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ORIGINAL ARTICLE

Head on collision of shock and breaking waves in degenerate hadronic plasmas

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Abstract Astronomical compact objects, like neutron stars, are made of Fermi-Dirac distributed hadronic matter which has a wide range of densities. In this work, non-relativistic dynamics of high density hadronic plasmas with shear and bulk viscosities are studied. The propagation of localized waves in media with hadronic gas equation of state is investigated. Initially, localized lumps are propagated as breaking and shock waves in inviscid media. It is shown that in the viscous case, the localized waves can travel longer distances before changing into shock or breaking profiles.

Keywords Fermi-Dirac distribution · Shock wave · Degenerate hadronic matter · Hadron gas · Neutron star · Burgers equation

1 Introduction

Astronomical phenomena include many examples where matter is found under conditions entirely different from those found in terrestrial environments. High density degenerate matter in "compact objects" is such an example. The original size of a progenitor star contracts appreciably during the collapse and the interior of the remnant compact star reaches sufficiently high densities, so that instead of thermal pressure the compact star is stabilized by the degeneracy pressure of interacting Fermi gas. Compact objects are indeed unique cosmic laboratories for studying the properties of matter at high densities.

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Investigation of the super-dense hadronic and quark matter is required for the understanding the physics of compact stars. Dense matter physics is important in the contexts of supernova explosions, merging of compact (neutron and black-hole) stars, gamma-ray bursts, etc. (Basu et al. 2014; Fraga et al. 2014; Ayvazyan et al. 2013). While the relevant density range in each of these system is uncertain, the densities may vary from very low values characteristic to ordinary stars up to densities several times the nuclear saturation density (Drago et al. 2014).

Neutron stars (NS) are the densest observable objects in our universe. They act as a window into the physics of matter under extreme conditions of high pressures, high densities, and strong electromagnetic and gravitational fields. It is clear that properties of the nuclear matter at high densities play a crucial role in building reasonable models of neutron stars. The behavior of hadronic matter in compact stars is described using the equations of state (EOS) of matter. The EOS depend substantially on the constituents of matter and may strongly interacting hadronic matter with different constituents, densities and temperatures are described with very different EOS. Density, temperature, and ingredients of NS are widely different from crust to its core, so it is very hard to model NS with a unique set of EOS. Nevertheless, one can present a model of EOS for every region of NS using the available information (or theoretical estimates) for that region (Nakazato et al. 2008; Miyatsu et al. 2014; Miyatsu et al. 2013; Gandol et al. 2014). It may be noted that the problem is harder than the above description because of complicated phase transitions which may occur in hadronic matter. Relations between EOS of a hadronic matter and NS' properties are bidirectional: one can find some constraints on the EOS of hadronic matter using observational information from NS too (Thomas et al. 2013).

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Propagation of localized defects in spatial distribution of plasma particle densities, or its energy densities due to perturbations have widely been investigated in different kinds of plasmas. Evolution of solitary waves in Fermi-Dirac plasmas also have recently been investigated (Ata-ur Rahman 2014; Zeba et al. 2012; Mahmood et al. 2013). Such plasmas are found in some astrophysical objects like white dwarfs. It is interesting to study the small amplitude localized acoustic waves in objects with greater densities like NS. Motivated by these arguments, we study the propagation and also head on collision of these localized small amplitude waves in such media.

In the next section we present a brief review of the equations of one-dimensional non relativistic fluid dynamics which is supposed to describe the evolution of hadronic matter constituents. The equation of state will be introduced as well. Small amplitude propagation of localized waves in energy density is derived in Sect. 3 for a non-interacting plasma of hadronic matter. Head on collision of localized waves is derived in Sect. 3. Numerical simulation for the evolution of localized waves and their properties are presented in Sect. 4. Finally, the results are summarized in Sect. 5.

2 Nonrelativistic viscous fluid dynamics

Consider a highly dense spatially unlimited hadronic plasma with shear viscosity ν and bulk viscosity ζ. For non-ideal fluids, the degrees of freedom are fluid's velocity, pressure and mass density: $\overrightarrow{v}, p, \rho$. The continuity and non relativistic Navier-Stokes equations in spherical coordinates are (Currie 1993):

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \overrightarrow{v}) = 0 \tag{1}$$

and

$$\rho \frac{Dv_r}{Dt} = -\frac{\partial p}{\partial r} + \left(\zeta + \frac{4}{3}\nu\right)\left[\nabla^2 v_r - \frac{2v_r}{r^2}\right] + f_r \tag{2}$$

where the material derivative $\frac{Dv_r}{Dt}$ and Laplace operator $\nabla^2 v_r$ of a scalar field v_r are defined as

$$\frac{Dv_r}{Dt} = \frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r}, \tag{3}$$

$$\nabla^2 v_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial v_r}{\partial r}\right) \tag{4}$$

and f_r represents the radial component of body forces, $\overrightarrow{f}$ (per unit volume) acting on the fluid. The vector field $\overrightarrow{f}$ typically consists of gravity, however may include other forces, such as electromagnetic force. The gravitational force in the Newtonian limit can be written as $\overrightarrow{\nabla}\phi$, where ϕ is the scalar gravitational potential. In this notation we may include it in the pressure term as a body force. As we suppose spatially unlimited fluid, the gravity (as well as any other external force) is absent. On the other hand collisional waves are travel in hadronic matter on scales of order of few fm and on such scales gravitational energy is almost constant. Therefore, the driving force has a hydrodynamical characters.

Above equations can be solved if we have an equation of state $p = p(\rho)$. To a first approximation, Eqs. (1) and (2) in spherical coordinates result in

$$\frac{\partial \rho_B}{\partial t} + v_r \frac{\partial \rho_B}{\partial r} + \rho_B \frac{\partial v_r}{\partial r} + \frac{2\rho_B v_r}{r} = 0 \tag{5}$$

$$\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{1}{\rho}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial^2 v_r}{\partial r^2} + \frac{2}{\rho r}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial v_r}{\partial r} - \frac{2}{\rho}\left(\zeta + \frac{4}{3}\nu\right)\frac{v_r}{r^2}, \tag{6}$$

The mass density and the baryon density are related to each other through $\rho = M\rho_B$ where M is the nucleon mass.

For cold nuclear matter, the equation of state in a relativistic heavy-ion collision and dense neutron stars can be derived from the Lagrangian density of non-linear Walecka model (Menezes et al. 2007; Serot and Walecka 1986)

$$\mathcal{L} = \overline{\psi}\left[\gamma_\mu\left(i\partial^\mu - g_v V^\mu\right) - (M - g_s\phi)\right]\psi + \frac{1}{2}\left(\partial_\mu\phi\partial^\mu\phi - m_s^2\phi^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2 V_\mu V^\mu - \frac{b}{3}\phi^3 - \frac{c}{4}\phi^4 \tag{7}$$

In Eq. (7) $F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ stands for the field strength tensor. The baryon field ψ, the neutral scalar meson field ϕ and the neutral vector meson field V_μ with the respective couplings and masses are the minimal degree of freedom. The equation of state is obtained using the mean-field approximation (Serot and Walecka 1986; Furnstahl 2004; Serot 2004; Fukushima and Sasaki 2013). The meson fields are considered as classical fields:

$$V_\mu \to \langle V_\mu \rangle \equiv \delta_{\mu 0}\, V_0, \qquad \phi \to \langle \phi \rangle \equiv \phi_0 \tag{8}$$

where V_0 and ϕ_0 are constant. We can use the classical approximation, if the following conditions are met: 1) the baryonic sources are intense, 2) their coupling to the meson field are strong and 3) the infinite nuclear matter is static, homogeneous and isotropic. Based on calculations done in Serot and Walecka (1986), Furnstahl (2004), Serot (2004), Fukushima and Sasaki (2013), the following equations of motion are obtained:

$$m_v^2 V_0 = g_v \psi^\dagger \psi \tag{9}$$

$$m_s^2 \phi_0 = g_s \overline{\psi}\psi - b\phi_0^2 - c\phi_0^3 \tag{10}$$

$$\left[i\,\gamma_\mu \partial^\mu - g_v \gamma_0 V_0 - (M - g_s\phi_0)\right]\psi = 0 \tag{11}$$

The baryon density, ρ_B, is given by

$$\psi^\dagger\psi \equiv \rho_B = \frac{\gamma_s}{6\pi^2}k_F^3 \qquad (12)$$

in which k_F is the Fermi momentum. $V_0 = g_v\rho_B/m_v$ can be obtained from Eq. (9) for the vector meson. The Dirac equation which is performed through Eq. (11) couples the nucleons to the vector mesons. It gives the fermion contribution to the energy density, which is given by

$$\varepsilon = \frac{g_v^2}{2m_v^2}\rho_B^2 + \frac{m_s^2}{2g_s^2(M-M^*)} + b\frac{(M-M^*)^3}{3g_s^3} + c\frac{(M-M^*)^4}{4g_s^4} + \frac{\gamma_s}{(2\pi)^3}\int_0^{k_F} d^3k\sqrt{\vec{k}^2 + M^{*2}} \qquad (13)$$

where $\gamma_s = 4$ is the nucleon degeneracy factor. We can define the effective mass of the nucleon as $M^* = M - g_s\phi_0$. The self-consistency relation obtained from the minimization of $\varepsilon(M^*)$ with respect to M^* determines the nucleon effective mass. According to Eq. (13) it is as following

$$M^* = M - \frac{g_s^2}{m_s^2}\frac{\gamma_s}{(2\pi)^3}\int_0^{k_F} d^3k\sqrt{\vec{k}^2 + M^{*2}} + \frac{g_s^2}{m_s^2}\left[\frac{b}{g_s^3}(M-M^*)^2 + \frac{c}{g_s^4}(M-M^*)^3\right] \qquad (14)$$

The density of a NS varies typically from around the density of nuclear matter (ρ_0) and up to about $2\rho_0$. Therefore the density is taken as $\rho_0 \le \rho_B \le 2\rho_0$. Equations (12) and (14) show that M^* depends on ρ_B. M^* is a function of ρ ($M^* = M^*(\rho_B)$) and can be found by solving Eq. (14) numerically. The energy density is obtained by inserting M^* into Eq. (13) as the following power series of the baryon density (Serot and Walecka 1986; Espindola and Menezes 2002; Santos and Menezes 2004)

$$\begin{aligned}\varepsilon = &\left(0.1\frac{m_s^2}{g_s^2} + 0.04\frac{b}{g_s^3} + 0.01\frac{c}{g_s^4}\right) \\ &+ \left(4 + 2\frac{m_s^2}{g_s^2} + \frac{b}{g_s^3} + 0.43\frac{c}{g_s^4}\right)\rho_B \\ &+ \left(-3.75 + \frac{g_v^2}{2m_v^2} + 8\frac{m_s^2}{g_s^2} + 7.6\frac{b}{g_s^3} + 5.42\frac{c}{g_s^4}\right)\rho_B^2 \\ &+ \left(21.26\frac{b}{g_s^3} + 30.35\frac{c}{g_s^4}\right)\rho_B^3 + \left(63.73\frac{c}{g_s^4}\right)\rho_B^4 \\ &- 1.22\rho_B^{\frac{8}{3}} + 2.61\,\rho_B^{\frac{5}{3}} - 1.4\rho_B^{2/3}\end{aligned} \qquad (15)$$

The masses and couplings have the following values $M = 939$ MeV, $m_v = 783$ MeV, $m_s = 550$ MeV, $b = 13.47\ \mathrm{fm}^{-1}$, $g_v = 9.197$, $g_s = 8.81$, and $c = 43.127$ and have been used in numerical calculations (Serot and Walecka 1986; Furnstahl 2004; Serot 2004; Fukushima and Sasaki 2013; Espindola and Menezes 2002; Santos and Menezes 2004). The energy density of the system can be written as

$$\varepsilon = \frac{m_s^2}{g_s^2}\left(0.1 + 2\rho_B + 8\rho_B^2\right). \qquad (16)$$

3 Hadron gas

As mentioned earlier, we need a relation between the pressure p and the density ρ (or energy ϵ) to solve Eqs. (1) and (2) which are depend on the EOS of the problem under investigation. The simplest form of EOS is the one of non-interacting point particles called the hadron gas. The first law of thermodynamic at zero temperature results in

$$d\varepsilon = \mu_B d\rho_B \qquad (17)$$

Then it is

$$\mu_B = \frac{d\varepsilon}{d\rho_B} \qquad (18)$$

After substituting the above equations into the Gibbs relation at zero temperature, we have

$$d\varepsilon + dp = \rho_B d\mu_B + \mu_B d\rho_B \qquad (19)$$

This yields to

$$dp = \rho_B d\mu_B \qquad (20)$$

and in conclusion

$$dp = \rho_B d\left(\frac{\partial\varepsilon}{\partial\rho_B}\right) \qquad (21)$$

$$\frac{\partial p}{\partial r} = \rho_B\frac{\partial}{\partial r}\left(\frac{\partial\varepsilon}{\partial\rho_B}\right) \qquad (22)$$

Substituting Eqs. (21) and (22) into (6) results in

$$\begin{aligned}&\rho_B\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} \\ &= -\frac{1}{M}\rho_B\frac{\partial}{\partial r}\left(\frac{\partial\varepsilon}{\partial\rho_B}\right) + \frac{1}{M}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial^2 v_r}{\partial r^2} \\ &\quad + \frac{2}{Mr}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial v_r}{\partial r} - \frac{2}{M}\left(\zeta + \frac{4}{3}\nu\right)\frac{v_r}{r^2},\end{aligned} \qquad (23)$$

The EOS which is encompassed by $\frac{\partial\varepsilon}{\partial\rho_B}$ can be calculated from Eq. (16). The result is:

$$\frac{\partial\varepsilon}{\partial\rho_B} = 2\frac{m_s^2}{g_s^2}(1 + 8\rho_B) \qquad (24)$$

Replacing Eq. (24) into Eq. (23) we have

$$\begin{aligned}&\rho_B\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} \\ &= -\frac{16}{M}\left(\frac{m_s^2}{g_s^2}\right)\rho_B\frac{\partial\rho_B}{\partial r} + \frac{1}{M}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial^2 v_r}{\partial r^2} \\ &\quad + \frac{2}{Mr}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial v_r}{\partial r} - \frac{2}{M}\left(\zeta + \frac{4}{3}\nu\right)\frac{v_r}{r^2},\end{aligned} \qquad (25)$$

This is the Navier-Stokes equation for the hadron phase (Fogaca et al. 2013).

Now we can find the small amplitude localized solution for the above non-linear equation. The Reductive Perturbation Method (RPM) is a technique which is usually used for non-linear wave equations. In this method, the effects of non-linear, dissipative and dispersive terms are preserved in the wave equations. We are going to investigate a head on collision between two localized solutions of the equation. Conventionally, in RPM the stretched coordinates are introduced as (Davidson 1972; Eslami et al. 2012)

$$\begin{cases} \xi = \sigma(r - c_1 t) + \sigma^2 P_0(\eta, \tau) + \sigma^3 P_1(\eta, \xi, \tau) + \cdots \\ \eta = \sigma(r + c_2 t) + \sigma^2 Q_0(\xi, \tau) + \sigma^3 Q_1(\eta, \xi, \tau) + \cdots \\ \tau = \sigma^3 t \end{cases} \tag{26}$$

where ξ and η denote the trajectories of two waves traveling in the right and left directions, respectively and σ is a small expansion parameter. The variables c_1 and c_2 are unknown phase velocities which will be calculated. Initially the dimensionless variables for the baryon density, the fluid velocity and the pressure are defined as:

$$\rho = \frac{\rho_B}{\rho_0}, \qquad v_r = \frac{v_r}{c_s}, \qquad p = \frac{p}{p_0} \tag{27}$$

where ρ_0, c_s and p_0 are, respectively, the background baryon density, the speed of sound and the background pressure in the medium where perturbation propagates. According to the equation set (27), Eqs. (5) and (25) can be rewritten as

$$\frac{\partial \rho}{\partial t} + c_s v_r \frac{\partial \rho}{\partial r} + c_s \rho \frac{\partial v_r}{\partial r} + \frac{2c_s}{r} \rho v_r = 0 \tag{28}$$

$$\begin{aligned} &\rho\left(\frac{\partial v_r}{\partial t} + c_s v_r \frac{\partial v_r}{\partial r}\right) \\ &\quad = \frac{\rho_0}{Mc_s}\left(-16\frac{m_s^2}{g_s^2}\right)\rho\frac{\partial \rho}{\partial r} + \frac{1}{M\rho_0}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial^2 v_r}{\partial r^2} \\ &\qquad + \frac{2}{M\rho_0 r}\left(\zeta + \frac{4}{3}\nu\right)\frac{\partial v_r}{\partial r} \\ &\qquad - \frac{2}{M\rho_0}\left(\zeta + \frac{4}{3}\nu\right)\frac{v_r}{r^2} \end{aligned} \tag{29}$$

The dimensionless baryon density and the fluid velocity are expanded around their equilibrium values as:

$$\rho = 1 + \sigma^2 \rho_1 + \sigma^3 \rho_2 + \sigma^4 \rho_3 + \cdots \tag{30}$$

$$v = \sigma^2 v_1 + \sigma^3 v_2 + \sigma^4 v_4 + \cdots \tag{31}$$

Substituting Eqs. (30) and (31) into Eqs. (28) and (29) and neglecting the terms proportional to $\sigma^{\geq 3}$, the first non-zero order of Eqs. (28) and (29) leads to

$$c_s \frac{\partial v_1}{\partial \xi} + c_s \frac{\partial v_1}{\partial \eta} - c_1 \frac{\partial \rho_1}{\partial \xi} + c_2 \frac{\partial \rho_1}{\partial \eta} = 0 \tag{32}$$

$$-c_1 \frac{\partial v_1}{\partial \xi} + c_2 \frac{\partial v_1}{\partial \eta} + 16\frac{m_s^2}{g_s^2}\frac{\rho_0}{Mc_s}\left(\frac{\partial \rho_1}{\partial \xi} + \frac{\partial \rho_1}{\partial \eta}\right) = 0 \tag{33}$$

The variables ρ_1 and v_1 can be grouped into two different terms, one depending on ξ and τ, and the other depending on η and τ as $\rho_1 = \rho_1^1(\xi, \tau) + \rho_1^2(\eta, \tau)$ and $v_1 = v_1^1(\xi, \tau) + v_1^2(\eta, \tau)$. If we apply these assumptions to Eqs. (32) and (33) then the following equations are obtained

$$c_s \frac{\partial v_1^1}{\partial \xi} + c_s \frac{\partial v_1^2}{\partial \eta} - c_1 \frac{\partial \rho_1^1}{\partial \xi} + c_2 \frac{\partial \rho_1^2}{\partial \eta} = 0 \tag{34}$$

$$-c_1 \frac{\partial v_1^1}{\partial \xi} + c_2 \frac{\partial v_1^2}{\partial \eta} + 16\frac{m_s^2}{g_s^2}\frac{\rho_0}{Mc_s}\left(\frac{\partial \rho_1^1}{\partial \xi} + \frac{\partial \rho_1^2}{\partial \eta}\right) = 0 \tag{35}$$

and the result is

$$v_1 = \frac{1}{c_s}\left(c_1 \rho_1^1(\xi, \tau) - c_2 \rho_1^2(\eta, \tau)\right) \tag{36}$$

The phase velocities are obtained as

$$c_1^2 = c_2^2 = \frac{16 m_s^2 \rho_0}{M g_s^2} \tag{37}$$

The second order equations (28) and (29) lead to the same result when the index "1" is replaced by "2". By inserting Eqs. (36) and (37) into Eqs. (28) and (29), and collecting the terms of the order σ^3 and considering small viscosities i.e. $\zeta = \sigma\widetilde{\zeta}$ and $\nu = \sigma\widetilde{\nu}$ we have

$$\begin{aligned} &\frac{\partial \rho_1^1}{\partial \tau} + \frac{\partial \rho_1^2}{\partial \tau} - c_1 \frac{\partial \rho_3}{\partial \xi} + c_2 \frac{\partial \rho_3}{\partial \eta} - 2c_2 Q_{0\xi} \frac{\partial \rho_1^2}{\partial \eta} \\ &\quad + 2c_1 P_{0\eta} \frac{\partial \rho_1^1}{\partial \xi} + c_s \frac{\partial v_3}{\partial \xi} + c_s \frac{\partial v_3}{\partial \eta} + 2c_1 \rho_1^1 \frac{\partial \rho_1^1}{\partial \xi} \\ &\quad - 2c_1 \rho_1^2 \frac{\partial \rho_1^2}{\partial \eta} + \frac{2}{\tau}\rho_1^1 - \frac{2}{\tau}\rho_1^2 = 0 \end{aligned} \tag{38}$$

and

$$\begin{aligned} &-c_1 \frac{\partial v_3}{\partial \xi} + c_2 \frac{\partial v_3}{\partial \eta} + \frac{2c_1^2}{c_s} P_{0\eta} \frac{\partial \rho_1^1}{\partial \xi} + \frac{2c_2^2}{c_s} Q_{0\xi} \frac{\partial \rho_1^2}{\partial \eta} + \frac{c_1}{c_s}\frac{\partial \rho_1^1}{\partial \tau} \\ &\quad - \frac{c_2}{c_s}\frac{\partial \rho_1^2}{\partial \tau} - \frac{1}{M\rho_0}\left(\widetilde{\zeta} + \frac{4}{3}\widetilde{\nu}\right)\left[\frac{c_1}{c_s}\frac{\partial^2 \rho_1^1}{\partial \xi^2} - \frac{c_2}{c_s}\frac{\partial^2 \rho_1^2}{\partial \eta^2}\right] \\ &\quad + \frac{c_1^2}{c_s}\frac{\partial \rho_3}{\partial \xi} + \frac{c_2^2}{c_s}\frac{\partial \rho_3}{\partial \eta} + \frac{c_1^2}{c_s}\left(\rho_1^1 \frac{\partial \rho_1^1}{\partial \xi} + \rho_1^2 \frac{\partial \rho_1^2}{\partial \eta}\right) \\ &\quad - \frac{c_1^2}{c_s}\left(\rho_1^1 \frac{\partial \rho_1^2}{\partial \eta} + \rho_1^2 \frac{\partial \rho_1^1}{\partial \xi}\right) = 0 \end{aligned} \tag{39}$$

By differentiating (38) and (39) with respect to ξ and η, we will find four different equations. Combining these four new equations, using Eq. (37) we obtain

$$\begin{aligned} &\frac{\partial}{\partial \xi}\left(2\frac{\partial \rho_1^1}{\partial \tau} + 3c_1 \rho_1^1 \frac{\partial \rho_1^1}{\partial \xi} - \frac{1}{M\rho_0}\left(\widetilde{\zeta} + \frac{4}{3}\widetilde{\nu}\right)\frac{\partial^2 \rho_1^1}{\partial \xi^2} + \frac{2}{\tau}\rho_1^1\right) \\ &\quad - \frac{\partial}{\partial \eta}\left(2\frac{\partial \rho_1^2}{\partial \tau} - 3c_1 \rho_1^2 \frac{\partial \rho_1^2}{\partial \eta} - \frac{1}{M\rho_0}\left(\widetilde{\zeta} + \frac{4}{3}\widetilde{\nu}\right)\frac{\partial^2 \rho_1^2}{\partial \eta^2} \right. \\ &\quad \left. - \frac{2}{\tau}\rho_1^1 + 2c_1 \rho_1^2 \frac{\partial \rho_1^1}{\partial \xi}\right) + c_1 \frac{\partial}{\partial \xi}\left(\left(4P_{0\eta} - \rho_1^2\right)\frac{\partial \rho_1^1}{\partial \xi}\right) \\ &\quad + c_1 \frac{\partial}{\partial \eta}\left(\left(4Q_{0\xi} - \rho_1^1\right)\frac{\partial \rho_1^2}{\partial \eta}\right) + 4c_1 \frac{\partial^2 \rho_3}{\partial \xi \partial \eta} = 0 \end{aligned} \tag{40}$$

Finally by considering the dependence of ρ_1^1 and ρ_1^2 on variables τ, η and ξ, we find

$$\frac{\partial\rho_1^1}{\partial\tau}+\frac{3c_1}{2}\rho_1^1\frac{\partial\rho_1^1}{\partial\xi}-\frac{1}{2M\rho_0}\left(\tilde{\zeta}+\frac{4}{3}\tilde{\nu}\right)\frac{\partial^2\rho_1^1}{\partial\xi^2}+\frac{1}{\tau}\rho_1^1=0 \tag{41}$$

$$\frac{\partial\rho_1^2}{\partial\tau}-\frac{3c_1}{2}\rho_1^2\frac{\partial\rho_1^2}{\partial\eta}-\frac{1}{2M\rho_0}\left(\tilde{\zeta}+\frac{4}{3}\tilde{\nu}\right)\frac{\partial^2\rho_1^2}{\partial\eta^2}-\frac{1}{\tau}\rho_1^1+c_1\rho_1^2\frac{\partial\rho_1^1}{\partial\xi}=0 \tag{42}$$

$$P_{0\eta}=\frac{1}{4}\rho_1^2 \tag{43}$$

$$Q_{0\xi}=\frac{1}{4}\rho_1^1 \tag{44}$$

Equations (41) and (42) are the viscous spherical Burgers equations in (ξ,τ) and (η,τ) spaces respectively. $P_{0\eta}$ and $Q_{0\xi}$ are the phase shifts of the localized waves after their head on collision. Using Eq. (26) the Burgers equations in (x,t) space for two shock waves will result, which move toward each other:

$$\frac{\partial\hat{\rho}_1^1}{\partial t}+c_1\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{3c_1}{2}\hat{\rho}_1^1\frac{\partial\hat{\rho}_1^1}{\partial r}-\frac{1}{2M\rho_0}\left(\zeta+\frac{4}{3}\nu\right)\frac{\partial^2\hat{\rho}_1^1}{\partial r^2}+\frac{\hat{\rho}_1^1}{t}+\frac{1}{4}\hat{\rho}_1^2\left[\frac{\partial\hat{\rho}_1^1}{\partial t}-c_1\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{1}{2M\rho_0}\left(\zeta+\frac{4}{3}\nu\right)\frac{\partial^2\hat{\rho}_1^1}{\partial r^2}+\frac{\hat{\rho}_1^1}{t}\right]=0 \tag{45}$$

This is the spherical Burgers equation for $\hat{\rho}_1^1\equiv\varepsilon^2\rho_1^1$, which is a small perturbation in the baryon density, moving to the right with spherical symmetry.

Equation (45) reduces to the following form, called breaking wave equation, in a medium in which $\zeta=\nu=0$.

$$\frac{\partial\hat{\rho}_1^1}{\partial t}+c_1\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{3c_1}{2}\hat{\rho}_1^1\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{\hat{\rho}_1^1}{t}+\frac{1}{4}\hat{\rho}_1^2\left[\frac{\partial\hat{\rho}_1^1}{\partial t}-c_1\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{\hat{\rho}_1^1}{t}\right]=0 \tag{46}$$

Similar equation are derived for $\hat{\rho}_1^2\equiv\varepsilon^2\rho_1^2$, which is a small perturbation in the baryon density moving to the left

$$\frac{\partial\hat{\rho}_1^2}{\partial t}-c_1\frac{\partial\hat{\rho}_1^2}{\partial r}-\frac{3c_1}{2}\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^2}{\partial r}-\frac{1}{2M\rho_0}\left(\zeta+\frac{4}{3}\nu\right)\frac{\partial^2\hat{\rho}_1^2}{\partial r^2}-\frac{\hat{\rho}_1^2}{t}+\frac{1}{4}\hat{\rho}_1^1\left[\frac{\partial\hat{\rho}_1^2}{\partial t}+c_1\frac{\partial\hat{\rho}_1^2}{\partial r}+\frac{1}{2M\rho_0}\left(\zeta+\frac{4}{3}\nu\right)\frac{\partial^2\hat{\rho}_1^2}{\partial r^2}-\frac{\hat{\rho}_1^2}{t}\right]+c_1\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{c_1}{4}\left[\hat{\rho}_1^1-\hat{\rho}_1^2\right]\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^1}{\partial r}=0 \tag{47}$$

and in the case of $\zeta=\nu=0$, it reduces to the following equation:

$$\frac{\partial\hat{\rho}_1^2}{\partial t}-c_1\frac{\partial\hat{\rho}_1^2}{\partial r}-\frac{3c_1}{2}\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^2}{\partial r}-\frac{\hat{\rho}_1^2}{t}+\frac{1}{4}\hat{\rho}_1^1\left[\frac{\partial\hat{\rho}_1^2}{\partial t}+c_1\frac{\partial\hat{\rho}_1^2}{\partial r}-\frac{\hat{\rho}_1^2}{t}\right]+c_1\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^1}{\partial r}+\frac{c_1}{4}\left[\hat{\rho}_1^1-\hat{\rho}_1^2\right]\hat{\rho}_1^2\frac{\partial\hat{\rho}_1^1}{\partial r}=0 \tag{48}$$

4 Discussion of numerical results

The Burgers equations (45) and (47) can be rewritten in the general form

$$\frac{\partial\hat{\rho}}{\partial t}+c\frac{\partial\hat{\rho}}{\partial r}+\alpha\hat{\rho}\frac{\partial\hat{\rho}}{\partial r}=\mu\frac{\partial^2\hat{\rho}}{\partial r^2} \tag{49}$$

where α and μ are the respective non-linear and dissipative coefficient. The non-linear coefficient for two moving waves are $\alpha=\pm\frac{3c_1}{2}$. The dissipative coefficient μ is related to the viscosity. The derived Eqs. (47) and (48) for moving waves contain additional terms in comparison with the standard Burgers equation (49) which comes from non-planar geometry of spherical symmetry. The more important term is $\frac{\hat{\rho}_1^1}{t}$ ($\frac{\hat{\rho}_1^2}{t}$). This term is singular at $t=0$ and therefore numerical calculations should be started from negative times (Javidan 2013). For $|t|\gg 1$ this term is sufficiently small so that Eqs. (47) and (48) are reduced to (49). For $|t|\to 0$, the term $\frac{\hat{\rho}_1^1}{t}$ becomes very large. At sufficiently large values of time, we can take localized solutions of the Burgers equation as initial value for numerical simulation of Eqs. (47) and (48). Unfortunately, the Burgers equation does not have well-known exact solution. We have used localized initial condition $\rho(r,t=0)=A\,\mathrm{sech}(\frac{r}{\Delta})$ where A is the initial amplitude and Δ is its width (Javidan 2013). It is the solution of the Korteweg-de Vries equation, (KdV), which is very similar to the Burgers equation. Two solitary wave profiles were propagated with the speed $c_1=c_2$ in opposite directions from different initial positions as initial condition for solving Eqs. (47) and (48) numerically.

The phase shifts in the waves, after the head on collision are negative which one can find from Eqs. (43) and (44). Also wave with smaller amplitudes finds greater absolute value of the phase shift.

Figure 1 shows the evolution of the propagating wave in an inviscid hadronic matter ($\zeta=\nu=0$). The initial positions of solitary waves are $r_{01}=10$ and $r_{02}=20$. The amplitude of the left (right) profile is $A=0.2$ ($A=0.4$). The shapes of the waves are distorted in time by oscillations until a shock profile is formed. At the final steps of simulation with $|t|\approx 0$ the wave amplitudes become very large as the term $\frac{\hat{\rho}_1^1}{t}$ is dominant.

Figure 2 demonstrates head on collision in-between two localized waves in a viscous medium. This figure shows that the viscosity is able to control the creation of shock waves.

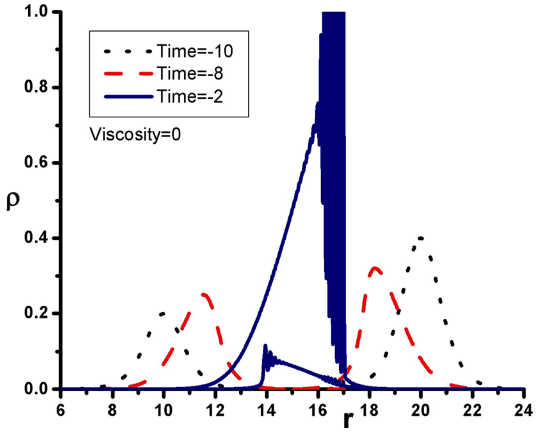

Fig. 1 Wave profiles before and after collision in non viscous medium

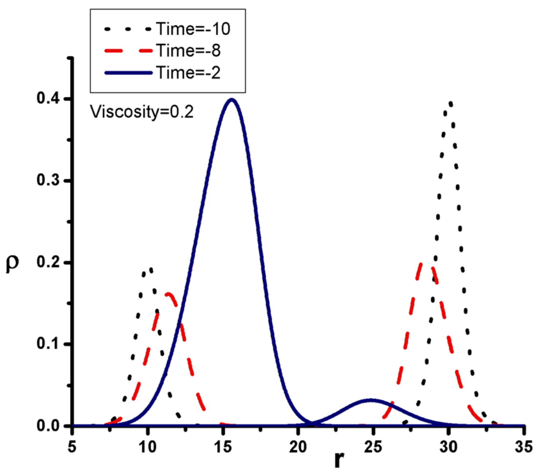

Fig. 2 Wave profiles before and after collision in viscous media

Indeed the term with second order of derivatives in Eqs. (41) and (42) reduces the non-linearity effects in a way that the shock profiles are created very late. But the amplitude of the wave is damped due to viscosity effects.

5 Conclusions and remarks

Astrophysical compact objects like neutron stars contain hadronic matter. We have studied the propagation of small amplitude localized waves in a viscous hadronic matter present in compact objects. The non-relativistic evolution equations of breaking waves during the head on collision in this medium are derived using the continuity and non relativistic Navier-Stokes equations. The hadronic matter under investigation is described by the simplest equation of state as hadronic gas. It is shown that the propagated waves in non-viscous medium can be described by inviscid Burgers equation which describes the creation of breaking waves. Viscosity plays an important role in the propagation of localized waves. It is shown that a propagated wave in viscous media can travel longer distances without getting shock profiles. However it will lose its amplitude.

There are many questions and problems which have to be investigated. As mentioned before, the equation of state of hadronic matter changes with its density and temperature. Understanding the effects of changing theses parameters needs serious studies. We have assumed a Fermi-Dirac distribution function for the matter under investigation. It is acceptable only for zero temperature condition. The effects of different distribution functions for the matter also need more attention. The most difficult problem is the modeling of different phase transitions which can occur in such complicated media.

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