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T-duality of O-plane action at order α'^2

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ABSTRACT

We use compatibility of O-plane action with nonlinear T-duality as a guiding principle to find all NS–NS couplings in the O-plane action at order α'^2. We find that the dilaton couplings appear in the string frame only via the transformation $\hat{R}_{\mu\nu} \to \hat{R}_{\mu\nu} + \nabla_\mu \nabla_\nu \phi$.

1. Introduction and result

The low energy effective field theory of D-branes in type II superstring theories consists of the Dirac–Born–Infeld (DBI) [1] and the Chern–Simons (CS) actions [2]. The effective theory of O-plane is an orientifold projection of above actions. The curvature corrections to the CS part have been found in [3–5] by requiring that the chiral anomaly on the world volume of intersecting D-branes (I-brane) cancels with the anomalous variation of the CS action. The curvature corrections to the DBI action, on the other hand, have been found in [6] by requiring consistency of the effective action with the $O(\alpha'^2)$ terms of the corresponding disk-level scattering amplitude [7,8]. For totally-geodesic embeddings of world-volume in the ambient spacetime, the corrections in the string frame for zero B-field and for constant dilaton are[1] [6]

$$S \supset \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x\, e^{-\phi}\sqrt{-\widetilde{G}}\Big[R_{abcd}R^{abcd} - 2\hat{R}_{ab}\hat{R}^{ab} - R_{abij}R^{abij} + 2\hat{R}_{ij}\hat{R}^{ij}\Big] \quad (1)$$

where $\hat{R}_{ab} = R^c{}_{acb}$, $\hat{R}_{ij} = R^c{}_{icj}$ and $\widetilde{G} = \det(\widetilde{G}_{ab})$ where $\widetilde{G}_{ab}$ is the pull-back of the bulk metric onto the word-volume, *i.e.*,

$$\widetilde{G}_{ab} = \frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b}G_{\mu\nu} \quad (2)$$

The orientifold projection projects out the Riemann curvature with odd number of transverse indices, so the above couplings are the curvature couplings on the world volume of both D-brane and O-plane depending on the tension T_p which is different for D-brane and O-plane.

In the presence of non-constant dilaton, the couplings (1) are not consistent with T-duality. For zero B-field, the compatibility with linear T-duality requires the following extension [9,10]:

$$S \supset \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x\, e^{-\phi}\sqrt{-\widetilde{G}}\Big[R_{abcd}R^{abcd} - 2\mathcal{R}_{ab}\mathcal{R}^{ab} - R_{abij}R^{abij} + 2\mathcal{R}_{ij}\mathcal{R}^{ij}\Big] \quad (3)$$

where $\mathcal{R}_{\mu\nu} = \hat{R}_{\mu\nu} + \nabla_\mu\nabla_\nu\phi$. The orientifold projection projects out dilaton with odd number of transverse derivatives, so the above couplings are valid for both D-brane and O-plane actions.

In the presence of non-zero B-field, the couplings (3) are not consistent with the T-duality. Using the compatibility of these couplings with linear T-duality as a guiding principle, the quadratic B-field couplings at order $O(\alpha'^2)$ have been found in [9] to be

$$S \supset \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x\, e^{-\phi}\sqrt{-\widetilde{G}}\Big[\frac{1}{2}\nabla_a H_{bci}\nabla^a H^{bci} - \frac{1}{6}\nabla_a H_{ijk}\nabla^a H^{ijk} - \frac{1}{3}\nabla_i H_{abc}\nabla^i H^{abc}\Big] \quad (4)$$

The above couplings have been confirmed with the disk level S-matrix calculations [9]. The orientifold projection projects out covariant derivatives of B-field with even number of transverse indices, so again the above couplings are valid for both D-brane and O-plane.

The quadratic couplings (3) and (4) are not however consistent with the full nonlinear T-duality transformations. In this paper, we are going to use the compatibility of the couplings (1) with nonlinear T-duality as a guiding principle to find the couplings of O-plane to all massless NS–NS fields at order α'^2. We find the quadratic

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[1] Our index conversion is that the Greek letters $(\mu, \nu, \cdots)$ are the indices of the space–time coordinates, the Latin letters $(a, d, c, \cdots)$ are the world-volume indices and the letters $(i, j, k, \cdots)$ are the normal bundle indices.

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couplings (3) and (4) as well as the following higher order couplings:

$$S \supset \frac{\pi^2\alpha'^2 T_p}{48}\int d^{p+1}x e^{-\phi}\sqrt{-\widetilde{G}}\Big[H^{abi}H_a{}^c{}_i\mathcal{R}_{bc} - \frac{3}{2}H^{abi}H_{ab}{}^j\mathcal{R}_{ij} + \frac{1}{2}H^{ijk}H_{ij}{}^l\mathcal{R}_{kl} - H^{abi}H^{cd}{}_iR_{abcd} + H^{abi}H_i{}^{jk}R_{abjk} - \frac{1}{4}H^{abi}H_{ab}{}^jH_i{}^{kl}H_{jkl} + \frac{1}{4}H^{abi}H_{ab}{}^jH^{cd}{}_iH_{cdj} + \frac{1}{8}H^{abi}H_a{}^{cj}H_b{}^d{}_jH_{cdi} - \frac{1}{6}H^{abi}H_a{}^{cj}H_{bc}{}^kH_{ijk} + \frac{1}{24}H^{ijk}H_i{}^{lm}H_{jl}{}^nH_{kmn}\Big] \quad (5)$$

The consistency of the effective actions with T-duality has been also used in [11–18] to find new couplings in the world volume and spacetime actions.

An outline of the paper is as follows. In the next section, we present an algorithm for calculating the world volume theory of D-brane/O-plane by imposing the action to be consistent with the T-duality transformations. In Section 3, we find the couplings of gravity and dilaton for O-plane which are invariant under a simplified T-duality transformations in which there is no B-field and the metric is diagonal. We find six multiplets which are invariant under the simplified T-duality transformations. These multiplets, however, are not invariant under the full T-duality transformations. Using the consistency of the multiplets with S-matrix elements, we argue that only two of these multiplets survive under the full T-duality transformations. In Section 4, we find the appropriate B-field couplings which make the two multiplets to be invariant under the full T-duality transformations.

2. T-duality constraint

The full set of nonlinear T-duality transformations for massless fields have been found in [19–23]. When the T-duality transformation acts along the Killing coordinate y, the transformations of NS–NS fields are

$$\begin{aligned} e^{2\phi} &\to \frac{e^{2\phi}}{G_{yy}}, & G_{yy} &\to \frac{1}{G_{yy}},\\ G_{\alpha y} &\to \frac{B_{\alpha y}}{G_{yy}}, & G_{\alpha\beta} &\to G_{\alpha\beta} - \frac{G_{\alpha y}G_{\beta y} - B_{\alpha y}B_{\beta y}}{G_{yy}},\\ B_{\alpha y} &\to \frac{G_{\alpha y}}{G_{yy}}, & B_{\alpha\beta} &\to B_{\alpha\beta} - \frac{B_{\alpha y}G_{\beta y} - G_{\alpha y}B_{\beta y}}{G_{yy}}, \end{aligned} \quad (6)$$

where $\alpha, \beta \neq y$. In above transformation the metric is given in the string frame. If y is identified on a circle of radius ρ, *i.e.*, $y \sim y + 2\pi\rho$, then after T-duality the radius becomes $\tilde{\rho} = \alpha'/\rho$. The string coupling is also transformed as $\tilde{g} = g\sqrt{\alpha'}/\rho$. It is known that the above transformations do not receive α' correction in the type II superstring theories in which we are interested.

If one defines the new field φ as $G_{yy} = e^{-\varphi}$ and uses the dimensional reduction to write the 10-dimensional metric and B-field as

$$G_{\mu\nu} = \begin{pmatrix} g_{\alpha\beta} + e^{\varphi}g_\alpha g_\beta & e^{\varphi}g_\alpha \\ e^{\varphi}g_\beta & e^{\varphi}\end{pmatrix},$$

$$B_{\mu\nu} = \begin{pmatrix} b_{\alpha\beta} + \frac{1}{2}b_\alpha g_\beta - \frac{1}{2}b_\beta g_\alpha & b_\alpha \\ -b_\beta & 0\end{pmatrix} \quad (7)$$

where $g_{\alpha\beta}$, $b_{\alpha\beta}$ are the metric and the B-field, and g_α, b_α are two vectors in the 9-dimensional base space, then the T-duality transformations (6) simplify to

$$\phi \to \phi - \frac{1}{2}\varphi \;,\; \varphi \to -\varphi \;,\; g_\alpha \to b_\alpha \;,\; b_\alpha \to g_\alpha \quad (8)$$

The 9-dimensional base space fields $g_{\alpha\beta}$ and $b_{\alpha\beta}$ remain invariant under the T-duality.

A method for finding the world volume couplings which are invariant under linear T-duality is given in [9]. This method may be used to show that the S-matrix elements satisfy the Ward identity corresponding to the T-duality (see *e.g.*, [24]). In this section, we are going to extend this method to find the world volume action at order α'^2 which is invariant under nonlinear T-duality. To this end, we first write all covariant couplings at order α'^2 with unknown coefficients. These couplings can be constructed by contracting the appropriate bulk tensors with the inverse of bulk metric $G^{\mu\nu}$ or with the world volume first fundamental form $\widetilde{G}^{\mu\nu}$. This tensors is defined as

$$\widetilde{G}^{\mu\nu} = \frac{\partial X^\mu}{\partial\sigma^a}\frac{\partial X^\nu}{\partial\sigma^b}\widetilde{G}^{ab} \quad (9)$$

where $\widetilde{G}^{ab}$ is inverse of the pull-back metric $\widetilde{G}_{ab}$. We call the action corresponding to these couplings S. Then we reduce the action to the 9-dimensional space which depends on whether the Killing coordinate y is a world volume or transverse direction. When y is a world volume direction we call the reduced action S^w, and when y is a transverse direction we call it S^t. The T-duality of S^w which we call it S^{wT} must be equal to S^t up to some total derivative terms, *i.e.*,

$$S^{wT} - S^t = 0 \quad (10)$$

The above constraint can be used to find the unknown coefficients in the original action S.

Using the reductions (7) and the reduction of $G^{\mu\nu}$ which is the inverse of $G_{\mu\nu}$ in (7), *i.e.*,

$$G^{\mu\nu} = \begin{pmatrix} g^{\alpha\beta} & -g^\alpha \\ -g^\beta & e^{-\varphi} + g_\chi g^\chi \end{pmatrix} \quad (11)$$

where $g^{\alpha\beta}$ is inverse of $g_{\alpha\beta}$, it is straightforward to reduce the bulk tensors which contract only with $G^{\mu\nu}$. One should first separate the 10-dimensional indices to the 9-dimensional indices and the y index. Then one should reduce the 10-dimensional field in them to the 9-dimensional fields according to (7) and (11).

For the couplings in which the bulk tensors contract with $G^{\mu\nu}$ and $\widetilde{G}^{\mu\nu}$, we also need the reduction of $\widetilde{G}^{\mu\nu}$. The reduction of the first fundamental form depends on weather the brane is along or orthogonal to the circle. In the former case, the reduction of $\widetilde{G}_{ab}$ is

$$\widetilde{G}_{ab} = \begin{pmatrix} \frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}g_{\alpha\beta} + e^{\varphi}\frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}g_\alpha g_\beta & e^{\varphi}\frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}g_\alpha \\ e^{\varphi}\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}g_\beta & e^{\varphi}\end{pmatrix} \quad (12)$$

where the world volume indices $\tilde{a}, \tilde{b} \neq y$. Inverse of this matrix is

$$\widetilde{G}^{ab} = \begin{pmatrix} \tilde{g}^{\tilde{a}\tilde{b}} & -\tilde{g}^{\tilde{a}} \\ -\tilde{g}^{\tilde{b}} & e^{-\varphi} + \tilde{g}_{\tilde{a}}\tilde{g}^{\tilde{a}}\end{pmatrix} \quad (13)$$

where $\tilde{g}^{\tilde{a}\tilde{b}}$ is inverse of the pull-back of the 9-dimensional bulk metric onto the word-volume, *i.e.*,

$$\tilde{g}_{\tilde{a}\tilde{b}} = \frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}g_{\alpha\beta} \quad (14)$$

and $\tilde{g}^{\tilde{a}} = \tilde{g}^{\tilde{a}\tilde{b}}\frac{\partial X^\alpha}{\partial\sigma^{\tilde{b}}}g_\alpha$. Therefore, the reduction of $\widetilde{G}^{\mu\nu}$ becomes

$$\widetilde{G}^{\mu\nu} = \begin{pmatrix} \frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}\tilde{g}^{\tilde{a}\tilde{b}} & -\frac{\partial X^\alpha}{\partial\sigma^{\tilde{a}}}\tilde{g}^{\tilde{a}} \\ -\frac{\partial X^\beta}{\partial\sigma^{\tilde{b}}}\tilde{g}^{\tilde{b}} & e^{-\varphi} + \tilde{g}_{\tilde{a}}\tilde{g}^{\tilde{a}}\end{pmatrix} \quad (15)$$

In the latter case where y is not a world volume index, the reduction of $\widetilde{G}_{ab}$ is

$$\widetilde{G}_{ab}=\begin{pmatrix}\frac{\partial X^\alpha}{\partial\sigma^{\tilde a}}\frac{\partial X^\beta}{\partial\sigma^{\tilde b}}g_{\alpha\beta} & 0\\ 0 & 0\end{pmatrix}\tag{16}$$

and its inverse is

$$\widetilde{G}^{ab}=\begin{pmatrix}\widetilde{g}^{\tilde a\tilde b} & 0\\ 0 & 0\end{pmatrix}\tag{17}$$

Therefore, the reduction of $\widetilde{G}^{\mu\nu}$ in this case becomes

$$\widetilde{G}^{\mu\nu}=\begin{pmatrix}\frac{\partial X^\alpha}{\partial\sigma^{\tilde a}}\frac{\partial X^\beta}{\partial\sigma^{\tilde b}}\widetilde{g}^{\tilde a\tilde b} & 0\\ 0 & 0\end{pmatrix}\tag{18}$$

To proceed further, we need to fix the world volume reparametrization invariance of the action. We fix it by choosing the static gauge where $X^a=\sigma^a$ and $X^i=2\pi\alpha'\Phi^i$. For O-plane at $X^i=0$ and D-brane at fixed position $X^i=0$, one finds $\frac{\partial X^\alpha}{\partial\sigma^{\tilde a}}=\delta^\alpha_{\tilde a}$. As a result, the pull-back metric becomes $\widetilde{g}_{\tilde a\tilde b}=g_{\tilde a\tilde b}$ and the above reductions become

$$\widetilde{G}^{\mu\nu}=\begin{pmatrix}g^{\tilde a\tilde b} & -g^{\tilde a}\\ -g^{\tilde b} & e^{-\varphi}+g_{\tilde a}g^{\tilde a}\end{pmatrix}\tag{19}$$

when brane is along the y direction, and

$$\widetilde{G}^{\mu\nu}=\begin{pmatrix}g^{\tilde a\tilde b} & 0\\ 0 & 0\end{pmatrix}\tag{20}$$

when brane is orthogonal to the y direction. Using the reductions (7) and (11), and the reductions (19), (20), it is then straightforward to reduce the bulk tensors which contract with $G^{\mu\nu}$ and with the first fundamental form $\widetilde{G}^{\mu\nu}$.

The T-duality requires the world volume action to have the following structure:

$$S=\int d^{p+1}x e^{-\phi}\sqrt{-\widetilde{G}}\mathcal{L}\tag{21}$$

Only the covariant derivatives of dilaton appears in $\mathcal{L}$. The reduction of $e^{-\phi}\sqrt{-\widetilde{G}}$ along a world volume direction can be read from (12) to be $e^{-\phi+\varphi/2}\sqrt{-\widetilde{g}}$ where $\widetilde{g}=\det(\widetilde{g}_{\tilde a\tilde b})$. It transforms to $e^{-\phi}\sqrt{-\widetilde{g}}$ under the T-duality (8). On the other hand, the reduction of $e^{-\phi}\sqrt{-\widetilde{G}}$ along a transverse direction can be read from (16) to be $e^{-\phi}\sqrt{-\widetilde{g}}$. So the constraint (10) can be written as

$$\int d^p e^{-\phi}\sqrt{-\widetilde{g}}\Big[\mathcal{L}^{wT}-\mathcal{L}^t\Big]=0\tag{22}$$

where $\mathcal{L}^{wT}$ is the T-duality of reduction of Lagrangian $\mathcal{L}$ when brane is along the y-direction, and $\mathcal{L}^t$ is the reduction of $\mathcal{L}$ when brane is orthogonal to the y-direction.

To satisfy the constraint (22) there are two possibilities. One is to consider all couplings with arbitrary covariant derivatives in $\mathcal{L}$ which are at order α'^2, *e.g.*, $\nabla^2 R$, and then to impose the constraint that the Lagrangian is T-duality invariant, *i.e.*, $\mathcal{L}^{wT}-\mathcal{L}^t=0$. Another possibility is to consider only couplings in which each term has at most two derivatives, *e.g.*, R^2. In this case, one may not have the strong constraint $\mathcal{L}^{wT}-\mathcal{L}^t=0$, however, using integration by part on the left-hand side of (22), one can fix the unknown coefficients to satisfy the constraint. In this paper we use this latter possibility.

3. Couplings without B-field

In this section we are going to apply the above T-duality constraint to find the world volume couplings of dilaton and graviton at order α'^2. Since the off-diagonal components of metric transforms to B-field under the T-duality transformations, we assume then the metric is diagonal and B-field is zero. Using the Mathematica package "xAct" [25], one can easily write all couplings with structures R^2, $R(\nabla\phi)^2$, $R\nabla\nabla\phi$, $(\nabla\nabla\phi)^2$, $\nabla\nabla\phi(\nabla\phi)^3$ and $(\nabla\phi)^4$ where R stands for scalar, Ricci and Reimann curvatures. In general, one should consider also the couplings involving the second fundamental form. However, such couplings are zero for O-plane. In writing the above couplings explicitly, one should use both $G^{\mu\nu}$ and $\widetilde{G}^{\mu\nu}$ for contracting the indices. There are too many of such couplings to be able to write them here.

To find the unknown coefficients of these couplings, we first consider the case that brane is along the y-direction. We then reduce it to the 9-dimensional space and use the T-duality transformation

$$\phi\to\phi-\frac{1}{2}\varphi\ ,\ \varphi\to-\varphi\ ,\tag{23}$$

The resulting couplings must be the reduction of the couplings when brane is orthogonal to the y-direction, *i.e.*, (22). This constraint produces many equations for the unknown coefficients. Since we are interested in finding an action which is invariant under the T-duality transformations, we use integration by part to reduce the constraints to independent ones. To do this last step, we write the curvatures and the covariant derivatives in terms of the 9-dimensional metric and then use the integration by part to find the independent structures. For example, the following terms are a total derivative:

$$\begin{aligned}\int d^p e^{-\phi}\sqrt{-\widetilde{g}}\Big[&\frac{1}{2}g^{\tilde a\tilde b}g^{\tilde c\tilde d}g^{\tilde e\tilde f}\partial_{\tilde a}\varphi\partial_{\tilde b}\varphi\partial_{\tilde c}\varphi\partial_{\tilde d}g_{\tilde e\tilde f}\\ &-g^{\tilde a\tilde b}g^{\tilde c\tilde d}\partial_{\tilde a}\phi\partial_{\tilde b}\varphi\partial_{\tilde c}\varphi\partial_{\tilde d}\varphi+2g^{\tilde a\tilde b}g^{\tilde c\tilde d}\partial_{\tilde a}\varphi\partial_{\tilde c}\varphi\partial_{\tilde d}\partial_{\tilde b}\varphi\\ &+g^{\tilde a\tilde b}g^{\tilde c\tilde d}\partial_{\tilde a}\varphi\partial_{\tilde b}\varphi\partial_{\tilde d}\partial_{\tilde c}\varphi-g^{\tilde a\tilde b}g^{\tilde c\tilde d}g^{\tilde e\tilde f}\partial_{\tilde a}\varphi\partial_{\tilde c}\varphi\partial_{\tilde e}\varphi\partial_{\tilde f}g_{\tilde b\tilde d}\\ &-g^{\tilde a\tilde b}g^{\tilde c\tilde d}g^{\tilde e\tilde f}\partial_{\tilde a}\varphi\partial_{\tilde b}\varphi\partial_{\tilde c}\varphi\partial_{\tilde f}g_{\tilde d\tilde e}\Big]\end{aligned}$$

So one can use it to write $g^{\tilde a\tilde b}g^{\tilde c\tilde d}\partial_{\tilde a}\varphi\partial_{\tilde c}\varphi\partial_{\tilde d}\partial_{\tilde b}\varphi$ in terms of other terms above. In this way the constraint imposed by the coefficient of $g^{\tilde a\tilde b}g^{\tilde c\tilde d}\partial_{\tilde a}\varphi\partial_{\tilde c}\varphi\partial_{\tilde d}\partial_{\tilde b}\varphi$ can be written in terms of constraints imposed by other terms. One has to use such total derivative terms to reduce the constraints to independent ones. Then the coefficients of all independent structures must be zero.

Another set of constraints on the coefficients of the couplings for O-plane is that in the static gauge the curvatures and the covariant derivatives of dilaton with odd number of transverse indices must be zero. In the 9-dimensional space, the orientifold projection is $\partial_i\varphi=\partial_i\partial_{\tilde a}\varphi=\partial_i\phi=\partial_i\partial_{\tilde a}\phi=0$ and $\partial_i\partial_{\tilde a}g_{\tilde b\tilde c}=\partial_i\partial_{\tilde a}g_{jk}=\partial_i g_{\tilde a\tilde b}=\partial_i g_{jk}=0$.

Using the above constraints, all of the coefficients can be written in terms of a few constants. They produce three type of terms which are invariant under the T-duality transformation (23). One type is the couplings which are zero using the cyclic symmetry of the Riemann curvature, *i.e.*,

$$\begin{aligned}\mathcal{L}\supset C_1&\Big[R_{acbd}R^{abcd}-\frac{1}{2}R_{abcd}R^{abcd}\Big]\\ &+C_2\Big[R_{ajbi}R^{aibj}+\frac{1}{2}R_{abij}R^{abij}-R_{aibj}R^{aibj}\Big]+\cdots\end{aligned}\tag{24}$$

where we have written the spacetime indices in terms of world-volume and transverse indices. Since the above terms are identities, it is safe to set their coefficients to zero. One may use the above identities to simplify the final result.

Another type of terms is the couplings which are total derivatives, *i.e.*,

$$\mathcal{L}\supset C_3\Big[\nabla_a\phi\nabla^a\phi\nabla_b\nabla^b\phi-\nabla_a\phi\nabla^a\phi\nabla_b\phi\nabla^b\phi+2\nabla^a\phi\nabla_b\nabla_a\phi\nabla^b\phi\Big]+C_4\Big[\nabla_a\nabla^a\phi\nabla_b\nabla^b\phi-\nabla_a\phi\nabla^a\phi\nabla_b\nabla^b\phi+\nabla^a\phi\nabla_b\nabla_a\phi\nabla^b\phi-\nabla_b\nabla_a\phi\nabla^b\nabla^a\phi-R^{ab}{}_a{}^c\nabla_b\phi\nabla_c\phi\Big]+\cdots \tag{25}$$

In the action, they can be ignored, so it is safe to set these constants to zero too. One may also use these total derivative terms to simplify the final result.

The remaining T-duality invariant multiplets are the following:

$$\begin{aligned}\mathcal{L}\supset C_5\Big[&R_{ab}R^{ab}+R_{ij}R^{ij}+4R^{ab}\nabla_b\nabla_a\Phi+4\nabla_a\nabla^a\Phi\nabla_b\nabla^b\Phi\\&-4\nabla_a\Phi\nabla^a\Phi\nabla_b\nabla^b\Phi+4\nabla^a\Phi\nabla_b\nabla_a\Phi\nabla^b\Phi\\&-4R^{ab}{}_a{}^c\nabla_b\Phi\nabla_c\Phi+4R^{ij}\nabla_j\nabla_i\Phi+4\nabla_j\nabla_i\Phi\nabla^j\nabla^i\Phi\Big]\\+C_6\Big[&R^a{}_aR^b{}_b+6R^a{}_a\nabla_b\nabla^b\Phi+9\nabla_a\nabla^a\Phi\nabla_b\nabla^b\Phi\\&-8\nabla_a\Phi\nabla^a\Phi\nabla_b\nabla^b\Phi-4R^a{}_a\nabla_b\Phi\nabla^b\Phi\\&+8\nabla^a\Phi\nabla_b\nabla_a\Phi\nabla^b\Phi+2R^a{}_a\nabla_i\nabla^i\Phi+6\nabla_a\nabla^a\Phi\nabla_i\nabla^i\Phi\\&-4\nabla_a\Phi\nabla^a\Phi\nabla_i\nabla^i\Phi+\nabla_i\nabla^i\Phi\nabla_j\nabla^j\Phi\Big]\\+C_7\Big[&R^a{}_aR+3R\nabla_a\nabla^a\phi-2R\nabla_a\phi\nabla^a\phi+4R^a{}_a\nabla_b\nabla^b\phi\\&+12\nabla_a\nabla^a\phi\nabla_b\nabla^b\phi-12\nabla_a\phi\nabla^a\phi\nabla_b\nabla^b\phi-4R^a{}_a\nabla_b\phi\nabla^b\phi\\&+16\nabla^a\phi\nabla_b\nabla_a\phi\nabla^b\phi+4R^a{}_a\nabla_i\nabla^i\phi+R\nabla_i\nabla^i\phi\\&+16\nabla_a\nabla^a\phi\nabla_i\nabla^i\phi-12\nabla_a\phi\nabla^a\phi\nabla_i\nabla^i\phi\\&+4\nabla_i\nabla^i\phi\nabla_j\nabla^j\phi\Big]\\C_8\Big[&R^2+8R\nabla_a\nabla^a\phi-8R\nabla_a\phi\nabla^a\phi+16\nabla_a\nabla^a\phi\nabla_b\nabla^b\phi\\&-16\nabla_a\phi\nabla^a\phi\nabla_b\nabla^b\phi+32\nabla^a\phi\nabla_b\nabla_a\phi\nabla^b\phi+8R\nabla_i\nabla^i\phi\\&+32\nabla_a\nabla^a\phi\nabla_i\nabla^i\phi-32\nabla_a\phi\nabla^a\phi\nabla_i\nabla^i\phi+16\nabla_i\nabla^i\phi\nabla_j\nabla^j\phi\Big]\\+C_9R_{abij}R^{abij}+C_{10}\Big[&R_{abcd}R^{abcd}-2R^{ab}{}_a{}^cR_{bdcd}\\&+2R^{ai}{}_a{}^jR^b{}_{ibj}-2\nabla_b\nabla_a\phi\nabla^b\nabla^a\phi-4R^{ab}{}_a{}^c\nabla_c\nabla_b\phi\\&+4R^{ai}{}_a{}^j\nabla_j\nabla_i\phi+2\nabla_j\nabla_i\phi\nabla^j\nabla^i\phi\Big]\end{aligned}$$

The above multiplets are invariant under the simplified T-duality transformations (23). The coefficients $C_5,\cdots,C_{10}$ should satisfy further constraint if one includes the B-field and demands that the couplings to be invariant under the full T-duality transformations (8). The dilaton couplings in the multiplet with coefficient C_{10} is exactly the couplings in (3) which are reproduced by the corresponding S-matrix element [9]. As a result, we expect the invariance under the full T-duality (8) constraints the coefficients of other multiplets which include the dilaton, to be zero, *i.e.*,

$$C_5=C_6=C_7=C_8=0 \tag{26}$$

In the next section, we use the above constraint and include the B-field couplings to the multiplets C_9 and C_{10}.

4. Couplings with B-field

In this section we are going to find the connection between the constants C_9, C_{10} and find the unknown coefficients of B-field couplings by constraining the whole couplings to be invariant under the T-duality transformation (8). The multiplets with coefficient C_9 and C_{10} in terms of 10-dimensional indices are the following:

$$\begin{aligned}\mathcal{L}\supset C_9\Big[&2R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-4R_{\alpha\gamma\beta\delta}R^{\alpha\beta\gamma\delta}-4R_\alpha{}^{\gamma\delta\epsilon}R_{\beta\gamma\delta\epsilon}\widetilde{G}^{\alpha\beta}\\&+8R_\alpha{}^{\gamma\delta\epsilon}R_{\beta\delta\gamma\epsilon}\widetilde{G}^{\alpha\beta}+2R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\beta\epsilon\delta\varepsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\\&-2R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\beta\varepsilon\delta\epsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}-4R_{\alpha\gamma\epsilon}{}^\zeta R_{\beta\varepsilon\delta\zeta}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\widetilde{G}^{\epsilon\varepsilon}\\&+2R_{\alpha\gamma\epsilon\zeta}R_{\beta\varepsilon\delta\eta}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\widetilde{G}^{\epsilon\varepsilon}\widetilde{G}^{\zeta\eta}\Big]\\+C_{10}\Big[&9R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-18R_{\alpha\gamma\beta\delta}R^{\alpha\beta\gamma\delta}\\&-16R_\alpha{}^{\gamma\delta\epsilon}R_{\beta\gamma\delta\epsilon}\widetilde{G}^{\alpha\beta}+32R_\alpha{}^{\gamma\delta\epsilon}R_{\beta\delta\gamma\epsilon}\widetilde{G}^{\alpha\beta}\\&-8R_{\alpha\gamma}{}^{\epsilon\varepsilon}R_{\beta\epsilon\delta\varepsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}+8R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\beta\epsilon\delta\varepsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\\&-8R_\alpha{}^\epsilon{}_\gamma{}^\varepsilon R_{\beta\varepsilon\delta\epsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}+2R_\alpha{}^\epsilon{}_\beta{}^\varepsilon R_{\gamma\epsilon\delta\varepsilon}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\\&+4R_{\alpha\gamma\beta}{}^\zeta R_{\delta\epsilon\varepsilon\zeta}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\widetilde{G}^{\epsilon\varepsilon}+2R_{\alpha\gamma\epsilon\zeta}R_{\beta\varepsilon\delta\eta}\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\widetilde{G}^{\epsilon\varepsilon}\widetilde{G}^{\zeta\eta}\\&+2\nabla_\beta\nabla_\alpha\phi\nabla^\beta\nabla^\alpha\phi-4\widetilde{G}^{\alpha\beta}\nabla_\gamma\nabla_\beta\phi\nabla^\gamma\nabla_\alpha\phi\\&+4R_\alpha{}^\gamma{}_\beta{}^\delta\widetilde{G}^{\alpha\beta}\nabla_\delta\nabla_\gamma\phi-8R_{\alpha\gamma\beta}{}^\epsilon\widetilde{G}^{\alpha\beta}\widetilde{G}^{\gamma\delta}\nabla_\epsilon\nabla_\delta\phi\Big]\end{aligned}$$

We have to add to the above couplings, the B-field couplings in which the tensors with structures H^4, RH^2, $\nabla\phi\nabla\phi H^2$ and $\nabla\nabla\phi H^2$ contract with $G^{\mu\nu}$ or $\widetilde{G}^{\mu\nu}$. Again there are too many of such couplings to be able to write them here. To find their coefficients we impose the T-duality constraint (22).

To simplify the calculation, we assume that the base fields $g_{\alpha\beta}=\eta_{\alpha\beta}$ and $b_{\alpha\beta}=0$. With this assumption, there are still too many terms to handle even with computer. Since the overall factor of all terms is $\int d^px\,e^{-\phi}\sqrt{-\det(g_{\tilde a\tilde a})}$ which is independent of $g_{\tilde a}$, $b_{\tilde a}$, we can consider the cases that the number of $g_{\tilde a}$, $b_{\tilde a}$ to be 0, 2, 4 and 6 separately. In each case one can use integration by part to find independent structures whose coefficients must be zero. In the case that the number of $g_{\tilde a}$, $b_{\tilde a}$ is 0, one finds no constraint because it is a simplified version of the previous section in which $g_{\alpha\beta}=\eta_{\alpha\beta}$. In the case that the number of $g_{\tilde a}$, $b_{\tilde a}$ is odd or is larger than 6, there is no constraint, *i.e.*, the left-hand side of (22) is zero. Moreover, for the case that the number of $g_{\tilde a}$, $b_{\tilde a}$ is 6, there is no term which has derivative of dilaton or φ, *i.e.*, all four derivatives are on $g_{\tilde a}$ or $b_{\tilde a}$. In this case, the strong condition $\mathcal{L}^{wT}-\mathcal{L}^t=0$ produces only independent constraints because the total derivative terms contains, among other things, the derivatives of dilaton which are not in the list of constraints produced by $\mathcal{L}^{wT}-\mathcal{L}^t=0$.

For O-plane action, the covariant derivatives of B-field with even number of transverse indices are projected out. So the coefficient of such term constraint to be zero. In the 9-dimensional space, the orientifold projection is $\partial_i\varphi=\partial_i\partial_{\tilde a}\varphi=\partial_i\phi=\partial_i\partial_{\tilde a}\phi=0$,

$\partial_{\tilde a} b_i = \partial_i b_{\tilde a} = \partial_i\partial_j b_k = \partial_i\partial_{\tilde a} b_{\tilde b} = 0$ and $\partial_{\tilde a} g_{\tilde b} = \partial_i g_j = \partial_i\partial_{\tilde a} g_j = \partial_i\partial_j g_{\tilde a} = 0$.

Using the above constraints, all of the coefficients can be written in terms of a few constants. In particular, the T-duality fixes $C_9 = -C_{10}$ which makes the above Lagrangian to be proportional to the Lagrangian in (3). The non-zero coefficients of the B-field produce two type of terms which are invariant under the T-duality transformation (8). One type is the couplings which are zero using the cyclic symmetry of the Riemann curvature or the Bianchi identity of B-field, *i.e.*,

$$\begin{aligned}\mathcal{L} \supset a_1\Big[&- 12H_i{}^{lm}H^{ijk}H_{jl}{}^nH_{kmn} + 9H_{ij}{}^lH^{ijk}H_k{}^{mn}H_{lmn}\\ &- H_{ijk}H^{ijk}H_{lmn}H^{lmn}\Big]\\ &+ a_2\Big[- \nabla_b H_{aci}\nabla^c H^{abi} + \frac{1}{2}\nabla_c H_{abi}\nabla^c H^{abi}\\ &- \frac{1}{6}\nabla_i H_{abc}\nabla^i H^{abc}\Big]\\ &+ a_3\Big[\frac{1}{2}H^{abi}H^{cd}{}_iR_{abcd} - H^{abi}H^{cd}{}_iR_{acbd}\Big]\\ &+ a_4\Big[\frac{1}{3}\nabla_a H_{ijk}\nabla^a H^{ijk} - \nabla^a H^{ijk}\nabla_k H_{aij}\Big] + \cdots \end{aligned} \tag{27}$$

These coefficients can be set to zero. The remaining terms have only one overall unknown coefficient, *i.e.*,

$$\begin{aligned}\mathcal{L} \supset C_{10}\Big[&\frac{1}{8}H_a{}^{cj}H^{abi}H_b{}^d{}_jH_{cdi} + \frac{1}{4}H_{ab}{}^jH^{abi}H_{cdj}H^{cd}{}_i\\ &- \frac{1}{6}H_a{}^{cj}H^{abi}H_{bc}{}^kH_{ijk} - \frac{1}{4}H_{ab}{}^jH^{abi}H_i{}^{kl}H_{jkl}\\ &+ \frac{1}{32}H_{ij}{}^lH^{ijk}H_k{}^{mn}H_{lmn} - \frac{1}{288}H_{ijk}H^{ijk}H_{lmn}H^{lmn}\\ &- \frac{3}{4}H^{abi}H^{cd}{}_iR_{abcd} + \frac{1}{2}H^{abi}H_i{}^{jk}R_{abjk} - \frac{3}{2}H^{abi}H^{cd}{}_iR_{acbd}\\ &+ H^{abi}H_i{}^{jk}R_{ajbk} + R_{abcd}R^{abcd} - R_{abij}R^{abij} + \frac{1}{2}H_{ij}{}^lH^{ijk}R^a{}_{kal}\\ &- 2H_a{}^{cj}H^{abi}R_{bcij} + 2H_a{}^c{}_iH^{abi}R_b{}^d{}_{cd} - 2R^{ab}{}_a{}^cR_b{}^d{}_{cd}\\ &+ H_a{}^{cj}H^{abi}R_{bicj} - H_a{}^{cj}H^{abi}R_{bjci} + 2R^{ai}{}_a{}^jR^b{}_{ibj}\\ &- \frac{3}{2}H_{ab}{}^jH^{abi}R^c{}_{icj} + \frac{1}{4}H_i{}^{lm}H^{ijk}R_{jklm} - \frac{1}{2}H_i{}^{lm}H^{ijk}R_{jlkm}\\ &- \frac{1}{6}\nabla_a H_{ijk}\nabla^a H^{ijk} - 2\nabla_b\nabla_a\phi\nabla^b\nabla^a\phi + \nabla_a H^{abi}\nabla_c H_b{}^c{}_i\\ &- H^{abi}\nabla_a\phi\nabla_c H_b{}^c{}_i + H_a{}^c{}_iH^{abi}\nabla_c\nabla_b\phi - 4R^{ab}{}_a{}^c\nabla_c\nabla_b\phi\\ &+ 3\nabla_b H_{aci}\nabla^c H^{abi} - \frac{1}{2}\nabla_c H_{abi}\nabla^c H^{abi} - H^{abi}\nabla_b H_{aci}\nabla^c\phi\\ &- \frac{3}{2}H_{ab}{}^jH^{abi}\nabla_j\nabla_i\phi + 4R^{ai}{}_a{}^j\nabla_j\nabla_i\phi\\ &+ 2\nabla_j\nabla_i\phi\nabla^j\nabla^i\phi + \frac{1}{2}H_{ij}{}^lH^{ijk}\nabla_l\nabla_k\phi\Big] \end{aligned} \tag{28}$$

The overall constant C_{10} can be fixed by comparing R^2 terms above with the corresponding terms in the action (1). The above result can be further simplified using the Bianchi identities in (27).

Since we have used the assumption $g_{\alpha\beta} = \eta_{\alpha\beta}$, the calculation in this section could not find the couplings that are total derivatives. However, using the identity

$$\begin{aligned}&H^\alpha{}_{di}R_{bac\alpha} - H^\alpha{}_{ci}R_{bad\alpha} + H^\alpha{}_{cd}R_{bai\alpha} + \nabla_a\nabla_b H_{cdi}\\ &- \nabla_b\nabla_a H_{cdi} = 0\end{aligned} \tag{29}$$

One can verify that the following is a total derivative term:

$$\begin{aligned}\int d^{p+1}x e^{-\phi}\sqrt{-\tilde G}\Big[&- \nabla_a H^{abi}\nabla_c H_b{}^c{}_i + H_a{}^{cj}H^{abi}R_{bcij}\\ &+ H^{abi}H^e{}_{ci}R_b{}^c{}_{ae} - H_a{}^c{}_iH^{abi}R_b{}^d{}_{cd} - H^{abi}\nabla_b\phi\nabla_c H_a{}^c{}_i\\ &- \nabla_b H_{aci}\nabla^c H^{abi} + H^{abi}\nabla_b H_{aci}\nabla^c\phi\Big]\end{aligned} \tag{30}$$

Using the above total derivative term and the Bianchi identities in (27), we have found that the T-dual couplings in (28) can be simplified to the couplings in (3), (4) and (5).

Our calculation indicates that the derivatives of dilaton in the world volume theory, *i.e.*, equations (3), (4) and (5), appear only through the replacement $\hat R_{\mu\nu} \to \mathcal{R}_{\mu\nu}$. Such replacement appears also in the Chern–Simons part at the quadratic order fields [10]. In fact $\mathcal{R}_{\mu\nu}$ is invariant under linear T-duality. We expect, apart from the overall dilaton factor $e^{-\phi}$, dilaton appears in the world volume theory only through this replacement.

We have seen that the couplings in (3), (4) and (5) are invariant under the full T-duality transformation (8). However, these couplings are not invariant under S-duality for O_3-plane. The S-duality requires adding appropriate R-R couplings. We expect all R-R couplings can be found by requiring the world volume action to be invariant under both T-duality and S-duality. The consistency of the couplings under S-duality and linear T-duality has been considered in [10] to find quadratic world-volume couplings. It would be interesting to extend the calculation in [10] to full nonlinear T-duality to find all NS-NS and R-R couplings on the world volume of O-plane. It would be also interesting to confirm the cubic couplings in (5) by the corresponding S-matrix element of three closed string states at projective plane level. Two-point function on projective plane has been calculated in [26].

5. Note added in proof

During the completion of this work, the preprint [27] appeared which has some overlaps with the results in this paper.

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