prices that would result.

In this paper, a cobweb theory based model is proposed for maintenance coordination process, in which an economic theoretic framework to include the price variation impact, is employed. The cobweb theory was first proposed in [11] and, later on, developed and extended as summarised in [12]. To the best of our knowledge, the only application of cobweb theory in an electricity market is reported in [13] for a bidding strategy.

The cobweb theory based model is a dynamical system that describes price fluctuations as a result of the interaction between demand functions; depending on current price, and supply function; depending on the expected price. In this model, based on initial market price expectation, the suppliers provide the best quantities. Based on these and with due attention to the demand function, a new market price is achieved. This, in turn, leads to a modification in initially proposed quantities. The process is repeated until an equilibrium point is reached. Based on how the price expectation and the demand function are modelled, the complexity of the cobweb-based model may vary, as follows:

- The price expectation modelling may be naive, forward looking, backward looking or a combination of the last two [14, 15].
- Each approach may be either deterministic or stochastic in providing the future expectation to the suppliers.
- The demand function may be, linear or non-linear, too.

In a cobweb theory based model, the decision variables are the quantities to be determined by the players. In a maintenance coordination problem, the GenCos (the players) should propose their maintenance capacities (the quantities) to the ISO. The maintenance schedule is coordinated through profit maximisation, whereas such a schedule is highly affected by the market price. Hence, if the price is properly modelled in terms of the available capacities, it would act as the inverse of the demand function. In this way, the cobweb theory may be effectively used in a maintenance coordination problem. Some improvements in theory are required that is further discussed in Section 2.

This paper is organised as follows. Conceptual framework of the model is introduced in Section 2. Maintenance scheduling formulation is described in Section 3. The cobweb theory based model elements are described in Sections 4–6. Numerical results are demonstrated in Section 7. Some concluding remarks are provided in Section 8.

2 Conceptual framework

The cobweb theory based model is commonly used to simulate the markets without any surveillance. In the

doi: 10.1049/iet-gtd.2012.0370

maintenance coordination problem, the cobweb theory-based coordination is carried out by the ISO; because of its responsibility in keeping system reliability. Therefore some modifications are required to consider the ISO role in the proposed coordination process. In this paper, the ISO responsibility is implemented through devising penalty/reward control signals based on the probabilistic evaluation of system reliability (see Section 5 for details).

The proposed maintenance coordination procedure, as depicted in Fig. 1, is as follows:

1. Initially and based on the weekly electricity price, provided by the IMO, the GenCos propose their generating units maintenance schedules by maximising their own profits.
2. Based on the proposal as given by the GenCos, the new weekly prices would be determined, by either the supplementary forward market or its proxy. Then, the IMO provides the GenCos with the new prices (as explained in Section 4).
3. The ISO would then calculate the reliability index and compare it with an acceptable level for each interval (as explained in Section 5). Based on the contribution of each GenCo in reliability reduction/increase of each interval, the ISO will send penalties/rewards to the GenCos to reschedule their maintenances.
4. By considering the penalties/rewards and the new prices, the GenCos would independently reschedule their maintenance proposals (as explained in Section 6).
5. The convergence criterion is checked to see if the whole process should be restarted from step (2). This criterion may be chosen to be similar solutions in consecutive iterations.

The convergence property of the proposed model is crucial in terms of reaching an equilibrium point. As the quantities in each iteration are not determined in a coordinated way, if proper precautions are not foreseen, the algorithm may fail to converge. The ISO is, however, the final decision maker. So, the players should use the past experience in using the control (penalty/reward) signals as sent by the ISO. Memory rate is introduced in stage (4) above for the proper convergence property. In this new approach, that makes the proposed cobweb theory based model different from the others, the expectation of the probable penalties/rewards is calculated based on the previous ISO signals observations. The details are given in Section 6.

3 GenCo maintenance scheduling formulation

The maintenance scheduling problem is formulated in this section. The objective function and the constraints are described as follows:

3.1 Objective function

An energy-only market is assumed in this paper; although, other market mechanisms may also be considered. Therefore a rational GenCo profit function, in each iteration, is as follows

$$\max B_i^r(X_i^r) = \sum_{t=1}^{T}\sum_{k=1}^{N_i}\big[(\rho_t^r - PC_{k,i})cmax_{k,i}u_{k,i,t}DP_t - \big(MC_{k,i,t} - Trew_{k,i,t}^r + Tpen_{k,i,t}^r\big)x_{k,i,t}^r\big] \quad \forall i \in \Delta \tag{1}$$

where the first term $\big(\big((\rho_t^r - PC_{k,i}\big)cmax_{k,i}u_{k,i,t}DP_t\big)\big)$ shows the interval-based GenCo profit.

$u_{k,i,t}$ is the utilisation factor of the generating unit k of GenCo i at interval t. It shows the portion of the energy

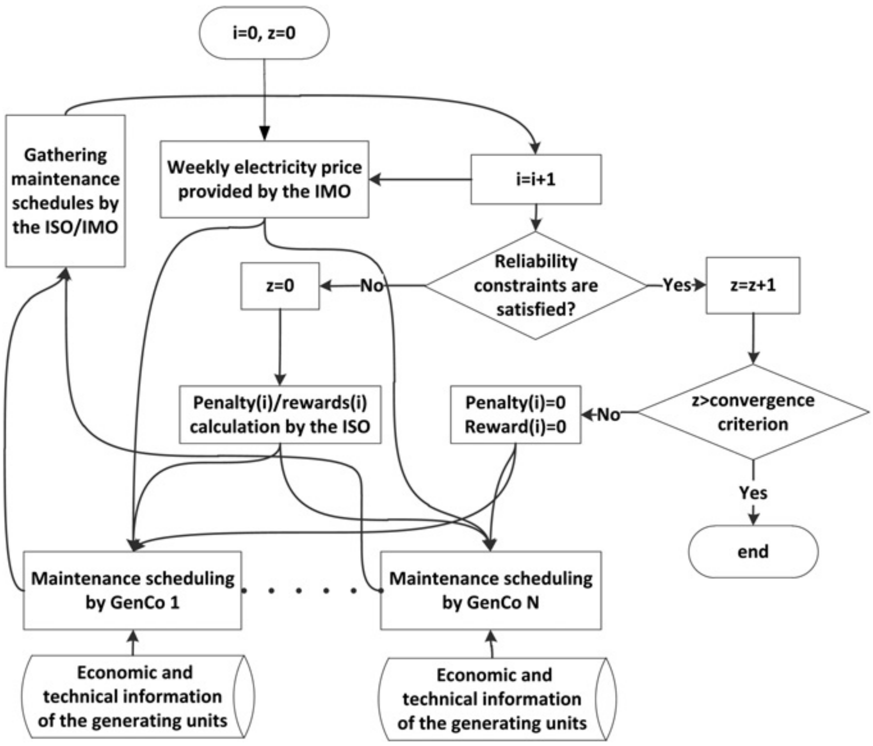

Fig. 1 *Generation maintenance coordination algorithm*

produced by the generating unit during an interval (e.g. one week). It, generally, depends on generating unit type, commitment policy and bidding strategy. As the first one is the dominant parameter, it is assumed here that the utilisation factor can be properly estimated by using historical data. Considering this item in profit calculation; making objective function different from the one used in [5, 6], is very essential to estimate a more realistic market-based profit.

The second term $\left(\left(MC_{k,i,t} - Trew^r_{k,i,t} + Tpen^r_{k,i,t}\right)x^r_{k,i,t}\right)$ in (1) is the cost incurred because of the maintenance of generating unit k of GenCo i at interval t in which the penalties/rewards as imposed by the ISO are also observed. During the first iteration of the cobweb theory based process, the penalties/rewards are considered to be zero. The penalties/rewards for the next iterations would be considered with the details given in Section 5.

It should be noted that (1), analogous to [5, 6], explains and simulates the rational GenCos behaviour during the process iterations. As seen in the following sections, to execute the assigned functions, the ISO and the IMO evaluate the GenCos maintenance proposals in each iteration, without concerning the objective of the GenCos. Therefore implementing the proposed coordination process does not require knowing the objectives of the GenCos.

In each iteration, the GenCos would propose their maintenance schedules, without being aware of the rivals' behaviours and the effect on the electricity prices. The main focus of this paper is on the price variation effects on the GenCos decision, not the market power exercise. It is assumed that, each GenCo has not enough market power to manipulate the price, but two or more GenCos may have such power. Therefore the price variation provides the GenCos with a communication line and carries the information of the other GenCos maintenance schedules effects, from one GenCo viewpoint. In this way, a GenCo uses the price variation to schedule the maintenance, not deciding to manipulate the prices. Studying market power exercise may be subject of another research.

3.2 Constraints

The set of constraints to be observed is specified below, as explained in details in [1, 4, 5]. Conejo *et al.* [5] justifies all these constraints by using illustrative examples.

3.2.1 Maintenance continuity: Equation (2) ensures that the maintenance of any generating unit must be completed once it begins; considering the required number of time intervals.

$$\begin{aligned} &\sum_{t=1}^{T} x^r_{k,i,t} = MD_{k,i} \\ &x^r_{k,i,t} - x^r_{k,i,t-1} \le x^r_{k,i,t+MD_{k,i}-1} \\ &\forall k \in \Lambda_i, i \in \Delta, \textit{ and } t \in [1,T] \end{aligned} \tag{2}$$

The first equation in (2) ensures that the number of time intervals of the maintenance of generating unit k of GenCo i is $\mathrm{MD}_{k,i}$. The second relation guarantees that the maintenance intervals are consecutive.

3.2.2 Maintenance priority: In some cases, a GenCo may wish to give a priority of maintenance to some of its units. This may be accomplished by using (3).

$$\sum_{t'=1}^{t} x^r_{k_1,i,t'-1} - x^r_{k_2,i,t} \ge 0 \; \forall k_1, k_2 \in \Lambda_i, i \in \Delta, \text{and } t \in [1,T] \tag{3}$$

Equation (3) enforces that if $x^r_{k_2,i,t_1} = 1$, which means generating unit k_2 is on maintenance in t_1, $x^r_{k_1,i,t}$ must take 1 in $[1, \; t_1 - 1]$. This means that generating unit k_1 maintenance must be started soon.

3.2.3 Maintenance coincidence: The maximum number of the generating units which can be maintained at an interval may be limited. This may be observed by using (4).

$$\sum_{k=1}^{N_i} x^r_{k,i,t} \le MN_{i,t} \quad \forall i \in \Delta \text{ and } t \in [1,T] \tag{4}$$

Equation (4) enforces that sum of $x^r_{k,i,t}$, that shows the maintenance status of generating units of GenCo i in interval t, is lower than $\mathrm{MN}_{i,t}$.

3.2.4 Maintenance exclusion: This constraint enforces the impossibility of scheduling two pre-specified generating units maintenances at the same interval.

$$x^r_{k_1,i,t} + x^r_{k_2,i,t} \le 1 \forall k_1, k_2 \in \Lambda_i, i \in \Delta, \text{ and } t \in [1,T] \tag{5}$$

Any other constraints such as those introduced in [5, 6] may also be observed and included.

A GenCo's maintenance scheduling problem described by (1)–(5) is a mixed integer linear programming problem and can be solved by using various available solvers, such as CPLEX [5, 6, 16].

In each iteration, the rational GenCos solve the scheduling problem described by (1)–(5). The coordination process is iterative, but may lead to oscillation. This means that the electricity price alone may not be sufficient in reaching the reasonable results, ensuring system reliability. This problem is shown by using a simple example in the first section of Appendix.

4 IMO coordination activities

The price, in this paper, is considered to be a function of the fundamental drivers. The basic model is already proposed in [7, 17] by which the long-term electricity price, considering the supply uncertainties, can be properly modelled.

The electricity market price, considering both the supply and the demand, as the main drivers, can be modelled as follows

$$\rho^r_t = e^{aL_t + b^r_t}, \forall t \in [1,T] \tag{6}$$

The above equation shows that the price in interval t is an exponential function of supply (b^r_t) and load (L_t). These two parameters exhibit, in general, stochastic behaviours, quantifying the uncertainty of the price movements. In general, the proposed price model captures the basic physical and economic relationships, present in the production and the trading of the electricity. Essential explanations and justifications can be found in [7, 17].

IET Gener. Transm. Distrib., 2013, Vol. 7, Iss. 11, pp. 1253–1262
doi: 10.1049/iet-gtd.2012.0370

A probabilistic model of b_t^r, that is generally a function of available capacity, is assumed as [7].

$$b_t^r = \mu_t - aC_t^r, \forall t \in [1,T] \quad (7)$$

where b_t^r has two fundamental elements, namely, μ_t and C_t^r. μ_t is a deterministic parameter (to be forecasted through historical data) which captures the average supply seasonality. In the high average price intervals, μ_t is higher than that of in low average price intervals. C_t^r is a random variable showing the available capacity. It is denoted as a summation of Bernoulli random variables, which is a common way of generation uncertainty modelling in the literature [18], as follows

$$C_t^r = \sum_{i=1}^{N}\sum_{k=1}^{N_i}\left(1 - x_{k,i,t}^r\right)Cmax_{k,i} \forall t \in [1,T] \quad (8)$$

in which

$$Cmax_{k,i} = \begin{cases} cmax_{k,i} & p(Cmax_{k,i} = cmax_{k,i}) = 1 - FOR_{k,i} \\ 0 & p(Cmax_{k,i} = 0) = FOR_{k,i} \end{cases}$$
$$\forall k \in \Lambda_i, \text{and } i \in \Delta \quad (9)$$

where $p(.)$ is the probability operator.

In both (6) and (7), a is a parameter, depending mainly on load levels (such as peak, off-peak etc.), reflecting proper seasonal price response to available capacity. L_t is assumed to be known that can be forecasted by using historical data.

5 ISO coordination activities

As shown in Appendix, although, the electricity price can help the GenCos to adjust the maintenance schedules in an efficiently economic way, it cannot be regarded as a sufficient signal to preserve the reliability within the acceptable limits. Therefore some appropriate penalty/reward signals should also be devised and proposed, so that the GenCos, individually, try to reschedule their maintenance proposals to the preferred ISO intervals.

From various reliability indices, categorised as being probabilistic and deterministic [18], the energy index of reliability (EIR), as an index from the former category is employed in this paper. It is calculated for each interval, as given below

$$EIR_t^r = 1 - \frac{EENS_t^r}{TE_t} \quad \forall t \in [1,T] \quad (10)$$

where $EENS_t^r$, as the expected energy not supplied (EENS) in interval t, is calculated as given in [18]. Initially, the interval-based load duration curves are drawn. Based on the available generating units technical characteristics, the capacity outage probability table (COPT) is generated for each interval. Therefore $EENS_t^r$ can be calculated. It is clear that the COPT may be changed in each iteration based on the maintenance proposals.

A higher EIR is more favorable. The higher the EIR, the more the system costs would be. For each reference EIR (EIR_{ref}), there would be a maximum EENS ($EENS_{t,\max}$), so that maintenance of generating units must not result in EENS exceeding $EENS_{t,\max}$. The contribution of the maintenance schedule of generating unit i in iteration r, in increasing EENS at interval t, is calculated as

$$Cont_{k,i,t}^r = \frac{\left(x_{k,i,t}^r cmax_{k,i}/(1 - FOR_{k,i})\right)\text{psgn}\left(EENS_t^r - EENS_{t,\max}\right)}{\sum_{i=1}^{N}\sum_{k=1}^{N_i} x_{k,i,t}^r cmax_{k,i}/\left(1 - FOR_{k,i}\right)}$$
$$\forall k \in \Lambda_i, \quad i \in \Delta, \text{ and } t \in [1,T] \quad (11)$$

where psgn(z) is equal to z, if z is positive; otherwise zero.

In (11), if generating unit i of GenCo k in interval t goes on maintenance ($x_{k,i,t}^r = 1$) and the reliability criteria is violated $\left(EENS_t^r > EENS_{t,\max}\right)$, the contribution of the generating unit in the violation (MWh) is proportional to a weighted maximum capacity ($cmax_{k,i}$) with the weight of $1/(1 - FOR_{k,i})$, to the sum of the weighted maximum capacity. In practice, the exact value for the contribution is more complicated than the one shown in (11) and needs the unit commitment to be taken into account. However, believing that the available capacity is the most important factor of ensuring the reliability level, the approach presented in (11) is sufficient for our purposes. Moreover, the approach guarantees that more unreliable generating units would pay more penalties (per MW).

Based on the value of $Cont_{k,i,t}^r$, the penalty is calculated as

$$Pen_{k,i,t}^r = CUE \times Cont_{k,i,t}^r \; \forall k \in \Lambda_i, i \in \Delta, \text{and } t \in [1,T] \quad (12)$$

where the cost of unserved energy (CUE) typically represents the cost of a substitute energy, which could be from an expensive generation or the ISO's payments for an interrupted power [6]. In general, CUE is valued differently in different hours of the year. In this paper, for the sake of simplicity, an annual average value is used.

As the ISO is an independent entity, the penalties in each iteration as imposed should be, somehow, prorated among generating units that improve the reliability index, as rewards. With aiming of equating the penalties with the rewards, the rewards are calculated as

$$Rew_{k,i,t}^r = \frac{x_{k,i,t}^r(1 - FOR_{k,i})cmax_{k,i}}{\sum_{i=1}^{N}\sum_{k=1}^{N_i} x_{k,i,t}^r(1 - FOR_{k,i})cmax_{k,i}} \times \frac{\left(1 - \prod_{i=1}^{N}\prod_{k=1}^{N_i}\left(1 - x_{k,i,t}^r\right)\right)\text{psgn}\left(EENS_{t,\max} - EENS_t^r\right)}{\sum_{t=1}^{T}\left(1 - \prod_{i=1}^{N}\prod_{k=1}^{N_i}\left(1 - x_{k,i,t}^r\right)\right)\text{psgn}\left(EENS_{t,\max} - EENS_t^r\right)} \times \sum_{i=1}^{N}\sum_{k=1}^{N_i}\sum_{t=1}^{T} Pen_{k,i,t}^r \quad (13)$$
$$\forall k \in \Lambda_i, i \in \Delta, \text{and } t \in [1,T]$$

The way (13) is formulated results in assigning rewards to each generating unit in proportion of its contribution in improving $EENS_t^r$ (with respect to $EENS_{t,\max}$) and only for those intervals for which proposals on maintenance exist.

The first term in (13) shows the contribution of each generating unit in each interval. The second term shows the

Table 1 Generating unit data for the IEEE-RTS

ID number	Size, MW	FOR	Maintenance duration, weeks/year	Production cost, \$/MWh	GenCo	Mint. Cand.
1–5	12	0.02	2	31.2	1	1,2
6–9	20	0.10	2	37.7	2	8,9
10–15	50	0.01	2	0	3	–
16–19	76	0.02	3	22.8	2	16,17
20–22	100	0.04	3	26	4	20,21
23–26	155	0.04	4	18.43	1	23,26
27–29	197	0.05	4	24.96	5	27,28
30	350	0.08	5	18.05	5	30
31–32	400	0.12	6	19	6	32

penalty contribution of those intervals for which there are some maintenance proposals $\left(\left(1-\prod_{i=1}^{N}\prod_{k=1}^{N_i}\left(1-x_{k,i,t}^{r}\right)\right)\right.$ is non-zero, provided, at least, one maintenance schedule, proposed at interval t exists$\left.\right)$. The third term is because of the total penalties allocated.

6 Penalties/rewards involvement in the GenCo's objective function

The key point in the involvement of penalties/rewards in the GenCo's objective function is that any rational GenCo avoids repetition of the earlier undesirable decisions and seeks the adoption of the earlier desirable decisions. This behaviour is modelled by considering a memory rate of the penalties/rewards in the GenCo's objective function. In this approach, the expectation of the probable penalty/reward is the weighted mean of the past observations with decreasing weights given by a normalised geometrical progression of parameter γ_i. Therefore the penalty/reward of generating unit k of GenCo i is calculated as follows

$$\begin{aligned}
&Tpen_{k,i,t}^{r} = Pen_{k,i,t}^{r} + \gamma_i Tpen_{k,i,t}^{r-1}\\
&Trew_{k,i,t}^{r} = Rew_{k,i,t}^{r} + \gamma_i Trew_{k,i,t}^{r-1}\\
&\forall k \in \Lambda_i, i \in \Delta, \text{ and } t \in [1,T]
\end{aligned} \tag{14}$$

The less the memory rate is, the faster forgetting would occur. In the direction towards the equilibrium point, through iterations, the effect of previous decisions is reduced by a factor of γ_i. For example, after R iterations, the effect of the first iteration decision appears as a multiple of $(\gamma_i)^R$ in the objective function.

Convergence of the cobweb theory based process is one of the interesting fields of study which has absorbed some researchers [12, 14, 15]. The main difference between the cobweb theory based maintenance coordination and the other developed cobweb theory based model is the binary nature (0 or 1) and the temporary interdependence (continuous maintenance) of decision variables that makes the proposed model very complicated for mathematical analysis. Therefore there is no way to give an analytical proof of the main contribution of the memory rate in reaching the equilibrium point in the process. Thus, to show an intuitive investigation, the effect of memory rate is studied by providing a simple example in the second section of Appendix. Moreover, the optimal estimation of the memory rate is not dealt with this paper. This may be addressed in further research.

7 Numerical results

Numerical results, based on IEEE reliability test system (IEEE-RTS), are reported in this section. The test system details are given in Tables 1 and 2 [7, 19]. The last column of Table 1 demonstrates the generating units that are considered to go on maintenance. Electricity market price is assumed to be as in (6). a is assumed to be 4.66×10^{-4} MW^{-1} for off-peak loads, 9.6×10^{-4} MW^{-1} for medium loads and 12.9×10^{-4} MW^{-1} for peak loads. Load pattern is characterised in Fig. 2. The values of μ_t are assumed to be as reported in Fig. 3, considering weekly intervals. μ_t can be calculated based on the historical cumulative bidding behaviour of the GenCos that shapes the supply curve [7]. Since there is no historical data of bidding on IEEE-RTS, we assume supply shift indices that could follow the load variations.

CUE is assumed to be \$2500/MWh [6]. EIR_{ref} is considered to be as 99.2%. The convergence criterion is chosen to be three similar solutions in consecutive iterations, whereas the reliability constraints in all weeks are preserved.

The model is implemented in MATLAB linked with GAMS 23.1.1 [16] on a computer equipped with Intel Corei5 CPU clocking at 2.4 GHz with 4 GB of RAM. For instance, the CPU time, required to attain the solution in six iterations, is about 10 s.

Considering three values for γ_i, (0.5, 0.75 and 0.9) for all GenCos, Fig. 4 shows the annual EENS variation in the iterations. The maintenance schedules all over the year for the first and the last iterations are shown in Fig. 5 for $\gamma_i = 0.75$. The weeks in which no maintenance schedules happen are not shown. For the first iteration, the annual EENS is increased to 94 TWh because of the high maintenance proposals in low load periods as seen in Fig. 5. The maintenance schedules are converged to the acceptable coordinated ones in 8, 7 and 7 iterations, considering three values for γ_i, 0.5, 0.75 and 0.9, respectively. As seen, higher memory rate enforces the process to converge sooner. It is obvious that there is no penalty to assign to the GenCos in the last iteration.

EIR evolution all over the year is shown in Fig. 6 for the first and last iterations with $\gamma_i = 0.75$. For instance, in weeks 30–34 and 42–45, the associated EIRs are improved

Table 2 Utilisation factors of generating units, %

	Winter	Spring	Summer	Autumn
base	100	100	100	100
mid	70	48	65	53
peak	40	20	36	25

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doi: 10.1049/iet-gtd.2012.0370

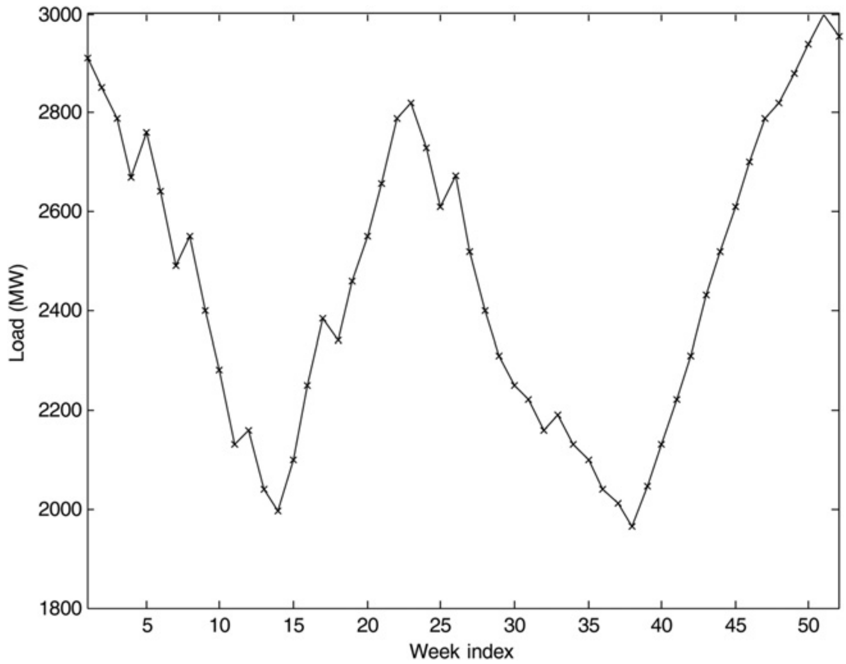

Fig. 2 *Weekly load level*

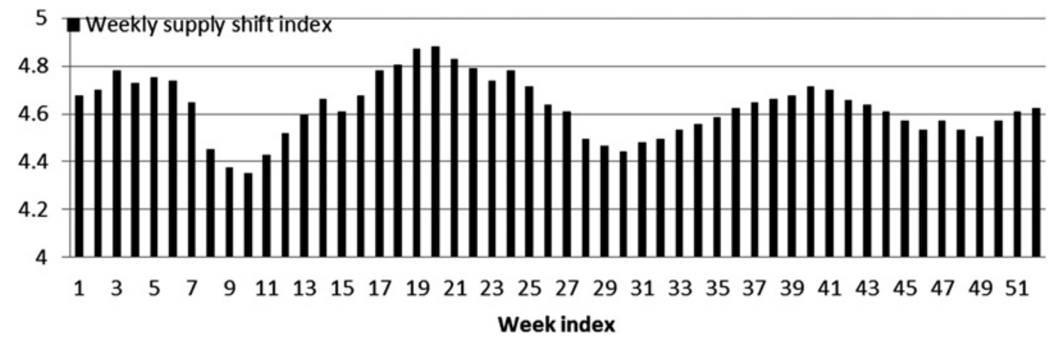

Fig. 3 *Values of weekly supply shift indices (μ_t)*

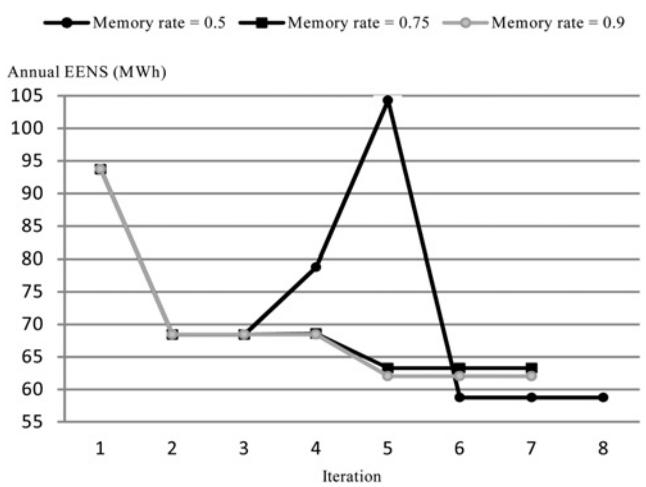

Fig. 4 *Annual EENS evolution in the cobweb process*

whereas for some remaining weeks (e.g. 8–12 and 16–18), they are slightly reduced. This has happened because of shifting of some of the maintenance schedules in weeks 30–34 and 42–45 (the low price periods) to weeks 8–12, 16–18 and 27–28 (the medium price weeks). Therefore excess trend of maintenance schedules towards low price periods, which may unduly affect the system reliability, is moderated, so that annual reliability index is improved for the last iteration (see Fig. 4).

Fig. 7 demonstrates the weekly energy weighted average price (WEWAP) evolution, considering three values of γ_i. This variable is computed as follows

$$WEWAP^r = \frac{\sum_{t=1}^{T} \rho_t^r TE_t}{\sum_{t=1}^{T} TE_t} \tag{15}$$

The price in each iteration is resulted from the maintenance schedules. The initial WEWAP, when no maintenance is proposed, is \$46.6/MWh (that is not shown). Fig. 7 shows that the WEWAP jumps, suddenly, to the high value (\$55.95/MWh) in the first iteration. As expected, all the GenCos choose the low price weeks to go on maintenance, resulting in maintenance schedules congestion and suddenly spike price. In the later iteration, the maintenance schedules

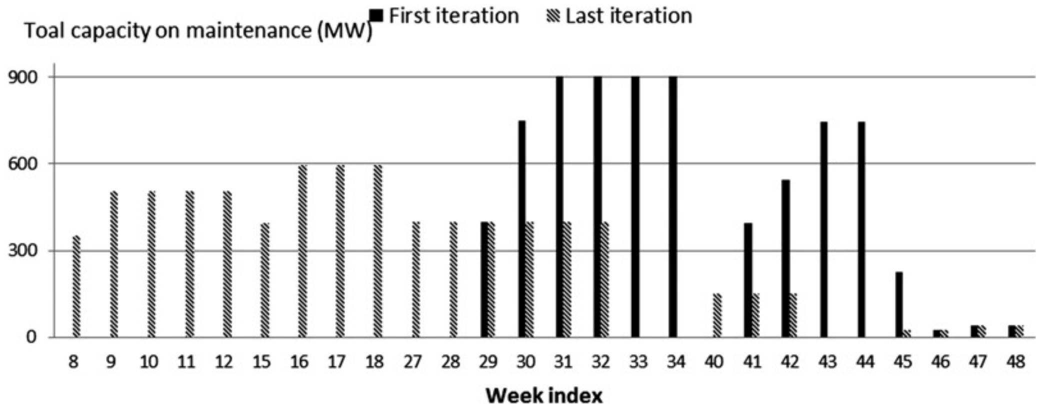

Fig. 5 *Evolution of total maintenance outages*

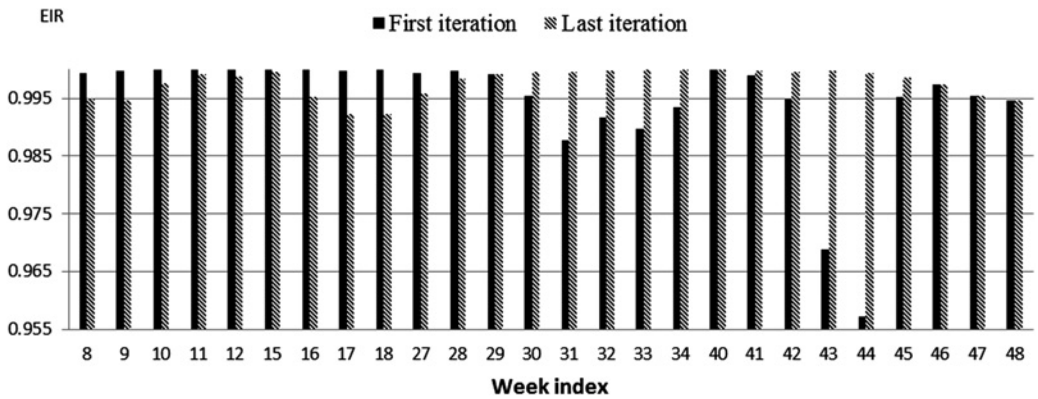

Fig. 6 *Evolution of EIR*

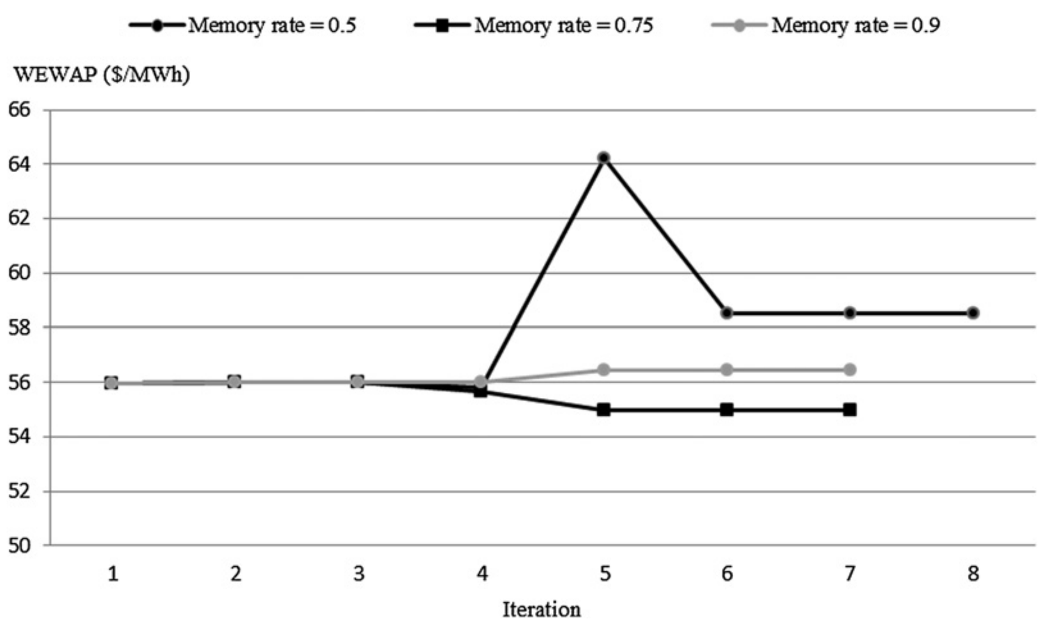

Fig. 7 *WEWAP ($/MWh) evolution in the cobweb process*

are readjusted in weeks in which the penalties are reduced and the rewards are caught. Therefore the maintenance schedules congestion is reduced, in comparison with the first iteration.

Assuming γ_i to be 0.75, Fig. 8 shows the initial and final profits of the GenCos. The initial and final profits stand for the maintenance schedules which are proposed in the first iteration and the last iteration, respectively. GenCos loose some profits in the iterations. These lost profits can be inferred as the annual cost of maintaining reliability level, imposed on GenCos.

doi: 10.1049/iet-gtd.2012.0370

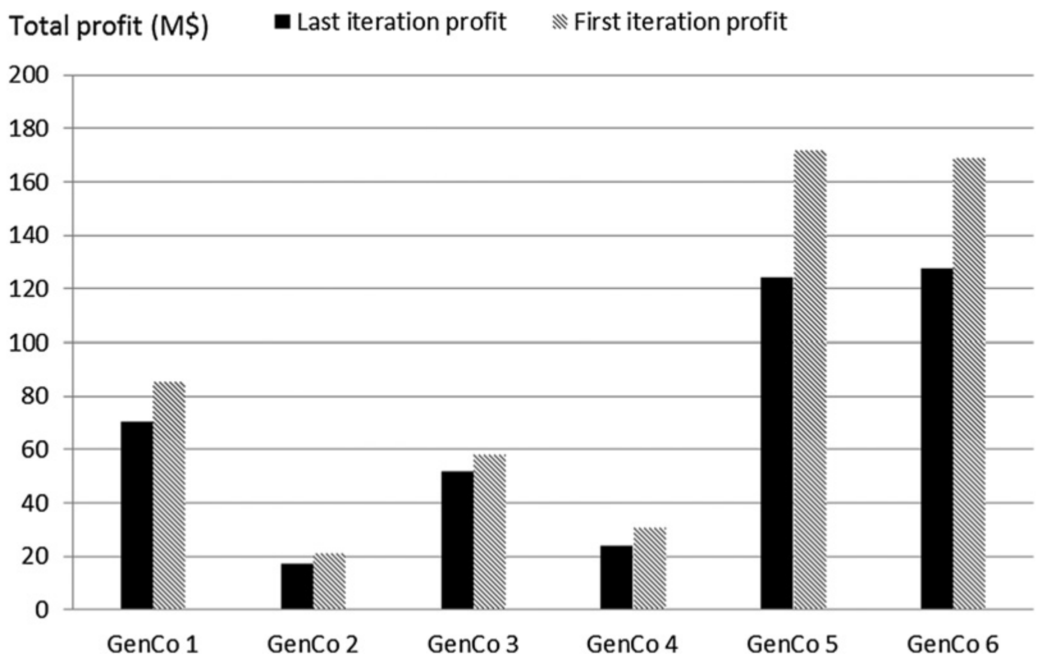

Fig. 8 *Total initial and final profits (M$) of the GenCos*

As the final study, we examine the price variation effects on coordinated maintenance schedules. Assume that the initial weekly price estimations are fixed throughout the iterations (i.e. maintenance proposals do not affect the weekly prices). Therefore the only incentive signals for maintenance rescheduling would be the penalties/rewards. In this way, the process is converged to the acceptable maintenance schedules in eight iterations. These schedules result in a WEWAP as \$55.62/MWh, which is higher than that is seen in Fig. 7 (\$55/MWh). This gives rise to an increased cost [about \$0.62/MWh × 21 450 000 MWh (annual served energy in Fig. 2) = \$13 299 000], imposed somehow on consumers. That is why the approach of observing the impact of maintenance schedules on electricity market price is introduced in this paper.

8 Conclusion

A cobweb theory based maintenance coordination algorithm was proposed in which the effect of maintenance on electricity price was taken into account. Iterative process was established between GenCos, from one side, and the ISO and the IMO, from the other side. Assigning the penalties/rewards signals from the latter side and observing a memory rate by the former side, guaranteed reaching an equilibrium point whereas the overall costs imposed on the consumers are reduced. Numerical results showed the effectiveness of the proposed coordination process to moderate the excess trend of maintenance schedules towards the low price periods in the first iteration; so that annual reliability index is improved in the last iteration. Moreover, it was shown that when the penalty cost rate is high enough, in the equilibrium point, no GenCo proposes the maintenance schedules that may violate the reliability criteria.

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10 Appendix

10.1 Flip-flop effect when no penalty/reward is considered

Consider a system comprising of two GenCos, A and B (Table 3). Assume a three-interval (week) maintenance coordination problem with the load and the price characterised in Table 4. At the first iteration, because of the low price of week 1, two GenCos decide to go on maintenance at week 1 in which the generation system is not able to supply the load. Electricity price in week 1 rises to CUE (e.g. \$5 000/MWh), whereas in other weeks remain unchanged. In the second iteration, two GenCos decide to go on maintenance at week 3. Thus, the electricity price in this week rises to CUE and the electricity price in week 1 is back to its initial value. In the third iteration, week 1 is the best candidate and therefore a flip-flop effect occurs.

10.2 Flip-flop effect considering penalty/reward and the effect of memory rate

Weekly served energy could be calculated by multiplying weekly load level and duration of a week (168 hours). $EENS_{t,\max}$ is calculated as 5, 6.4 and 5.6 MWh in weeks 1, 2 and 3, respectively. Studying the first iteration in the previous subsection showed that 42 000 MWh cannot be supplied. Contributions of generating units 1 and 2 in increasing EENS are 6719.2 and 35 275.8 MWh, respectively. Therefore a significant amount of penalties are assigned to the GenCos by considering (12) and (13). Since the maintenance outage durations of the generating units are 1 week and the two GenCos choose the same week to go on maintenance, there is no reward considering (14). If the maintenance outage duration of generating unit 2 is two weeks, GenCo B could obtain the reward of week 2.

Studying the second iteration result shows that 47 040 MWh is not supplied in week 3. Contributions of generating units 1 and 2 in increasing EENS are calculated as 6719.2 and 40 315.2 MWh, respectively. If the memory rate is not considered in the process ($\gamma_i = 0$ for all GenCos), the flip-flop effect, similar to one shown in the previous subsection occurs. Now, assume that γ_i is 0.6 for two GenCos. Therefore two GenCos choose the second week to go on maintenance in the third iteration. It can be shown that the flip-flop effect can occur again. This example demonstrates that imposing penalty cannot be sufficient to direct the GenCos to schedule their maintenances in weeks that would be acceptable by the ISO.

Consider that the maintenance outage duration of generating unit 2 is two weeks (assumption of different maintenance outage durations is more realistic). In the first iteration, two GenCo choose week 1 to start the maintenances. Since there is no unacceptable EENS in week 2, GenCo B could obtain the reward of this week which is equal to the total penalty of week 1. In the next iteration, GenCo B chooses week 2 to start the maintenance and GenCo A chooses week 3. In this iteration, weekly electricity price would be 35, 1096 and 5000 \$/MWh in three weeks, respectively. Assuming γ_i to be zero and following the iterations show that flip-flop effect is emerged again. Table 5 demonstrates that considering γ_i to be 0.6 results in that the maintenance schedules converge to the acceptable ones.

Table 3 Generating units characteristics owned by GenCos, *A* and *B*

GenCo	Production cost, \$/MWh	Maintenance duration, week	FOR	Size, MW	ID number
A	23	1	0	400	1
B	18	1	0	300	2

Table 4 Load and price curves characteristics

Week	Load, MW	Supply shift index	Price, \$/MWh	a, MW^{-1}	$u_{1,1,t}$%	$u_{1,2,t}$%
1	250	8.05	35	0.01	10	70
2	320	7.8	54.6		12.5	90
3	280	7.9	40		10	80

Table 5 Cobweb process summary considering memory rate

Iteration	Maintenance starting week of GenCo *A*	Maintenance starting week of GenCo *B*	Total EENS, MWh
1	1	1	42 000
2	3	2	47 040
3	2	1	53 760
4	1	1	42 000
5	3	1	0.00019

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doi: 10.1049/iet-gtd.2012.0370