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Improving the performance of water balance equation using fuzzy logic approach

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SUMMARY

It is a common practice to conduct the water budget or water balance analysis in a given area within a specified time in order to investigate the balance between the inputs and outputs of the water system. Such an analysis can be used for water management and water allocation in a designated study area. Due to appearance of an error in water balance equation because of difficulty in accurate estimation of its individual components, the main objective of the current paper was to apply a set of fuzzy coefficients to the components of the water balance equation in order to reduce this error. The fuzzy coefficients reflect the uncertainty and imprecision in evaluating each component, and minimize the overall error of the water balance equation. These coefficients are adjusted by an error minimization procedure, based on fuzzy regression concepts and using available recorded data for a given study area within a specified time scale. The adjusted coefficients can effectively estimate the water balance components in the future. In this study, four different models, representing different types of fuzzy coefficients, were considered and used for annual water balance of Azghand catchment in Khorasan Razavi Province, Iran as a case study. Analysis of results showed that all models were effective in reducing water balance error in Azghand catchment. The best model reduced the error up to 79% in terms of mean absolute error compared with error in water balance equation when conventional (with no correction coefficients) water balance analysis was conducted. Moreover, the results indicated that the performance of the proposed fuzzy models was not significantly sensitive to selection of confidence level in data (h) and improved slightly as h increased.

1. Introduction

Due to high demand of water in developed societies, precise strategies are required to manage water resources in order to achieve optimized patterns for distribution and consumption. Water resources planning for future consumption requires precise data, however managers and experts usually encounter an unavoidable challenge known as uncertainty in the field of hydrology.

In hydrological modeling, uncertainty may have two sources: (1) structural uncertainty, and (2) parameter uncertainty. Structural uncertainty is associated with uncertain cognition of the overall functioning of any hydrological system and the uncertainty in the model structures used for modeling the system. Parameter uncertainty is related to the uncertainty of the inputs and parameters of a model (Eder et al., 2005).

Water balance is one of the fundamental principles in water resources management and hydrology. There are many methods of balancing water in different temporal and spatial scales. In fact, the water balance study is the hydrological application of the principle of conservation of mass for water, namely the continuity equation (Sokolov and Chapman, 1974). Study of groundwater, surface water, and storage variations are also dependent on the results of water balance analysis in an aquifer or catchment (Todd and Mays, 2005).

The uncertainty, vagueness, and inaccuracy of hydrologic data result from errors of measurement equipments and estimation models. Therefore, random or epistemic errors are present in the water balance equations results. These errors indicate that water balance studies are fairly unreliable and therefore water allocations using hydrologic data can also be inaccurate for the future studies. Accuracy of the studies can be improved in many cases through upgrading, replacing or adding the equipment and methods required to collect and process the data, however it especially

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might be time-consuming and costly in countries with budget limitation. Enhancing the accuracy of water balance equations through increasing the efficiency of computational methods can fairly alleviate unreliability concern.

Fuzzy logic proposed by Zadeh (1965) is a powerful approach that has been successfully applied to various fields of science and engineering which deal with inaccurate data. Probability theory can determine the likelihood of an event, however it may not be capable to identify the uncertainty originated from vagueness in input data, parameters, and system behavior. In order to solve this problem, possibility theory using fuzzy numbers instead of crisp numbers can capture vagueness in system parameters or its complexity. In other words, fuzzy numbers can express imprecise, vague, intuitive, inconsistent, experimental, or subjective information in the form of uncertainty that can be used in fuzzy modeling (Zadeh, 1978). Several approaches have been developed based on fuzzy set theory (Zadeh, 1978, 1986; Dubois and Prade, 1994; Yager and Kelman, 1996) to model uncertainty of phenomena (Faybishenko, 2010).

Fuzzy modeling has been applied to several problems in hydrology (Bogardi et al., 1983; Bardossy and Disse, 1993; Ozelkan and Duckstein, 2001; Luchetta and Manetti, 2003; Maskey et al., 2004; Shrestha and Rode, 2008; Guler et al., 2012) and water resources management (Esogbue et al., 1992; Sutardi et al., 1995; Wu et al., 1997; Bender and Simonovic, 2000; Chang, 2005; Li et al., 2009; Wang and Huang, 2011) for modeling uncertainty of hydrological events or phenomena. However, Bardossy (1996) studied application of fuzzy logic in modeling water cycle for the first time. He used a fuzzy rule-based methodology to describe three elements of the hydrological cycle: surface runoff, infiltration, and unsaturated flow of water in soil.

Eder et al. (2005) presented a formulation for a lumped water balance model based on fuzzy logic in Upper Enns catchment in Austria and evaluated the efficiency of fuzzy logic approach in modeling the system complexity, predictive uncertainty and accuracy of predictions. In another study, Faybishenko (2010) studied water balance uncertainty using fuzzy probabilistic approach in Hanford site in USA. He combined the probability and possibility theories to model the soil water balance and assessed the associated uncertainty in the components of the water balance equation. Nasseri et al. (2013) proposed a new method based on fuzzy extension principle to assess uncertainty of the water balance models. They calibrated two non-linear monthly water balance models for two catchments in Iran and France. The results were compared with those of five different models. The suggested models showed well performance in uncertainty analysis of water balance model in all selected levels of confidence. In a more recent study, Nasseri et al. (2014) suggested a hybrid fuzzy-probabilistic model for monthly prediction or simulation of hydrological components in water balance equation. The suggested methodology was used to simulate stream flows of Roudzard and Karoon III basins in South-West of Iran.

For long, linear regression analysis has been used to analyze hydrological problems with the main objective of developing predictive equations by minimizing the deviations of the estimated values from the corresponding observed values. Fuzzy linear regression analysis proposed by Tanaka et al. (1982) is based on the linear programming formulation. It interprets these deviations as the indefiniteness of the system structure and fuzziness of system parameters. The objective of this type of fuzzy regression models is to minimize the fuzziness of a system in which fuzzy parameters follow a possibility distribution. Different versions of fuzzy regression models based on possibility theory have been introduced by Tanaka et al. (1989), Bardossy (1990) and Bardossy et al. (1990) trying to resolve some of the defects of the Tanaka et al.'s approach.

Tanaka et al.'s model has also been modified to determine the fuzzy coefficients of fuzzy linear relationship based on the least square approach (Celmins, 1987; Savic and Pedrycz, 1991; Tanaka and Ishibuchi, 1991; Chang and Ayyub, 1997). The other type of fuzzy regression is based on interval analysis, only requiring that in each degree of uncertainty, each predicted interval to intersect the associated observed interval of data (Peters, 1994; Ozelkan and Duckstein, 2000; Sakawa and Yano, 2000; Hojati et al., 2005).

The objective of this study was to apply a set of fuzzy coefficients to different components of water balance equation in order to minimize the existing error in water balance equation. The two following models were applied: fuzzy regression models based on possibility theory and fuzzy regression models based on interval analysis. Applying the error minimization procedure to a set of available data in each model, adjusts the fuzzy coefficients in the water balance equation. In the current case study, four models were generally developed and compared on water balance data of Azghand catchment in Khorasan Razavi Province, Iran. The models are presented and explained in the methodology section.

2. Background on fuzzy numbers and fuzzy linear regression models used in this study

2.1. Fuzziness and fuzzy numbers

The term fuzziness is generally referred to the class of objects or processes without sharp boundaries that may result from imprecision in definitions, estimations or measurements in order to model the system. Fuzzification of crisp values results in fuzzy numbers, which are expressed in the shape of fuzzy membership functions that can assign different intervals to a definition in different degrees of its membership. These intervals are defined by the information taken from measurements, intuition or perception of the parameter in the study. The most significant role of fuzzy numbers in possibility theory is the ability of expressing uncertainty of real values and crisp numbers in phenomena and providing a fuzzy set in R that arithmetic or algebraic operations can easily be performed (Zadeh, 1978, 2002).

Different shapes of fuzzy membership functions including triangular, trapezoidal, Gaussian, and sigmoid are commonly used to represent a fuzzy number. However, the triangular fuzzy number is one of the most widely used types in fuzzy mathematics. It is formulated as (Wang, 1997; Ross, 2004; Moller and Beer, 2004):

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leqslant x < b \\ (c-x)/(c-b), & b \leqslant x < c \\ 0, & x > c \text{ or } x < a \end{cases} \tag{1}$$

where $\mu_{\tilde{A}}(x)$ denotes membership degree of element x to set $\tilde{A}$ and a, b and c are left, center and right values of triangular membership

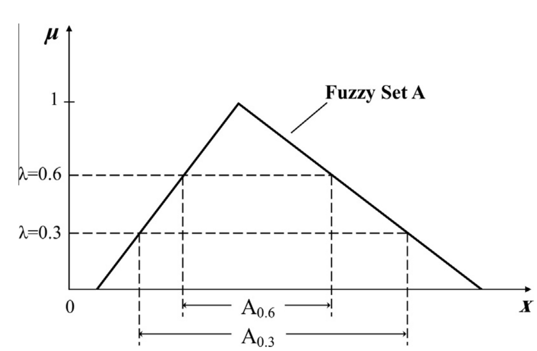

Fig. 1. Different λ-cut sets for a triangular fuzzy number (Ross, 2004).

functions on x axis. A key feature of fuzzy numbers is the notion of λ-cut sets, illustrated in Fig. 1. Consider fuzzy set $\tilde{A}$, then a λ-cut set of $\tilde{A}$, is a crisp set A_λ, where $A_\lambda = \{x|\ \mu_{\tilde{A}}(x) \geqslant \lambda,\ 0 \leqslant \lambda \leqslant 1\}$. Any λ-cut set defines an interval that includes all elements of $\tilde{A}$ with a degree of membership that is greater than or equal to the value λ (Ross, 2004).

2.2. Fuzzy linear regression

One of the applications of fuzzy logic is developing fuzzy linear regression models for forecasting and modeling purposes. The following formulation was presented by Tanaka et al. (1982) for a fuzzy linear regression:

$$\tilde{y} = A_0x_0 + A_1x_1 + \cdots + A_kx_k \tag{2}$$

where regression coefficients A_j, $j = 0,1,\ldots,k$ are assumed to be symmetric triangular fuzzy numbers centered at α_j (having degree of membership = 1) and half-width c_j, $c_j \geqslant 0$ (Fig. 2).

Depending on input and output variables, two cases can be considered for fuzzy regression models:

- Case 1: independent input variables (x) are crisp and dependent input variable (y) is also crisp.
- Case 2: independent input variables (x) are crisp and dependent input variable (y) is fuzzy.

In both cases, the input data are n sets of values (y_i, x_{i0}, $x_{i1},\ldots,$ x_{ik}), $i = 1,2,\ldots,n$, $n \geqslant k + 1$ except for the dependent input variable in Case 2, where the dependent variables are symmetric triangular fuzzy numbers with center y_i and half-width e_i, $e_i \geqslant 0$ (Fig. 3). A specific part of value y_i with minimum confidence level h, where $0 \leqslant h \leqslant 1$, is defined by interval $[y_i - (1 - h)e_i,\ y_i + (1 - h)e_i]$ and will be referred as *h-certain observed interval* in this paper. This interval is the bold line segment in Fig. 3.

Similarly, the support of the independent variables corresponding to a specific set of values which have a membership value of at least h is $\left[\sum_{j=0}^{k}(\alpha_j - (1-h)c_j)x_{ij}, \sum_{j=0}^{k}(\alpha_j + (1-h)c_j)x_{ij}\right]$ and will be referred as *h-certain predicted interval* (Hojati et al., 2005).

2.2.1. Model-I

Tanaka et al. (1982) have suggested a model to estimate A_j, $j = 0,$ $1,\ldots,k$ using linear programming formulation, as follows:

$$\text{Minimize} \sum_{i=1}^{n}\sum_{j=0}^{k} c_j|x_{ij}| \tag{3}$$

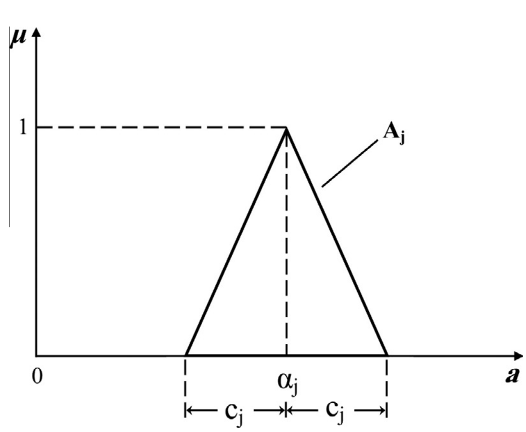

Fig. 2. Triangular fuzzy regression coefficients (Hojati et al., 2005).

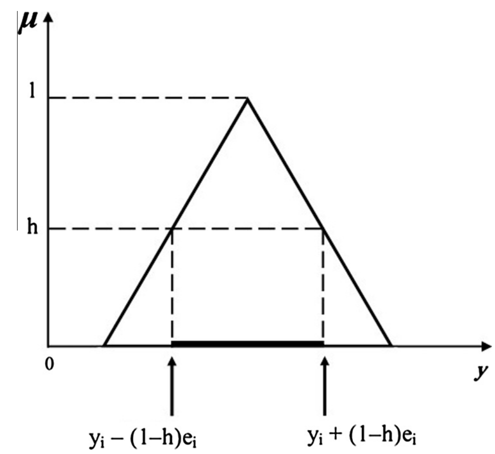

Fig. 3. Triangular fuzzy regression dependent variables and h-certain observed interval (Hojati et al., 2005).

Subject to

$$\sum_{j=0}^{k}(\alpha_j + (1-h)c_j)x_{ij} \geqslant y_i + (1-h)e_i, \quad i = 1,\ \ldots,\ n, \tag{4}$$

$$\sum_{j=0}^{k}(\alpha_j - (1-h)c_j)x_{ij} \leqslant y_i - (1-h)e_i, \quad i = 1,\ \ldots,\ n, \tag{5}$$

$$\alpha_j = \text{free}, \quad c_j \geqslant 0, \quad j = 0,\ \ldots,\ k. \tag{6}$$

This model provides a condition that corresponds to the data in Case 2, described above. For Case 1, e_i is considered zero for each i, $i = 1,2,\ldots,n$. Note that fuzzy regression Model-I forces the h-certain predicted interval to include the h-certain observed interval.

2.2.2. Model-II

To estimate A_j, $j = 0,1,\ldots,k$ the following linear programming formulation was suggested by Hojati et al. (2005).

$$\text{Minimize} \sum_{i=1}^{n}(d_{iU}^{+} + d_{iU}^{-} + d_{iL}^{+} + d_{iL}^{-}) \tag{7}$$

Subject to

$$\sum_{j=0}^{k}(\alpha_j + (1-h)c_j)x_{ij} + d_{iU}^{+} - d_{iU}^{-} = y_i + (1-h)e_i, \quad i = 1,\ \ldots,\ n, \tag{8}$$

$$\sum_{j=0}^{k}(\alpha_j - (1-h)c_j)x_{ij} + d_{iL}^{+} - d_{iL}^{-} = y_i - (1-h)e_i, \quad i = 1,\ \ldots,\ n, \tag{9}$$

$$d_{iU}^{+}, d_{iU}^{-}, d_{iL}^{+}, d_{iL}^{-} \geqslant 0, \quad i = 1,\ \ldots,\ n, \tag{10}$$

$$\alpha_j = \text{free}, \quad c_j \geqslant 0, \quad j = 0,\ \ldots,\ k. \tag{11}$$

Similarly, this model provides a condition that corresponds to the data in Case 2. For Case 1, e_i is considered zero for i, $i = 1,2,\ldots,n$. For each i, $i = 1,2,\ldots,n$, d_{iU}^{+} and d_{iU}^{-} represent positive and negative deviations of upper values of the predicted intervals from observed intervals in level of confidence h. Similarly, d_{iL}^{+} and d_{iL}^{-} are positive and negative differences between lower values of the predicted intervals and those of the observed intervals. It should be noted that in the formulation of the Hojati et al. fuzzy regression model, objective function and constraints were set so that for each i, $i = 1,2,\ldots,n$ at most one of d_{iU}^{+} and d_{iU}^{-} could be positive and at most one of d_{iL}^{+} and d_{iL}^{-} could be positive.

3. Water balance concepts and water balance in the study area

Water budget or water balance study is to analyze the conservation of the water mass in a specified time interval for any hydrologic system of interest. In terms of size, the hydrologic system can range from a local scale to a regional scale or from a watershed to the earth. The analysis is done by a continuity or water balance equation for the system.

As it is shown in Fig. 4, water balance equation can be expressed in three conditions depending on the control volume: surface water balance equation, groundwater balance equation, and system water balance equation (Todd and Mays, 2005). In fact, if the first two equations are written simultaneously, the water balance equation changes into the third one.

The overall system water balance equation for the system shown in Fig. 4, is written as:

$$\textit{Inflow Rate} - \textit{Outflow Rate} = \textit{Rate of Change in Storage} \qquad (12)$$

Eq. (12), in terms of different hydrologic components considered in the system, can be expressed as:

$$P + (Q_{in} - Q_{out}) + (ET_s + ET_g) + (G_{in} - G_{out}) + (\Delta S_s + \Delta S_g) = 0 \qquad (13)$$

where P is the precipitation, Q_{in} is the surface water flow into the system, Q_{out} is the surface water flow out of the system, ET_s is the surface evapotranspiration, ET_g is the groundwater evapotranspiration, G_{in} is the groundwater flow into the system, G_{out} is the groundwater flow out of the system, ΔS_s is the change in water storage of the surface water in the system, and ΔS_g is the change in groundwater storage of the system (Todd and Mays, 2005).

Eq. (13) consists of different hydrologic components and each of the components is usually evaluated one by one. However, it is practically hard to have an accurate evaluation of these components and error of each component is an inherent element of the evaluation process. Errors can arise from different sources such as inadequate or unevenly distributed measurement stations, outdated and inaccurate measurement instruments, human errors, errors associated with influence of natural factors, and errors in modeling and parameterization of the water balance components. The outcome of using these inaccurate component values in the water balance equation is that the right-hand side of Eq. (13) would not be zero and therefore an overall error always exists in the results of any water balance equation. Therefore, Eq. (13) can be written as:

$$P + (Q_{in} - Q_{out}) + (ET_s + ET_g) + (G_{in} - G_{out}) + (\Delta S_s + \Delta S_g) = \eta \qquad (14)$$

where η refers to the overall error in the water balance equation (Sokolov and Chapman, 1974). When Eq. (14) is used for water balance calculations, evapotranspiration values are assigned as negative values. Any changes in water storage values would also be negative when there is an increase in water level or storage.

The overall error depends on the quality and quantity of data and can be especially significant in countries with a budget limitation in order to enhance measurement networks.

3.1. Water balance for the study area

Azghand catchment is located at the west side of Torbat-e Heydarieh in the central part of Khorasan Razavi Province, Iran (Fig. 5). In Fig. 5, Mashhad refers to the center of the province. Azghand catchment is about 80 km long from 58°35′ to 59°8′ longitude and 23 km wide from 35°8′ to 35°35′ latitude. The area is 1875 km^2 that includes 661 km^2 of flat areas and 1214 km^2 of mountainous areas. Azghand catchment has an average elevation of 1526 m.

The horizontal area of Azghand aquifer is approximately 16% of the catchment (291 km^2). Azghand aquifer is located at the center of Azghand catchment with no water exchange with adjacent groundwater sources.

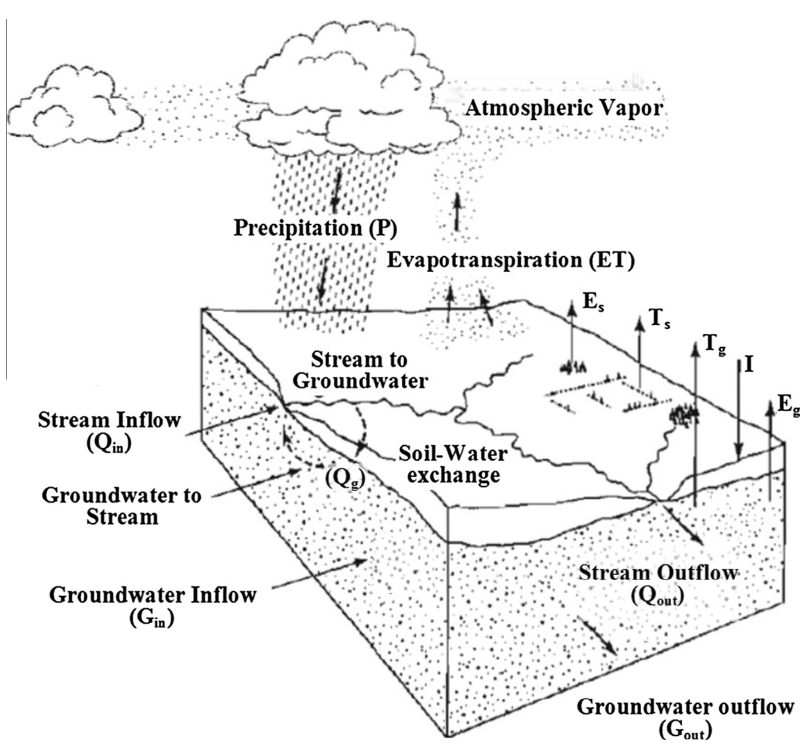

Fig. 4. Components of water balance equation in an open system (Todd and Mays, 2005).

Fig. 5. Azghand catchment in Khorasan Razavi Province, Iran.

Considering physiographical and hydrological features of the Azghand catchment, system water balance equation of the study area was considered as:

$$P + Q_{out} + ET_s + \Delta S_g = \eta \tag{15}$$

Eq. (15) indicates that neither there is surface water flow into the system nor evapotranspiration from the groundwater system due to deep level of groundwater in the area. It was also assumed that the observed change in surface water storage was negligible. Moreover, Eq. (15) describes that the water balance equation of the area is mainly an interaction between precipitations, actual evapotranspiration from the surface, surface water outflow, and groundwater storage, which is a characteristic of the arid or semi-arid regions.

3.3.1. Water balance components of the study area

As shown in Eq. (13), water balance equation of Azghand catchment consists of four components: precipitation, surface water flow out of the system, surface evapotranspiration, and change in groundwater storage. Although groundwater-monitoring piezometric wells are denser, lack of sufficient number of stations to measure other required parameters is noticeable in the area. In the current study, the water balance components were calculated using available data and conventional methods.

As shown in Fig. 6, there were only two stations to measure precipitation, which are located in the central part of the catchment. Thiessen polygons method was used to estimate the average precipitation of the catchment, therefore the area was divided into two parts, consisting of the area around Namegh station with 1099.08 km^2 and the area around Fadieh station with 775.72 km^2.

In general any equation or method for estimating actual evapotranspiration in any study area is vulnerable to uncertainty, and one should select the appropriate method for any specific climatological condition based on quantity and quality of available data. A comprehensive search was made in the literature (Rahimikhoob et al., 2012; Shiri et al., 2014; Sokolov and Chapman, 1974; Valipour, 2014a,b) to find the appropriate method for estimating actual evapotranspiration in the study area. Turc method (Sokolov and Chapman, 1974) was selected to estimate the actual evapotranspiration of Azghand catchment because this method is easily applicable to available data in the study area. Turc method requires precipitation and temperature data. Precipitation was calculated as previously mentioned, and temperature data was collected from Senobar station in the northeast of the study area (Fig. 6).

Since there is no hydrometric station in the catchment, regional analysis was used to estimate the surface water flow out of the catchment, therefore the available runoff-precipitation data for thirty-six years from seven nearby stations were used to create a linear equation between discharge and product of precipitation and area using regression analysis. The result indicated a runoff coefficient of 0.14, which was used to estimate annual surface outflow in Azghand catchment.

Annual change in groundwater storage was calculated as the product of the change in average groundwater level, area of the aquifer, and the storage coefficient of the aquifer. There were seven piezometric wells throughout Azghand aquifer to monitor groundwater levels (Fig. 6). The relevant data were available from 1992 water year onward. Average groundwater levels were estimated using Thiessen polygons method and the local hydrogeologists suggested the storage coefficient of 0.07.

3.3.2. Error in water balance equation

Table 1 reports the water balance components in million cubic meters and indicates the separately estimated values of the water balance components in Azghand catchment for eighteen consecutive water years. Obviously the uncertainty is an ingredient of these estimated values; therefore, they do not satisfy the

Fig. 6. Locations of different measurement stations in Azghand catchment.

Table 1
Water balance components for Azghand catchment.

Water year	Precipitation (mcm)	Surface water outflow (mcm)	Evapotranspiration (mcm)	Change in groundwater storage (mcm)	Error in water balance equation (mcm)
1992	893.541	−125.096	−741.483	−24.141	2.822
1993	869.102	−121.674	−748.963	−58.085	−59.620
1994	303.977	−42.557	−312.079	20.473	−30.186
1995	545.823	−76.415	−533.650	19.102	−45.140
1996	593.639	−83.109	−578.075	12.079	−55.466
1997	409.031	−57.264	−414.564	17.988	−44.809
1998	838.429	−117.380	−748.873	−11.006	−38.830
1999	547.863	−76.701	−538.087	−3.623	−70.547
2000	214.213	−29.990	−223.132	15.030	−23.878
2001	278.762	−39.027	−287.960	33.674	−14.551
2002	475.193	−66.527	−475.026	25.250	−41.110
2003	622.366	−87.131	−593.794	26.172	−32.388
2004	495.919	−69.429	−489.579	21.733	−41.356
2005	560.634	−78.489	−541.344	12.767	−46.432
2006	277.925	−38.910	−287.200	19.214	−28.970
2007	475.327	−66.546	−471.979	18.659	−44.539
2008	175.132	−24.518	−183.014	18.122	−14.279
2009	535.676	−74.995	−523.889	15.871	−47.336

continuity equation. The last column in the table shows the error values as were defined in Eq. (15). According to Table 1, the error can be significant in some years compared to the values of the hydrologic components.

4. Proposed methodology for reducing error in water balance equation with application to the study area

Hypothetically the error in water balance equation is mainly related to the fuzziness in modeling the system and bias in measurements and estimations. The correction coefficients were first considered for each water balance component in order to reduce the error. In other words, the application of these coefficients would supposedly reduce the error of water balance equation, therefore the first fifteen years of data from water balance of Azghand catchment were used to calibrate or adjust the correction coefficients for the water balance components. The coefficients were then applied to the last three years of data from water years of 2007–2009 in order to investigate the capability of the proposed methodology in reducing error in the forthcoming years. Four different approaches based on fuzzy linear regression concepts were used to calibrate the correction coefficients.

4.1. Fuzzy regression models for Azghand water balance equation

With reference to Section 2, four different approaches resulting from the combination of two models and two cases were used to calibrate the correction coefficients in the water balance equation. In other words, two cases were considered for each of the two fuzzy linear regression models in order to develop the four approaches. In Case 1, the error of the water balance equation was assumed to be a crisp number while in Case 2, it was assumed to be a symmetric triangular fuzzy number. The data collected from the first fifteen years (Table 1) were used to construct different mathematical models and the data from the three following years were used to evaluate the performance of the different approaches.

According to Eq. (15), fuzzy linear regression Model-I for Azghand water balance was formulated as:

$$\text{Minimize} \sum_{i=1}^{15} c_P|(P)_i| + c_{Q_{out}}|(Q_{out})_i| + c_{ET_s}|(ET_s)_i| + c_{\Delta S_g}|(\Delta S_g)_i| \quad (16)$$

Subject to

$$\begin{aligned}&\alpha_P(P)_i + \alpha_{Q_{out}}(Q_{out})_i + \alpha_{ET_s}(ET_s)_i + \alpha_{\Delta S_g}(\Delta S_g)_i \\ &\quad + (1-h)\left[c_P(P)_i + c_{Q_{out}}(Q_{out})_i + c_{ET_s}(ET_s)_i + c_{\Delta S_g}(\Delta S_g)_i\right] \\ &\quad \geqslant \eta_i + (1-h)e_{\eta_i}, \quad i = 1,2,\ \ldots,15,\end{aligned} \quad (17)$$

$$\begin{aligned}&\alpha_P(P)_i + \alpha_{Q_{out}}(Q_{out})_i + \alpha_{ET_s}(ET_s)_i + \alpha_{\Delta S_g}(\Delta S_g)_i \\ &\quad - (1-h)\left[c_P(P)_i + c_{Q_{out}}(Q_{out})_i + c_{ET_s}(ET_s)_i + c_{\Delta S_g}(\Delta S_g)_i\right] \\ &\quad \leqslant \eta_i - (1-h)e_{\eta_i}, \quad i = 1,2,\ \ldots,\ 15,\end{aligned} \quad (18)$$

$$\begin{aligned}&0.865 \leqslant \alpha_P \leqslant 1.135, \quad 0.649 \leqslant \alpha_{Q_{out}} \leqslant 1.351, \\ &0.688 \leqslant \alpha_{ET_s} \leqslant 1.312, \quad 0.490 \leqslant \alpha_{\Delta S_g} \leqslant 1.510,\end{aligned} \quad (19)$$

$$c_P \geqslant 0.215, \quad c_{Q_{out}} \geqslant 0.431, \quad c_{ET_s} \geqslant 0.438, \quad c_{\Delta S_g} \geqslant 0.701, \quad (20)$$

$$e_{\eta_i} = 18.150\ \text{mcm}, \quad i = 1,2,\ \ldots,\ 15. \quad (21)$$

In Eqs. (16)–(21), α and c refer to the center and half-width of the fuzzy number for the correction coefficient considered for each component in the water balance equation. For example, α_P and c_P are center and half-width of the fuzzy coefficient considered for the precipitation. As introduced before, η is the error in the water balance equation and considering the main objective of this research, which was to minimize the error in using the water balance equation, it was considered zero. In Eqs. (17), (18) and (21), e_η is the half-width of the fuzzy number of the error term and it was considered to be zero in Case 1. In Case 2, the standard deviation of the error values reported in Table 1 for the first 15 years of data, which was calculated 18.150 million cubic meters, was considered for e_η.

With reference to Section 2, the following formulation presents the fuzzy linear regression Model-II for Azghand water balance:

$$\text{Minimize} \sum_{i=1}^{15} (d_{iU}^+ + d_{iU}^- + d_{iL}^+ + d_{iL}^-) \quad (22)$$

Subject to

$$\begin{aligned}&\alpha_P(P)_i + \alpha_{Q_{out}}(Q_{out})_i + \alpha_{ET_s}(ET_s)_i + \alpha_{\Delta S_g}(\Delta S_g)_i \\ &\quad + (1-h)\left[c_P(P)_i + c_{Q_{out}}(Q_{out})_i + c_{ET_s}(ET_s)_i + c_{\Delta S_g}(\Delta S_g)_i\right] \\ &\quad + d_{iU}^+ - d_{iU}^- = \eta_i + (1-h)e_{\eta_i}, \quad i = 1,2,\ldots,\ 15,\end{aligned} \quad (23)$$

$$\begin{aligned}&\alpha_P(P)_i + \alpha_{Q_{out}}(Q_{out})_i + \alpha_{ET_s}(ET_s)_i + \alpha_{\Delta S_g}(\Delta S_g)_i \\ &\quad - (1-h)\left[c_P(P)_i + c_{Q_{out}}(Q_{out})_i + c_{ET_s}(ET_s)_i + c_{\Delta S_g}(\Delta S_g)_i\right] \\ &\quad + d_{iL}^+ - d_{iL}^- = \eta_i - (1-h)e_{\eta_i}, \quad i = 1,2,\ldots,15,\end{aligned} \quad (24)$$

$$d_{iU}^+, d_{iU}^-, d_{iL}^+, d_{iL}^- \geqslant 0, \quad i = 1,2,\ldots,\ 15, \quad (25)$$

$$\begin{aligned}&0.865 \leqslant \alpha_P \leqslant 1.135, \quad 0.649 \leqslant \alpha_{Q_{out}} \leqslant 1.351, \\ &0.688 \leqslant \alpha_{ET_s} \leqslant 1.312, \quad 0.490 \leqslant \alpha_{\Delta S_g} \leqslant 1.510,\end{aligned} \quad (26)$$

$$c_P \geqslant 0.215, \quad c_{Q_{out}} \geqslant 0.431, \quad c_{ET_s} \geqslant 0.438, \quad c_{\Delta S_g} \geqslant 0.701, \quad (27)$$

$$e_{\eta_i} = 18.150\ \text{mcm}, \quad i = 1,2,\ldots,\ 15. \quad (28)$$

where for each i, $i = 1,2,\ldots,15$, d_{iU}^+ and d_{iU}^- measure the positive and negative differences between upper values of the predicted water balance error interval and the associated observed error interval. Also, d_{iL}^+ and d_{iL}^- are the differences between the lower values of the corresponding predicted and observed intervals of the water balance error. In this formulation, predicted error refers to the values reported in last column of Table 1 and the observed error was considered zero in Case 1 and a fuzzy number around zero in Case 2. Definition and values of the other variables are the same as those used in Eqs. (16)–(21).

In fact, Model-I estimates the coefficients of the water balance equation components by minimizing the absolute overall fuzziness in the entire system. However, Model-II estimates the coefficients by minimizing the overall deviation of the predicted response variables, which is the error in water balance equation from the corresponding observed response variables.

As Eqs. (16)–(28) show, the constraints and the ranges considered for the model parameters are the essential elements in developing Model-I and Model-II. This aspect of the study is presented in detail in the next section.

4.2. Interpretation of the constraints

In practical problems, such as engineering problems, there are sometimes limitations to restrict the domain of optimized answers such as political, economic, social, environmental, and physical limitations, which need special attention. One of the advantages of the linear programming is that the new constraints can be mathematically involved in the analysis.

The coefficients behind all components in any water balance equation must be obviously one if the components are accurately estimated and although it is hard to achieve in many cases. As it was mentioned before, the main objective of this research is to apply correction coefficients to individual components in the water balance equation, therefore, constraints in the form of Eqs. (19) and (26) impose a central value of one for the correction coefficients with a possibility of a limited deviation from one. Similarly, constraints presented in the form of Eqs. (20) and (27) were used to impose that the correction coefficients to be fuzzy numbers rather than crisp numbers with $c = 0$.

Several criteria are available to determine the boundaries of different constraints. However, in this research, the error in estimating individual water balance components was calculated, and a particular amount of the error was considered as a criterion to determine the constraints' boundaries. Therefore, the overall procedure consisted of two steps as follow: the first step was to assess the estimation errors for the individual components in the water balance equation and the second step was to apply a reasonable criterion to define the constraints' boundaries.

One of the main sources of estimation error is interpolation error. For example, such error can occur to estimate average precipitation or average groundwater level in an area. Leave-one-out cross validation procedure was used to estimate the interpolation error for any parameter of interest, which involves removing one observation from the data set, and fitting the estimation model with this observation removed. This model is then used to predict the estimated value and the corresponding error for the removed observation. This process is repeated for each observation in the data set and the errors obtained from this process can be used to find the validation error. Mean absolute error defined by Eq. (29) was considered as a representative error in this study.

$$MAE = \frac{1}{p}\sum_{l=1}^{p}|Z_{obs} - Z_{l,est}| \tag{29}$$

where p is number of observation points, Z_{obs} is observed value of parameter, and $Z_{l,est}$ is estimated value of parameter after removing point l.

In some cases, the component of interest can be a function of several parameters. For example, in Turc method, actual evapotranspiration is expressed as a nonlinear function of precipitation and temperature. In these cases, Eq. (30) was used to find the overall estimation error of each component in the water balance equation in every year.

$$\Delta(f(x_1, x_2, \ldots, x_m, r)) = \sum_{j=1}^{m}\left|\frac{\partial f}{\partial x_j}\Delta x_j\right| \tag{30}$$

where x_j, $j = 0, 1, \ldots, m$ are involved variables, Δx_j, $j = 0, 1, \ldots, m$ are the corresponding estimation error for each x_j and Δ is the overall error in estimating the parameter of interest. In Eq. (30), r refers to the constants in the function (Bevington and Robinson, 2002). Eq. (30) was used to find the errors associated with the components in every year. The percentage of the errors was calculated for different components through dividing the calculated errors using Eq. (30) by the estimated values of the components. For every component, an average calculated error over the entire period was considered as a representative error. In the following four paragraphs, the application of the above-mentioned error analysis methods to the data of the study area is presented.

The available records of precipitation only for two stations were used to find the average annual precipitation depth in the study area for each year using Thiessen polygons method. The corresponding estimation errors were obtained using cross validation (Table 2).

In Turc method, actual evapotranspiration is expressed as a function of precipitation and temperature. To find an estimation error for temperature taken from the generalization of Senobar-station temperature data to the entire catchment, the nearby stations were first located. Then, cross validation was applied similarly to precipitation in order to find the temperature estimation error. Camillo and Gurney (1984) reported 10% error for Turc method or similar evapotranspiration estimation methods compared to the direct measurement of evapotranspiration. In this study, the 10% error was attributed to the modeling uncertainty in the equation developed by Turc. The overall estimation error for the estimation of the actual evapotranspiration was used using Eq. (30) (Table 2). The evapotranspiration errors reported in the table include the 10% error pertinent to the model structure in Turc method.

Annual surface outflow is the product of runoff coefficient, precipitation and area. As mentioned earlier, the average annual runoff coefficient was estimated as 0.14 in the study area. An estimation error of 0.03 was obtained for the runoff coefficient, which was developed using regional analysis. The overall error in the annual surface outflow was calculated by incorporating errors in precipitation and runoff coefficient using Eq. (30).

Change in groundwater storage in any time interval is a product of the change in groundwater level, storage coefficient, and area of the aquifer. Cross validation was applied to the available data for the seven piezometric wells in Azghand catchment in order to find an estimation error for the annual change in groundwater storage in each year. The average estimated value of the storage coefficient for the Azghand aquifer was 0.07. An upper value of 0.09 and a lower value of 0.05 were considered for the storage coefficient. This variation in storage coefficient with about 29% error was selected based on the geological data and the information from the local hydrogeologists. Overall error in estimating the annual change in groundwater storage was calculated using Eq. (30) (Table 2).

Table 2
Percentage error in estimation of water balance components.

Water year	Precipitation (%)	Surface water outflow (%)	Evapotranspiration (%)	Change in groundwater storage (%)
1992	9.861	31.436	34.261	44.810
1993	7.280	28.856	30.358	34.490
1994	5.859	27.434	17.878	56.400
1995	6.698	28.273	21.553	41.488
1996	19.028	40.602	35.451	36.120
1997	7.104	28.679	19.306	36.750
1998	11.292	32.867	32.086	48.731
1999	5.903	27.477	19.715	67.114
2000	7.658	29.232	18.405	43.121
2001	7.062	28.636	18.268	33.635
2002	13.809	35.383	27.011	33.961
2003	6.175	27.750	21.385	37.725
2004	6.616	28.190	20.812	37.239
2005	14.129	35.703	30.991	37.391
2006	13.660	35.234	25.240	33.758
Mean	9.476	31.050	24.848	41.516
St. Dev	4.007	4.007	6.302	9.534

After the data related to percentage error in estimating all components of the water balance equation had been obtained, the second step was to define the reasonable criteria for setting the constraints' boundaries. Therefore, the interval covering 68% of the data was considered for the center of the correction coefficient fuzzy numbers, and the interval covering 99% of the data was considered for the minimum half-width of the fuzzy numbers. From a probabilistic point of view, the coverage of 68% and 99% indicate coverage of a mean plus one standard deviation and a mean plus three standard deviations, respectively. Table 3 shows the boundaries considered for the correction coefficient fuzzy numbers used as constraints in the fuzzy linear regression models.

4.3. Sensitivity analysis

Although selection of h, indicating the level of confidence to the data, is a major issue in fuzzy regression analysis, there is no well-defined criterion to determine h. Bardossy et al. (1990) suggested $0.5 \leqslant h \leqslant 0.7$ for hydrologic studies. In this study, different values of 0.5, 0.7, and 0.9 were considered for h and the sensitivity of the results to its variation was studied.

5. Results and discussion

Four fuzzy linear regression models were applied to the annual data of Azghand catchment. Table 4 shows the fuzzy correction coefficients obtained for water balance components at different values of h. The values of α and c are the center and half-width of the symmetric fuzzy triangular numbers.

Sensitivity analysis shows that center of evapotranspiration and change in groundwater storage fuzzy numbers, as well as, half-width of evapotranspiration fuzzy numbers are sensitive to variations of h value. It is interesting to mention that in all models the precipitation and actual evapotranspiration are overestimated while surface water outflow is underestimated in Azghand catchment. Change in groundwater storage is about right in Model-I and overestimated in Model-II. The underestimation and

Table 3
Boundaries determined for the center and half-width of the fuzzy numbers.

Fuzzy number characteristics	Precipitation (mcm)	Surface water outflow (mcm)	Evapotranspiration (mcm)	Change in groundwater storage (mcm)
Minimum half-width of fuzzy number	0.215	0.431	0.438	0.701
Left boundary for center of fuzzy number	0.865	0.649	0.688	0.490
Right boundary for center of fuzzy number	1.135	1.351	1.312	1.510

Table 4
Fuzzy correction coefficients for different models and different h values.

Model		h	Precipitation		Surface water outflow		Evapotranspiration		Change in groundwater storage	
			α	c	α	c	α	c	α	c
Model-I	Case 1	0.5	0.865	0.556	1.351	0.431	0.712	0.438	1.029	0.701
		0.7	0.865	0.617	1.351	0.431	0.718	0.438	1.017	0.701
		0.9	0.865	0.918	1.351	0.431	0.724	0.438	1.006	0.701
	Case 2	0.5	0.865	0.586	1.351	0.431	0.712	0.438	0.876	0.701
		0.7	0.865	0.646	1.351	0.431	0.718	0.438	0.929	0.701
		0.9	0.865	0.948	1.351	0.431	0.724	0.438	0.975	0.701
Model-II	Case 1	0.5	0.865	0.633	1.351	0.431	0.720	0.610	0.828	0.701
		0.7	0.865	0.633	1.351	0.431	0.720	0.610	0.828	0.701
		0.9	0.865	0.633	1.351	0.431	0.720	0.610	0.828	0.701
	Case 2	0.5	0.865	0.541	1.351	0.431	0.720	0.479	0.828	0.701
		0.7	0.865	0.545	1.351	0.431	0.720	0.484	0.828	0.701
		0.9	0.865	0.545	1.351	0.431	0.720	0.484	0.828	0.701

Table 5
Modified water balance errors after application of correction coefficients (h = 0.5).

Water year	Error in conventional water balance equation (mcm)	Modified Error (mcm)			
		Model-I		Model-II	
		Case 1	Case 2	Case 1	Case 2
1992	2.822	51.13	54.83	50.05	50.12
1993	−59.62	−5.64	3.25	0.04	0.22
1994	−30.186	4.31	1.18	−2.3	−2.36
1995	−45.14	8.6	5.67	0.49	0.43
1996	−55.466	2.06	0.21	−5	−5.03
1997	−44.809	−0.21	−2.96	−7.14	−7.2
1998	−38.83	22.14	23.82	18.36	18.39
1999	−70.547	−16.57	−16.01	−20.14	−20.13
2000	−23.878	1.37	−0.93	−3.43	−3.48
2001	−14.551	18.03	12.87	8.95	8.85
2002	−41.11	8.93	5.06	0.05	−0.02
2003	−32.388	24.78	20.78	14.77	14.69
2004	−41.356	8.95	5.63	0.67	0.6
2005	−46.432	6.61	4.66	−0.29	−0.33
2006	−28.97	3.12	0.18	−3.04	−3.09
2007	−44.539	4.41	1.55	−3.12	−3.18
2008	−14.279	6.71	3.93	1.6	1.55
2009	−47.336	5.36	2.94	−2.02	−2.06
Total error (mcm)	−676.62	154.09	126.66	48.50	47.97
Total absolute error (mcm)	682.26	198.93	166.46	141.46	141.73
MAE (mcm)	37.90	11.05	9.25	7.86	7.87
RMSE (mcm)	41.32	16.25	15.96	14.27	14.28
MRE (%)	–	29.16	24.40	20.73	20.77

Table 6
Modified water balance errors after application of correction coefficients (h = 0.7).

Water year	Error in conventional water balance equation (mcm)	Modified Error (mcm)			
		Model-I		Model-II	
		Case 1	Case 2	Case 1	Case 2
1992	2.822	46.97	49.1	50.05	50.05
1993	−59.62	−9.44	−4.32	0.04	0.04
1994	−30.186	2.19	0.39	−2.3	−2.3
1995	−45.14	5.17	3.48	0.49	0.49
1996	−55.466	−1.56	−2.62	−5	−5
1997	−44.809	−2.92	−4.5	−7.14	−7.14
1998	−38.83	17.78	18.75	18.36	18.36
1999	−70.547	−19.75	−19.43	−20.14	−20.14
2000	−23.878	−0.14	−1.47	−3.43	−3.43
2001	−14.551	15.9	12.93	8.95	8.95
2002	−41.11	5.77	3.55	0.05	0.05
2003	−32.388	20.9	18.6	14.77	14.77
2004	−41.356	5.76	3.84	0.67	0.67
2005	−46.432	3.21	2.09	−0.29	−0.29
2006	−28.97	1.17	−0.52	−3.04	−3.04
2007	−44.539	1.35	−0.29	−3.12	−3.12
2008	−14.279	5.39	3.8	1.6	1.6
2009	−47.336	2.03	0.63	−2.02	−2.02
Total error (mcm)	−676.62	99.78	84.01	48.50	48.50
Total absolute error (mcm)	682.26	167.40	150.31	141.46	141.46
MAE (mcm)	37.90	9.30	8.35	7.86	7.86
RMSE (mcm)	41.32	14.62	14.45	14.27	14.27
MRE (%)	–	24.54	22.03	20.73	20.73

overestimation are indication of bias in measurements and estimations. Additionally, based on the c values, the change in groundwater storage and surface water outflow are the most and least uncertain parameters, respectively.

Tables 5–7 show the modified errors in water balance equation after applying the fuzzy correction coefficients for 0.5, 0.7, and 0.9 levels of confidence. The second columns in the tables show the initial error in conventional water balance equation when no correction coefficient is considered (the same values reported in Table 1) while the last four columns show the modified errors when correction coefficients, generated by each proposed model, were applied to the components of water balance equation. As shown in Tables 5–7, overall error criteria such as total error, total absolute error, mean absolute error (MAE) and root mean square error (RMSE) were also calculated for the conventional water balance equation and proposed models in order to highlight the overall efficiency of different models in reducing error in annual water balance equation. It must be emphasized that in the context of this

Table 7
Modified water balance errors after application of correction coefficients ($h = 0.9$).

Water year	Error in conventional water balance equation (mcm)	Modified Error (mcm)			
		Model-I		Model-II	
		Case 1	Case 2	Case 1	Case 2
1992	2.822	42.789	43.54	50.05	50.05
1993	−59.62	−13.291	−11.49	0.04	0.04
1994	−30.186	0.096	−0.54	−2.3	−2.3
1995	−45.14	1.754	1.16	0.49	0.49
1996	−55.466	−5.158	−5.53	−5	−5
1997	−44.809	−5.601	−6.16	−7.14	−7.14
1998	−38.83	13.404	13.75	18.36	18.36
1999	−70.547	−22.94	−22.83	−20.14	−20.14
2000	−23.878	−1.649	−2.11	−3.43	−3.43
2001	−14.551	13.797	12.75	8.95	8.95
2002	−41.11	2.647	1.86	0.05	0.05
2003	−32.388	17.054	16.24	14.77	14.77
2004	−41.356	2.58	1.91	0.67	0.67
2005	−46.432	−0.18	−0.58	−0.29	−0.29
2006	−28.97	−0.765	−1.36	−3.04	−3.04
2007	−44.539	−1.69	−2.27	−3.12	−3.12
2008	−14.279	4.09	3.53	1.6	1.6
2009	−47.336	−1.29	−1.78	−2.02	−2.02
Total error (mcm)	−676.62	45.65	40.09	48.50	48.50
Total absolute error (mcm)	682.26	150.78	149.39	141.46	141.46
MAE (mcm)	37.90	8.38	8.30	7.86	7.86
RMSE (mcm)	41.32	13.53	13.48	14.27	14.27
MRE (%)	–	22.10	21.90	20.73	20.73

study, all the mentioned error criteria were calculated based on the difference between the calculated and desired errors. As explained in Section 3, the desired error must approach to a value of zero in an accurate water balance analysis. The last row of the tables show an index referred to as model relative error in this study. Model relative error (MRE) in percentage was calculated by dividing the total absolute error resulting from each proposed model by the total absolute error of the conventional water balance equation. Model relative error values for all suggested models are found in Tables 5–7. The less is the value of MRE, the better is the performance of the model.

The following results are significant in Tables 5–7:

1 In almost all cases, the modified errors have significantly reduced compared to the initial errors, except for the five underlined data in the tables. Although the first-year data could have been considered as an outlier and excluded from the analysis, however it was not performed in the analysis.
2 All models perform well under validation data as it is observed in the shading rows in the tables.
3 Although all models have performed well based on overall error criteria, the least efficient model is identified as Model-I, Case 1 ($h = 0.5$).
4 Among the models, Model-I, Case 1 is more sensitive to the selection of h. For this model, the error decreases as h increases. Other models are not so sensitive to the selection of h.
5 There is no significant difference between Model-II, Case 1 and Model-II, Case 2 especially under higher h values.
6 Model-II is more efficient than Model-I in reducing error in water balance equation at 0.5 and 0.7 levels of confidence based on all selected overall error criteria reported in Tables 5 and 6. However, at $h = 0.9$, the response of the models to different error criteria is slightly different. In this case, total error and RMSE measures indicate that Model-I performs better while total absolute error, MAE and MRE show a better performance for Model-II.

The above-mentioned items indicate that Model-II performs better than Model-I especially at lower levels of confidence, which are more realistic in hydrological modeling. The better performance of Model-II than Model-I can be attributed to the objective functions considered for these models. As described in Section 2, the objective function of Model-I is minimizing uncertainty of parameters while that of Model-II is minimizing the deviations between observed and predicted values.

6. Summary and conclusions

Water balance study is the first step for water management in any region. It is believed that there is always an error in using water balance equation especially in the areas where the measurement networks are not well designed. The error in the water balance equation could hypothetically be reduced by applying correction coefficients to different components of the water balance equation. In this study, symmetric triangular fuzzy numbers were considered for the correction coefficients. Four different models were considered to adjust the correction coefficients in an error minimization procedure based on fuzzy linear regression concepts. The models were applied to the data collected from Azghand catchment as the study area of interest. In the formulation of the models for Azghand catchment, the constraints' boundaries were set based on a comprehensive review of the possible error that could occur in the process of estimation of individual water balance components. Data taken from fifteen consecutive years were used for model development and data taken from three following years were used as validation data set. The main conclusions of application of the models to the Azghand catchment data are as follows:

- Models show excellent performance in error reduction for the validation data.
- Although all models perform reasonably well in reducing the error in the water balance equation, Model-II has a better performance since it reduced the total absolute error up to 79% and also was not significantly sensitive to the selection of h. The 79% reduction in error for Model-II was calculated as (100-MRE (%)) for this model, using MRE values reported in the last rows of Tables 5–7.
- Models with higher values of h are more effective in reducing the error in using water balance equation than the models with lower h values. Additionally, different models showed nearly the same results under higher values of h.
- The current study shows that precipitation and evapotranspiration are overestimated and surface water outflow is underestimated in Azghand catchment. Change in groundwater storage is about right or overestimated based on Model-II. The overestimation or underestimation of the water balance components indicates poor performance of the current measurement networks and the methods to process the measured data.

The following suggestions are made for future studies:

- If data are available, the methodology proposed in current study can be applied to surface water and groundwater balance equations simultaneously, and results can be compared with those of overall system water balance equation.
- If data are available, the procedure suggested in this study for error reduction in annual water balance analysis can be applied to other time scales such as monthly or seasonal studies.
- The procedure suggested in this study for error reduction in water balance equation can be conducted using fuzzy numbers having shapes other than symmetric triangular fuzzy numbers.

- In a different approach from what presented in this study, correction coefficients in water balance equation can be considered as crisp numbers, and their values can be adjusted using conventional procedures that yield the least sum of square of errors.

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