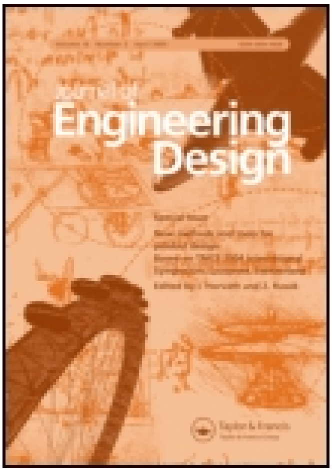

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Propagation of part tolerances through the assembly process affects the quality and overall performance of the products. Therefore, it is crucial to have a comprehensive model in order to analyse the relationship between part tolerances and final assembly errors. Assembly processes are often complex and nonlinear in nature. In sheet metal assemblies, the most important factor that makes the process nonlinear is contact interaction between mating parts during the assembly process. Another important feature in sheet metal variation analysis is the effect of geometric covariance. In sheet metal components, covariance always occurs since the surface continuity conditions force the deformation of the neighbouring points to be correlated. This paper aims to develop a new methodology for variation analysis of compliant sheet metal assemblies focusing on nonlinear contact analysis and including the effect of geometric covariance. The proposed methodology integrates a nonlinear finite element analysis with an improved sensitivity-free probability analysis in order to predict the final assembly variation. The efficiency of the developed approach is evaluated by an experimental case study as well as Monte Carlo simulation.

Keywords: nonlinear tolerance analysis; finite element analysis; geometric covariance; sensitivity-free probability analysis; principal component analysis

1. Introduction

The propagation of dimensional and geometric tolerances through the assembly process affects the quality and overall performance of the products. Errors arising from these tolerances may lead to several drawbacks such as product failure, additional production costs, warranty costs and end-user dissatisfaction. In this regard, it is crucial to have a comprehensive model in order to analyse the relationship between part tolerances and final assembly errors. Hence, the tolerance analysis procedures have been widely taken into consideration by designers.

One of the most common issues in the field of tolerance analysis is the variation analysis of compliant assemblies which is extensively focused on sheet metal systems (Shiu, Ceglarek, and Shi 1997). There are also a few other types of mechanical systems like kinematic linkages to which the flexible tolerance analysis was applied (Imani and Pour 2009). Sheet metal systems which are the main concept of this research are widely used in industry. Automotive bodies and plane fuselage are the most common examples constructed from sheet metals. Traditional methods of tolerance analysis are based on the assumption of rigid body analysis (Mantripragada

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and Whitney 1999; Ding, Ceglarek, and Shi 2000; Söderberg and Lindkvist 2002) and unable to estimate the effect of part flexibility in the process. The newer methodologies have been gradually extended to the compliant assemblies (Liu and Hu 1997; Camelio, Hu, and Ceglarek 2003; Camelio, Hu, and Ceglarek 2004; Camelio, Hu, and Marin 2004; Camelio and Yim 2006). Pioneers in the area of compliant tolerances analysis are Liu and Hu (1997) who introduced the method of influence coefficient (MIC) in order to express a linear relationship between incoming part deviation (input variables) and the final assembly deformation (key characteristics (KCs)) through a sensitivity matrix. The analysis is based on the linear force–displacement relationship in assembly processes which are, however, nonlinear in reality due to contact interactions between mating parts and tools. Various factors such as nonlinear material behaviour, large deformations, and contact interaction can introduce nonlinearities in the assembly process. The recent studies focus on nonlinear analysis considering the contact interaction between mating parts as the most important factor that makes the assembly process nonlinear (Cai et al. 2006; Dahlstrom and Lindkvist 2007; Liao and Wang 2007; Xie et al. 2007). The first research in this area was conducted by Cai et al. (2006). Recently, Xie et al. (2007) developed a new methodology which analyses the entire process as a set of sequential operations using nonlinear finite element method (FEM).

In a tolerance analysis procedure, a variation limit is defined for each input variable. The limit is equal to the variable's tolerance zone so that the variable obtains a random value within this range. In mass production, a series of random values are available for each variable which are scattered within its corresponding tolerance zone. Random nature of the variables suggests that the statistics should be employed to explain a relationship between input tolerances and variation of the assembly's KCs. The simplest method for this purpose is using Monte Carlo simulation (MCS) which is often too expensive to achieve reliable results. For example, for a sigma level of 3 and reliability of 90% MCS needs around 100,000 samples (Law 1996). Therefore, MCS in many cases is replaced by other methods such as root sum squares (RSS). By defining a sensitivity matrix, the RSS method determines the mean value and variance of the assembly's KC in terms of the part tolerances through a sensitivity probability analysis (SPA) (Chase and Parkinson 1991). However, in nonlinear analysis, there is no linear relationship between input variables and the assembly's KC. Therefore, no sensitivity matrix can be defined to apply sensitivity analysis. In this regard, the sensitivity-free probability analysis (SFPA) (Rahman and Xu 2004; Youn, Xi, and Wang 2008) must be employed to calculate the specifications of KC's distribution by means of statistical moments. On the other hand, this analysis entails computing multi-dimensional integrals and the problem remains unsolved since no analytical solution does generally exist for such integrals. Furthermore, due to the fact that the dimension of the integrals is equal to the number of input variables, direct numerical integration is not economically reasonable. To resolve the problem, Rahman and Xu (2004) proposed the dimension reduction (DR) method which simplifies a multi-dimensional integral into multiple one-dimensional integrals using an additive decomposition. Later on, Youn, Xi, and Wang (2008) developed an enhancement of the DR method which is referred to as the enhanced dimension reduction (EDR). Their approach incorporates three main modules of one-dimensional response approximation, eigenvector sampling and stabilised Pearson system.

One of the important points in any statistical analysis is the correlation between variables. Assuming independence of the input variables, especially when they are highly correlated, leads to large errors in the analysis. In a sheet metal part, surface continuity condition maintains a level of correlation between deformations of the neighbouring points. This correlation, which is also referred to as the geometric covariance, was first investigated in sheet metal variation analysis by Merkley (1998). Camelio, Hu, and Marin (2004), by means of the principal component analysis (PCA) (Johnson and Wichern 2007), theoretically identified the dominant variation patterns or deformation mode shapes of an individual sheet metal part from the geometric covariance matrix.

Table 1. Comparison of previous publications and the proposed methods in the area of sheet metal variation analysis.

Published by	Sources of variation	Finite element analysis	Statistical analysis
Liu and Hu (1997)	Independent	• MIC • Non-contact model • Linear FE analysis	• SPA • Covariance effect is *not* included
Camelio, Hu, and Marin (2004)	Correlated	• MIC • Non-contact model • Linear FE analysis	• SPA • Covariance effect is included • PCA is implemented to reduce computational efforts
Liao and Wang (2007)	Independent	• Contact model • Nonlinear FE analysis	• Statistical analysis is *not* applied
Xie, Wells, Camelio, and Youn (2007)	Independent	• Contact model • Nonlinear FE analysis	• SFPA is applied by means of EDR method • Covariance effect is *not* included
Proposed methodology of this paper	Correlated	• Contact model • Nonlinear FE analysis	• Improved SFPA which incorporates PCA into EDR method • Covariance effect is included • PCA is implemented to reduce computational efforts of EDR method

Given these dominant patterns and by disregarding the other patterns that have minor contributions to the part variation, the computational effort can be significantly reduced. However, this method has been only applied to linear analysis and a new methodology has yet to develop in order to account for geometric covariance in nonlinear analysis.

The objective of the current research is to present a comprehensive methodology for variation analysis of compliant sheet metal assemblies which not only accounts for nonlinearities arising from contact interactions but also includes the effect of surface continuity or geometric covariance in components. In comparison with the previous research, the main contribution of this work is focusing on correlated variables which have not been earlier dealt with in nonlinear variation analysis of sheet metal assemblies. Table 1 shows the previous publications in this area and compares how the sources of variation were selected and finite element and statistical analyses were applied. The last row of the table also presents the proposed methodology of this article. In the developed approach, a nonlinear finite element analysis is integrated with an improved version of SFPA, which incorporates PCA into EDR method, in order to assess the final assembly variation. The new approach reduces the computational effort of the analysis, so that time efficiency of the procedure promisingly increases.

Henceforth, the content of this paper is organised as follows. Section 2 describes the nonlinear finite element modelling of the assembly process followed by Section 3 which presents the improved SFPA in order to include the effect of geometric covariance in variation analysis of sheet metal assemblies. In Section 4, the accuracy and efficiency of the developed methodology is investigated by an experimental case study and Section 5 concludes the article.

2. Nonlinear finite element analysis of assembly process

Generally, the assembly process of sheet metal parts can be divided into four main steps: (1) locating the parts in desirable assembly position using fixtures; (2) moving the welding guns towards each other to close the gap; (3) joining the parts together by resistance spot-welding process; and (4) releasing the guns so that the assembly springs back. The process is schematically demonstrated in Figure 1.

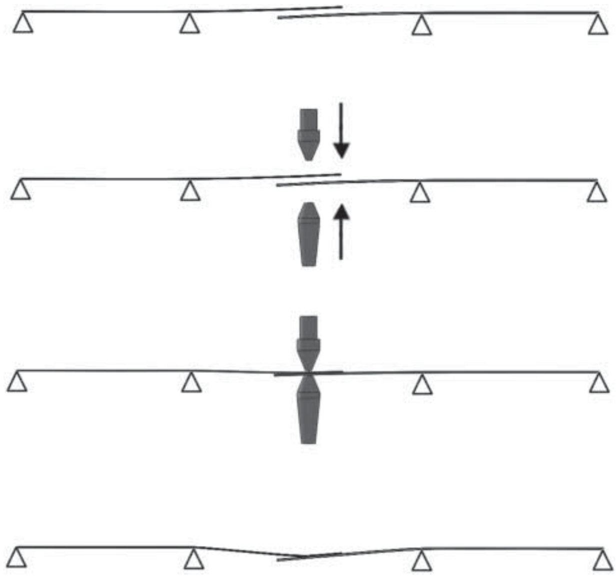

Figure 1. Assembly process of compliant sheet metal parts.

As stated earlier, the main factor that makes the assembly process nonlinear is the contact interaction. During the assembly process, flexible components deform linearly under the Hooke's law ($F = K\Delta$). This equation governs the parts' deformation under clamping or spring-back as long as they are not in contact. Once the mating parts come in contact, the linear force–displacement equation would not be valid anymore and it is required to apply the contact analysis in order to have a more accurate estimation of the assembly variation. Contact analysis, intrinsically, is a nonlinear analysis based on solving the energy equation (Hills 1992; Mijar and Arora 2000; Yue et al. 2007). Figure 2 shows a simple example in which the gap between two parts should be closed by welding guns. The difference between the non-contact and contact models is demonstrated graphically in Figure 3. As shown in the figure, the main limitation of non-contact model (linear assembly modelling) is penetration of the mating parts which is physically impossible while the contact model yields more realistic estimation. It is worth mentioning that normally all deflections and deformations remain in the elastic zone of the components' stress–strain diagram so that there is no nonlinearity due to plastic deformation.

The nonlinear FEM, which is implemented in this paper to express the relationship between incoming part deviation and final assembly variation, focuses on two general objectives: first, to model the assembly process as close to the real process as possible; and second, to consider the contact interactions of mating parts and also the contact between parts and welding guns. In the current research, the ANSYS environment is integrated with LS-DYNA solver in order to simulate the assembly process. LS-DYNA provides fast explicit solutions for any nonlinear problems concerning large deformation, quasi-static analysis, and complex contact/impact interaction. Therefore, it will be of interest to incorporate this solver into ANSYS environment in order to perform sequential explicit–implicit analysis which is one of the best ways of simulating the assembly process of sheet metal components. The idea of performing this type of analysis is taken from metal forming processes. In implicit solutions, ANSYS solves the equations using the Newmark approximation which requires inversion of equivalent stiffness matrix. It is unconditionally stable for linear problems, but convergence is not guaranteed for nonlinear problems. For the explicit method, a central difference time integration method is used. No inversion of the stiffness matrix is required and the equations become uncoupled so that they can be

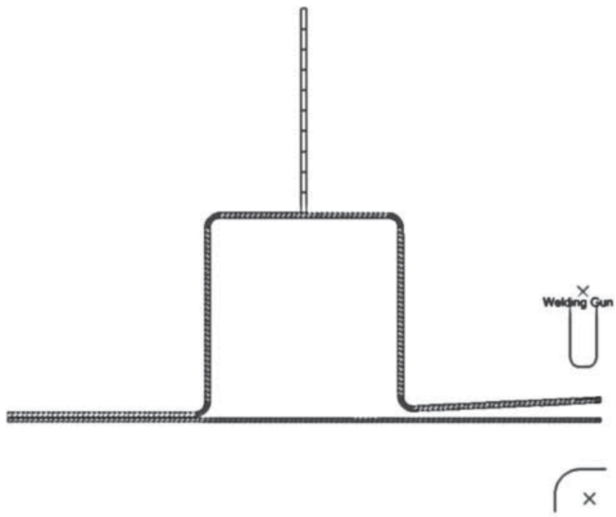

Figure 2. Hood bracket assembly model (Xie et al. 2007).

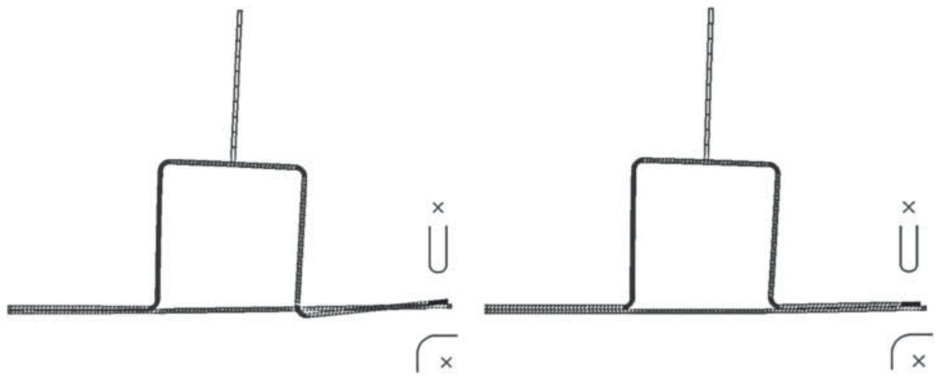

Figure 3. Left: non-contact model, physically impossible; right: contact model, close to reality (Xie et al. 2007).

solved directly (explicitly). Additionally, no convergence checks are needed since the equations are uncoupled (ANSYS 2009a). In this research, the step of moving welding guns and deforming the parts in order to close the gap is solved explicitly using LS-DYNA while the release of elastic energy and assembly spring-back is modelled implicitly by ANSYS modules. The steps of simulating sheet metal assembly process in ANSYS LS-DYNA environment through which input part deviations are related to final assembly's KC are listed below and also presented in the flow chart of Figure 4.

(1) Importing the CAD model or creating the model internally using ANSYS Pre-processor.
(2) Defining material properties (E, ν) and real constants (e.g. thickness for plates).
(3) Generating explicit elements (SHELL163 for plates and SOLID164 for welding guns). Although welding guns are assumed to be rigid bodies, they should be meshed as well since they will be moving during the analysis.
(4) Defining fixtures.
(5) Defining contact interactions between mating surfaces (e.g. plates and guns) by means of EDCGEN function. The automatic surface-to-surface contact, which is the most general

type of contact and commonly used for parts with arbitrary shapes and relatively large contact areas, is suggested. There are no additional parameters for this type of contact.

(6) Moving the welding guns by means of EDLOAD function (motions should be defined in terms of time arrays).
(7) Solving the model using LS-DYNA solver (explicit solution).
(8) Converting explicit elements into implicit ones using ETCHG function (explicit shell and solid elements will automatically convert into implicit SHELL181 and SOLID185, respectively).
(9) Updating geometry and importing residual stress from the explicit solution (UPGEOM function will create a new geometry for implicit analysis in accordance with deflections calculated in Step vii while RIMPORT function will transfer the residual stress to the new geometry).
(10) Defining spot-weld elements at the welding points using SWGEN function.
(11) Removing the welding guns and solving for assembly spring-back using ANSYS implicit solver.

Commonly, a pure implicit approach to contact analysis procedure has some crucial phases such as properly defining the contact elements (Target and Contactor) and their respective normal vectors, setting key options and real constants, and so on. (Liao and Wang 2007; ANSYS 2009b). Inappropriate definition of these parameters may cause limitation or even lack of convergence in the solution. However, applying an explicit–implicit analysis in ANSYS LS-DYNA environment resolves many of these restrictions and not only avoids convergence problems but also achieves reliable results. This type of analysis would be much easier than the conventional implicit contact analyses because the user can easily define contact interactions by only choosing the mating parts and no additional effort or knowledge is needed. Therefore, practitioners may benefit more when they try to apply the presented methodology to their real-world problems.

3. Statistical variation analysis of sheet metal assemblies

The objective of the current research is to develop a methodology to include the effect of geometric covariance of components in the nonlinear variation assessment of flexible sheet metal assemblies. It is interesting to note that the effect of geometric covariance has been earlier included only in linear variation analysis of sheet metal components (Camelio, Hu, and Marin 2004). In the developed methodology of this article, the SFPA is improved by means of PCA in order to account for geometric covariance in estimation of final assembly variation in a nonlinear analysis. Before describing the proposed approach, it may be helpful to have a short review of the existing probability analyses.

3.1. *Probability analysis review*

3.1.1. *Conventional sensitivity analysis*

In the conventional linear tolerance analysis of sheet metal assemblies, the sensitivity matrix **S** relates the assembly's response u and the input variables $\mathbf{V} = [v_1, v_2, \ldots, v_N]^t$ using the direct expression of $u = \mathbf{SV}$ (Liu and Hu 1997). Therefore, the mean and variance of the assembly's KC can be determined as follows where μ_i and σ_i are the mean and standard deviation of the ith

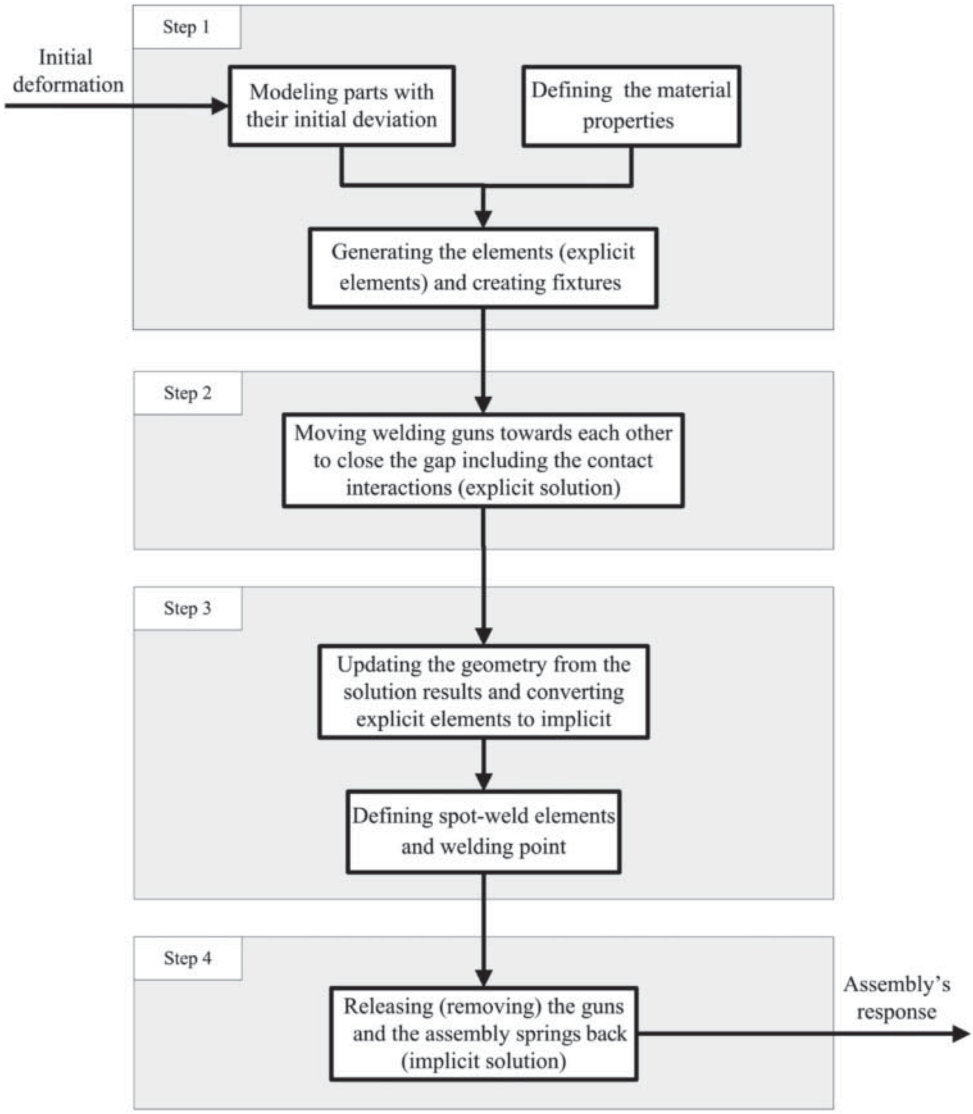

Figure 4. Four-step modelling of the assembly process in ANSYS LS-DYNA environment.

input variable, respectively (Chase and Parkinson 1991):

$$\mu_u = \sum_{i=1}^{N} S_i \mu_i, \tag{1}$$

$$\sigma_u^2 = \sum_{i=1}^{N} {S_i}^2 {\sigma_i}^2. \tag{2}$$

It is also clear that as a result of a linear relationship, if input variables have normal distributions, the KC's distribution will be normal as well.

3.1.2. *Sensitivity-free probability analysis*

As opposed to the above-mentioned sensitivity analysis, we can hardly find a direct relationship between input and output variables in nonlinear systems. In other words, there is no sensitivity matrix to calculate μ_u and σ_u by means of Equations (1) and (2). Moreover, due to system's nonlinearity, it is not guaranteed that the output distribution will be normal even if all input variables have normal distributions. Generally, if $u = u(v_1, v_2, \dots, v_N)$ is an arbitrary response function of input variables, its statistical distribution or the so-called probability density function (PDF) will be determined by statistical moments. In multivariate statistics, the rth moment of u is calculated as (Zhou and Nowak 1988):

$$m_r = E[u^r(v_1, v_2, \dots, v_N)]$$
$$= \underbrace{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}}_{N} u^r(v_1, v_2, \dots, v_N)\, f_{\mathbf{V}}(v_1, v_2, \dots, v_N)\mathrm{d}v_1\, \mathrm{d}v_2 \dots \mathrm{d}v_{\mathrm{N}}, \tag{3}$$

where E is the expectation operator and $f_{\mathbf{V}}(v_1, v_2, \dots, v_N)$ is the joint PDF of the input variables. Once moments are calculated, the statistical specifications of the response function will be obtained from the first four moments as follows (Papoulis and Pillai 2002):

$$\text{mean} = \mu = m_1, \tag{4}$$

$$\text{variance} = \sigma^2 = m_2 - \mu^2, \tag{5}$$

$$\text{skewness} = \gamma_1 = \frac{m_3 - \mu^3 - 3\mu\sigma^2}{\sigma^3}, \tag{6}$$

$$\text{kurtosis} = \beta_2 = \frac{m_4 - \mu^4 - 6\mu^2\sigma^2 - 4\mu\sigma^3\gamma_1}{\sigma^4}. \tag{7}$$

Finally, the Pearson system can be used to construct the PDF of the response function from these four specifications (Johnson, Kotz, and Balakrishnan 1995).

The above procedure entails computing multi-dimensional integral of Equation (3) which generally has no analytical solution. The direct numerical integration is also unreasonable when the number of input variables increases. In order to overcome this difficulty, Rahman and Xu (2004) proposed the DR method which converts a multi-dimensional integral into a summation of multiple one-dimensional integrals. Using an additive decomposition and according to definition of the expectation operator, DR method calculates the statistical moments by the following formula:

$$m_r \cong E\left[\sum_{j=1}^{N} u^r(\mu_1, \dots, v_j, \dots, \mu_N)\right] - (N-1)\, u^r(\mu_1, \mu_2, \dots, \mu_N). \tag{8}$$

According to the definition of the expectation operator and by designating $f(v_j)$ as the marginal PDF of variable v_j, one can write:

$$E\left[\sum_{j=1}^{N} u^r(\mu_1, \dots, v_j, \dots, \mu_N)\right] = \sum_{j=1}^{N}\int_{-\infty}^{\infty} u^r(\mu_1, \dots, v_j, \dots, \mu_N)\, f(v_j)\, \mathrm{d}v_j. \tag{9}$$

Youn, Xi, and Wang (2008) developed an enhancement of the DR method which is referred to as the EDR method. Their approach incorporates three main modules of one-dimensional response approximation, eigenvector sampling, and stabilised Pearson system. The first module

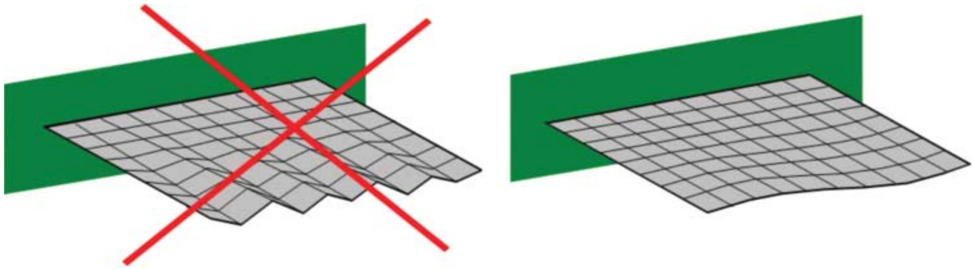

Figure 5. Surface continuity condition in a sheet metal component.

enhances computing one-dimensional integrals of Equation (8) in which instead of using the original response function, its one-dimensional approximations can be treated as the integrand. It is a practical process in stochastic problems when the original response function does not exist. The accuracy of the response approximation increases as the number of data points. However, dealing with a large number of data points, it is not economically reasonable since, particularly in the current research, the value of the response function at each data point should be obtained by FEM. Eigenvector sampling scheme suggests the use of $2N + 1$ or $4N + 1$ sampling points depending on the system nonlinearity. These sample points should be selected along the principal directions (eigenvectors) of covariance matrix of input variables. The application of the stabilised Pearson system is in special cases where the original Pearson system fails to construct the PDF of the response function. The singularity problem of the Pearson system may be resolved by fixing the first three statistical moments and slightly increasing or decreasing the kurtosis of the response function until the PDF is successfully constructed. More details on EDR method can be found in (Youn, Xi, and Wang 2008).

3.2. *Improved SFPA*

In sheet metal components, the geometric covariance always arises from surface continuity condition that makes the deformations of the neighbouring points correlated (Figure 5). However, Equation (8) cannot be directly used for correlated variables, since the correlation between variables is not included in the additive decomposition. In order to improve the above-mentioned SFPA to account for geometric covariance of sheet metal components, this research suggests the use of PCA incorporated into EDR method. PCA has two main advantages: (1) to convert correlated variables into uncorrelated ones; and (2) to identify the dominant variation patterns of sheet metal parts which have greater contribution to the part variation. Therefore, PCA not only improves the SFPA method to deal with a set of correlated variables but also reduces the computational efforts of the EDR method by neglecting variables with minor contributions.

3.2.1. *Principal component analysis*

In multivariate statistics, PCA transforms a set of correlated variables $\mathbf{V} = [v_1, v_2, \ldots, v_N]^{\mathrm{t}}$ into an independent set of $V' = [v'_1, v'_2, \ldots v'_N]^t$ by means of the following equation (Johnson and Wichern 2007):

$$\mathbf{V} = \mathbf{T}\,\mathbf{V}'. \tag{10}$$

The transformation matrix $\mathbf{T} = [\mathbf{t}_1\ \mathbf{t}_2\ \ldots\ \mathbf{t}_N]$ is constructed from the eigenvectors of the input covariance matrix. In other words, the vector $\mathbf{t}_i$ represents ith eigenvector of $\mathbf{\Sigma}_V$. According to

Equation (10) and from the statistical definition of covariance, one can write:

$$\boldsymbol{\Sigma}_{\mathrm{V}} = \mathbf{T}\,\boldsymbol{\Sigma}_{\mathrm{V}'}\,\mathbf{T}^{\mathrm{t}}. \tag{11}$$

Using matrix algebra, it is inferred from the above equation that all off-diagonal terms of $\boldsymbol{\Sigma}_{V'}$ are zero, so that there is no correlation between the components of V'. It means that using PCA, the correlated input variables $v_1, v_2, \ldots, v_N$ are transformed into the independent set of $v'_1, v'_2, \ldots v'_N$. Moreover, the eigenvalues of $\boldsymbol{\Sigma}_{V'}$, or in other words, the principal variances of $\mathbf{V}$ are actually the variances of V'. In such a case, the components of V' are referred to as the principal components of $\mathbf{V}$ (Johnson and Wichern 2007).

In compliant sheet metal assemblies, $\mathbf{V}$ may be defined as the initial deformation of individual parts and $\boldsymbol{\Sigma}_V$ as the geometric covariance matrix. Thus, eigenvectors of $\boldsymbol{\Sigma}_V$ correspond to deformation mode-shapes or variation patterns of the components. Furthermore, each eigenvalue indicates the contribution of the corresponding variation pattern to the overall deformation of the components. Some common variation patterns in a sheet metal component are illustrated in Figure 6.

3.2.2. *Incorporating PCA into EDR method*

Referring to Equation (10), the assembly response function $u(v_1, v_2, \ldots, v_N)$ can also be expressed in terms of independent variables as $U(v'_1, v'_2, \ldots v'_N)$. The statistical moments of U will then be calculated as follows:

$$m_r = E\left[\sum_{j=1}^{N} U^r(\mu'_1, \ldots, v'_j, \ldots, \mu'_N)\right] - (N-1)\,U^r(\mu'_1, \mu'_2, \ldots, \mu'_N). \tag{12}$$

Once the input variables are converted into an independent set, Equation (12) can be used to calculate the statistical moments in terms of independent variables taking into account the effect of geometric covariance. Another advantage of PCA is that in a multivariate statistical distribution, commonly a remarkable number of principal variances are zero or very close to zero. If the variance of v'_j is zero, its corresponding term in integrals of Equation (12) will vanish. As a result, in the domain of transformed independent variables V', those variables which have negligible variances can be excluded from the analysis. The remaining independent variables

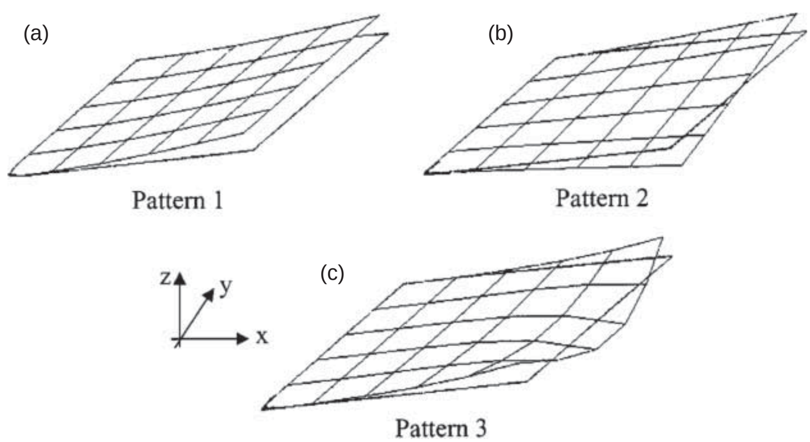

Figure 6. Three example variation patterns in a sheet metal component (Camelio, Hu, and Marin 2004).

with non-zero variances, which are referred to as dominant variables in this article, will then be involved in computation. Therefore, PCA significantly improves the time efficiency of SFPA by reducing the number of involved variables in the integrals of the EDR method. Referring to a basic criterion in tolerance analysis, a variable that has a smaller tolerance (variance) would have a lower contribution in the assembly response function (Chase and Parkinson 1991). In other words, the influence of input variables in the assembly function, to a great extent, depends on their variances. In sheet metal systems, it can also be inferred that variables with negligible variances will have a minute contribution to the deformation of the component. As a result, the assembly response function, which inherently depends on the component deformations, will almost be unaffected by those variables and, therefore, will only be a function of dominant variables. It is interesting to note that the applicability of removing variables with negligible variance has been earlier suggested and performed for sensitivity analysis of sheet metal systems when a linear force–displacement rule is implemented (Camelio, Hu, and Marin 2004).

By incorporating PCA into the EDR method, the statistical moments can be calculated by Equation (13) in terms of M dominant independent variables. The other variables with negligible variances will only participate in the equation with their mean values.

$$m_r = E\left[\sum_{j=1}^{M} U^r(\mu'_1, \ldots, v'_j, \ldots, \mu'_N)\right] - (M-1)\, U^r(\mu'_1, \mu'_2, \ldots, \mu'_N) \quad M << N. \tag{13}$$

In sheet metal assemblies, where generally a few variation patterns contribute to the deformation of a compliant part, PCA identifies the dominant patterns from the geometric covariance matrix. As demonstrated in Figure 7, considering the M dominant patterns and by disregarding

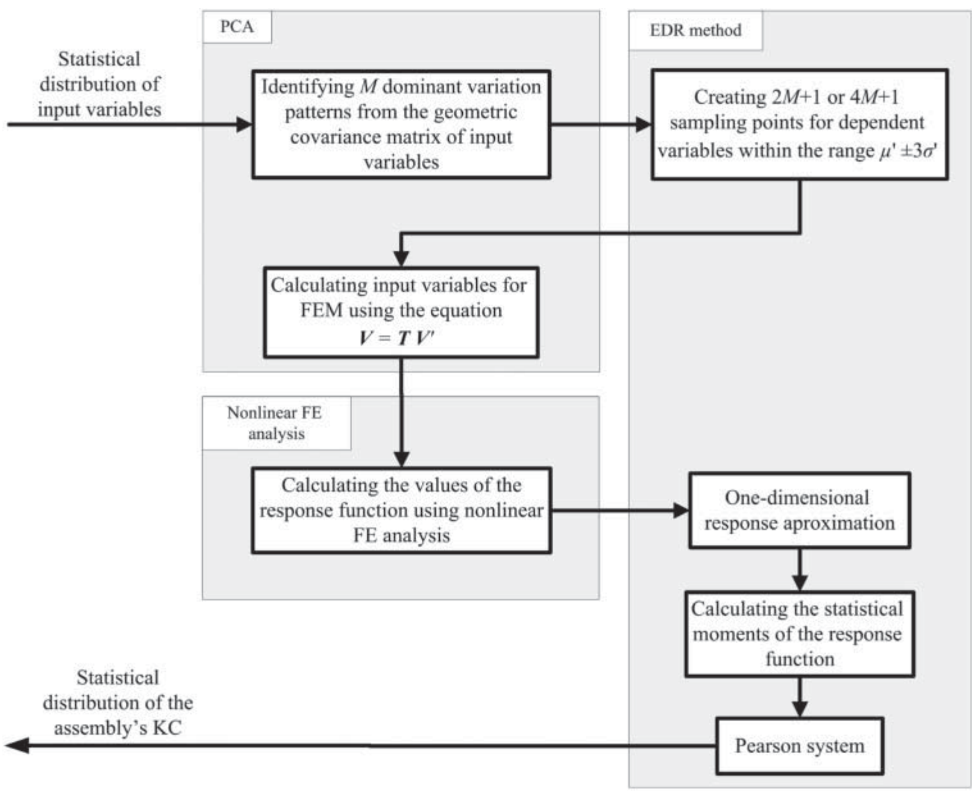

Figure 7. Overall variation analysis procedure of sheet metal assemblies.

the other patterns that have minor contributions, the EDR method creates $2M + 1$ or $4M + 1$ sampling points within the range of $\mu' \pm 3\sigma'$. The next step is to calculate the values of the assembly response function by implementing the nonlinear FEM which has been described in Section 2. Then, the EDR method is called again to construct one-dimensional response approximations so that the statistical moments can be computed using a numerical integration. In this paper, the Gauss–Kronrod quadrature rule, which is among the most effective methods for numerical integration, is suggested as an integration method. The Gauss–Kronrod quadrature rule typically uses an n-point Gaussian rule G_n paired with a $(2n+1)$-point Kronrod rule K_{2n+1} whose integration points are optimally chosen subject to the constraint that all points of G_n are reused in K_{2n+1} (Piessens and Branders 1974). It is commonly suggested to use a pair of G_7 and K_{15} rules in order to preserve a good accuracy in results (Piessens et al. 1983). Once the statistical moments are computed, the PDF of the assembly's KC will be estimated by Pearson system. Using the proposed methodology, the number of FEM runs as well as numerical integration will considerably decrease since some random variables are excluded from the analysis.

4. Experimental case study

The application of the proposed methodology in variation analysis of compliant sheet metal assemblies is investigated by studying a set of 30 assemblies of two flat steel plates. The plates have identical dimensions of $300 \times 240\,\text{mm}^2$ but different thicknesses of 1 and 1.5 mm. The Young's module and Poisson's ratio of plates are measured as of $200 \times 10^3\,\text{N/mm}^2$ and 0.3, respectively. As illustrated in Figure 8, an overlap of 60 mm is considered in the assembly where four joining points are marked with white dots. The initial deviation of each individual part from the ideal position is measured with an accuracy of 0.01 mm by the RENISHAW® 3D contact digitising CMM (Figure 9). Once the surface profiles are digitised, a routine was written to extract data points from the point clouds and generate a code in APDL (ANSYS Parametric Design Language) format for further processing.

Figure 8. Two sheet metal test pieces before being assembled.

Figure 9. Surface profile digitising of a test piece.

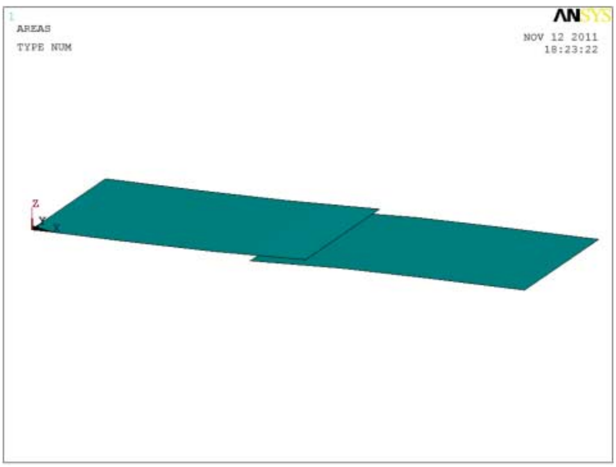

Figure 10. FE model of one experimental example before being assembled.

4.1. *Accuracy evaluation of nonlinear FEA*

Before statistically assessing the assembly variation of the current case study, it would be of interest to evaluate the accuracy of the proposed nonlinear FE analysis. Figure 10 presents the 3D model of one experimental example in ANSYS environment before being assembled. The spot-welding points are designated by W1–W4 in Figure 11 and positioned along the line $x = 270$ mm with intermediate distance of 60 mm. The displacement of a set of 23 points located along the line $x = 240$ mm on Plate A in final assembly is measured by CMM and compared with proposed nonlinear and conventional linear FE analyses. The result of this comparison is demonstrated in Figure 12 for three different assemblies. In this figure, the ideal state of the assembly, which shows no variation, is also illustrated. The difference between these assemblies is the input deviation of respective plates. For better interpretation, the average deviation of plates A and B at spot-welding points are written as δ_A and δ_B, respectively, at the top of each graph. The accuracy of the nonlinear explicit–implicit FE analysis is also quantitatively evaluated and tabulated in Table 2. Compared with error calculated from conventional non-contact (linear) analysis, the

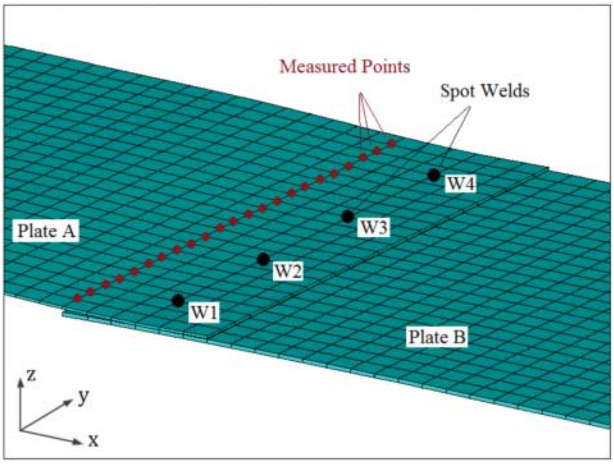

Figure 11. Positions of spot welds and measured points.

results show that the developed nonlinear contact analysis of this paper is in a great accordance with experimental data.

4.2. *Statistical variation assessment of final assembly*

In sheet metal variation analysis, generally, the sources of variation are initial deviations of components. For ease of computation, it is common to select a few points on components and consider their corresponding deviation as the input variables. In the current study, the initial deviation of the points W1 to W4 on two individual plates are input variables. These variables are named as v_1 to v_4 for Plate A and v_5 to v_8 for Plate B. The assembly's KC is assumed to be the final displacement of one of measured points mentioned in Section 4.1. This point is located at the position of $x = 240$ mm and $y = 120$ mm on Plate A (Figure 13). The location of assembly's KC is selected as far from fixtures as possible so that it will reflect the assembly variation to a great degree. However, to prevent any probable local impact by spot-welding process, this point should not be too close to spot welds.

As stated earlier, the deviation of neighbouring points on a plate, which creates the covariance structure, cannot be independent. It means that the variation of input variables v_1–v_4 on plate A and also v_5–v_8 on Plate B are dependent or they are statistically correlated. The mean vector ($\boldsymbol{\mu}$) and covariance matrix ($\boldsymbol{\Sigma}$) of the input variables are tabulated in Table 3 where the order of elements in respective arrays is in accordance with Figure 13. The diagonal elements of $\boldsymbol{\Sigma}_{\mathrm{A}}$ and $\boldsymbol{\Sigma}_{\mathrm{B}}$ represent the variance (square of standard deviation) of input variables, whereas the off-diagonal terms indicate the covariance. It is worth mentioning that the distributions of all variables are reported as normal. In addition, using the transformation formula ($\mathbf{V} = \mathbf{T}\mathbf{V}'$), the mean vector and covariance matrix of independent variables can be determined as presented in Table 4. The elements of diagonal matrices $\boldsymbol{\Lambda}_{\mathrm{A}}$ and $\boldsymbol{\Lambda}_{\mathrm{B}}$ represent the principal variances (eigenvalues) of respective covariance matrices $\boldsymbol{\Sigma}_{\mathrm{A}}$ and $\boldsymbol{\Sigma}_{\mathrm{B}}$. As stated in Section 3.2.1, these values are in fact the variances of independent variables and indicate the contribution of the corresponding variation patterns to the overall deformation of the components. It is inferred from the table that both plates have only one dominant pattern and the other patterns are not decisive. In order to have a better comparison, the matrices can also be shown through bar charts (Figures 14 and 15). It should be noted that variations of two plates are assumed to be completely independent since, in the current experimental case study, the plates are from different sheet rolls. But, practically, it

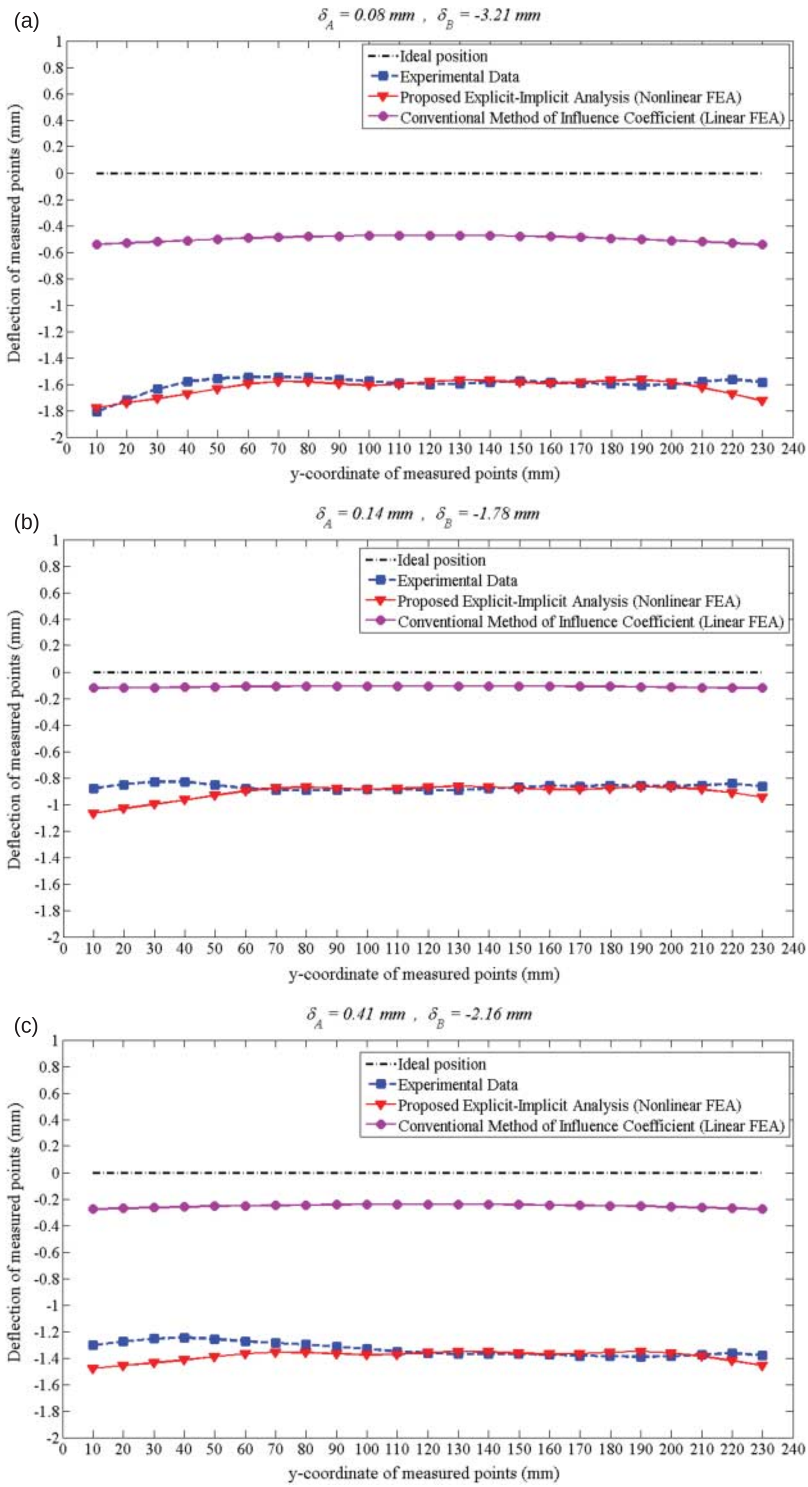

Figure 12. Deflection of measured points in three different assemblies with average input deviations of (a) $\delta_A = 0.08$ mm, $\delta_B = -3.21$ mm; (b) $\delta_A = 0.14$ mm, $\delta_B = -1.78$ mm; (*c*) $\delta_A = 0.41$ mm, $\delta_B = -2.16$ mm.

Table 2. Error comparison of proposed nonlinear and conventional linear FE analyses for assemblies of Figure 12.

	$\delta_A = 0.08$ mm, $\delta_B = -3.21$ mm		$\delta_A = 0.14$ mm, $\delta_B = -1.78$ mm		$\delta_A = 0.41$ mm, $\delta_B = -2.16$ mm	
y-Coordinate of measured points	% error in proposed nonlinear FEA	% error in conventional linear FEA	% error in proposed nonlinear FEA	% error in conventional linear FEA	% error in proposed nonlinear FEA	% error in conventional linear FEA
10	1.63	72.35	13.34	91.29	11.97	81.27
20	1.12	71.57	13.20	91.29	12.49	81.30
30	4.20	70.75	12.75	91.31	13.00	81.35
40	5.51	70.35	10.12	91.49	12.13	81.58
50	4.62	70.50	5.74	91.79	9.41	82.07
60	3.24	70.80	1.39	92.08	6.57	82.57
70	2.00	71.23	1.12	92.27	4.69	82.98
80	2.08	71.58	1.58	92.37	4.03	83.31
90	2.16	72.06	1.04	92.44	3.56	83.64
100	1.88	72.46	0.16	92.45	2.89	83.92
110	0.52	72.84	0.37	92.49	1.56	84.17
120	1.00	72.98	1.26	92.51	0.15	84.33
130	1.77	72.93	1.88	92.50	1.23	84.38
140	0.75	72.60	0.96	92.42	0.85	84.29
150	0.54	72.29	0.71	92.31	0.13	84.19
160	0.48	72.19	2.01	92.19	0.19	84.12
170	0.30	71.98	1.83	92.10	0.97	84.02
180	1.54	71.67	1.44	91.96	1.82	83.82
190	2.37	71.32	0.61	91.82	2.46	83.58
200	1.07	70.74	0.75	91.67	1.65	83.26
210	2.46	69.85	2.35	91.48	0.73	82.82
220	6.53	68.91	5.20	91.23	3.77	82.33
230	8.16	68.68	5.74	91.19	4.94	82.17

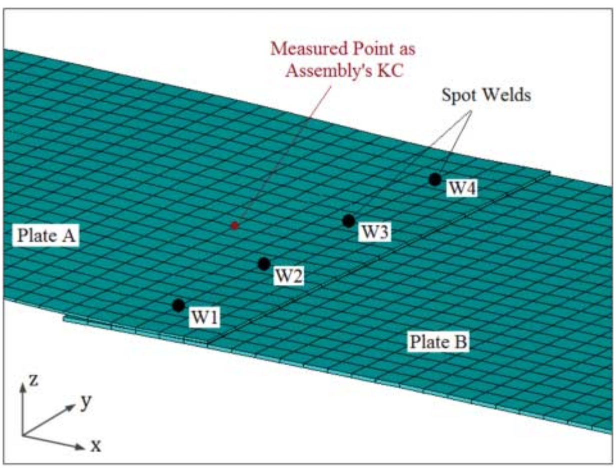

Figure 13. Position of assembly's KC in experimental case study.

is possible to find some assemblies in which the deviations of plates A and B are correlated. For example, if the plates are parts of a same sheet roll or manufactured by a same die.

PCA shows that for plate A, the eigenvector $\boldsymbol{t}_4 = [0.430\ 0.501\ 0.531\ 0.532]^t$ corresponds to the dominant variation pattern whose respective principal variance is considerably larger than the others (Figure 15). It is also concluded that the eigenvectors $\boldsymbol{t}_3 = [0.736\ 0.222\ -\ 0.196\ -\ 0.609]^t$ and $\boldsymbol{t}_2 = [-0.459\ 0.436\ 0.528\ -\ 0.566]^t$ are related to the second and third variation

Table 3. Mean vectors and covariance matrices of input variables.

	Vector of mean values (mm)	Covariance matrix (mm^2)
Test pieces A	$\boldsymbol{\mu}_A = [0.549 \quad 0.456 \quad 0.479 \quad 0.582]^t$	$\boldsymbol{\Sigma}_A = \begin{bmatrix} 0.4001 & 0.4268 & 0.4277 & 0.4094 \\ 0.4268 & 0.4904 & 0.5123 & 0.5022 \\ 0.4277 & 0.5123 & 0.5497 & 0.5518 \\ 0.4094 & 0.5022 & 0.5518 & 0.5772 \end{bmatrix}$
Test pieces B	$\boldsymbol{\mu}_B = -[1.268 \quad 1.443 \quad 1.485 \quad 1.488]^t$	$\boldsymbol{\Sigma}_B = \begin{bmatrix} 0.6629 & 0.7560 & 0.7478 & 0.6365 \\ .7560 & 0.9252 & 0.9454 & 0.8118 \\ 0.7478 & 0.9454 & 0.993 & 0.8846 \\ 0.6365 & 0.8118 & 0.8846 & 0.8571 \end{bmatrix}$

Table 4. Mean vectors and covariance matrices of independent variables.

	Vector of mean values	Covariance matrix
Test pieces A	$\boldsymbol{\mu}_A{}' = [0.021 - 0.130 \quad 0.058 \quad 1.028]^t$	$\boldsymbol{\Lambda}_A = \begin{bmatrix} 0.0003 & 0 & 0 & 0 \\ 0 & 0.0085 & 0 & 0 \\ 0 & 0 & 0.0765 & 0 \\ 0 & 0 & 0 & 1.9323 \end{bmatrix}$
Test pieces B	$\boldsymbol{\mu}_B{}' = [0.017 \quad 0.134 \quad 0.042 - 2.844]^t$	$\boldsymbol{\Lambda}_B = \begin{bmatrix} 0.0003 & 0 & 0 & 0 \\ 0 & 0.0291 & 0 & 0 \\ 0 & 0 & 0.1316 & 0 \\ 0 & 0 & 0 & 3.2773 \end{bmatrix}$

patterns, respectively, which are not, however, as decisive as $\boldsymbol{t}_4$. Comparing these three eigenvectors with three patterns in Figure 6 shows that $\boldsymbol{t}_4$ is related to pattern 1 (bending along y-axis) while $\boldsymbol{t}_3$ and $\boldsymbol{t}_2$ are related to patterns 2 (twisting) and 3 (bending along x-axis), respectively. Same results can also be concluded for plate B. It means that all test pieces of this case study remarkably follow pattern 1 and the other patterns have less contributions.

As illustrated in Figure 15, PCA reports variables v'_4 of plate A and v'_8 of plate B as the principal variables in order to be used in the EDR method and the other variables with quite smaller variances can be neglected. The nonlinear FE analysis will be applied to estimate the values of the response function. Referring to Figure 7, the one-dimensional response functions in terms of v'_4 and v'_8 can be used to calculate the statistical moments as:

$$m_r = E[U^r(\mu'_1, \ldots, v'_4, \ldots, \mu'_8)] + E[U^r(\mu'_1, \ldots, \mu'_7, v'_8)] - U^r(\mu'_1, \ldots, \mu'_8). \tag{14}$$

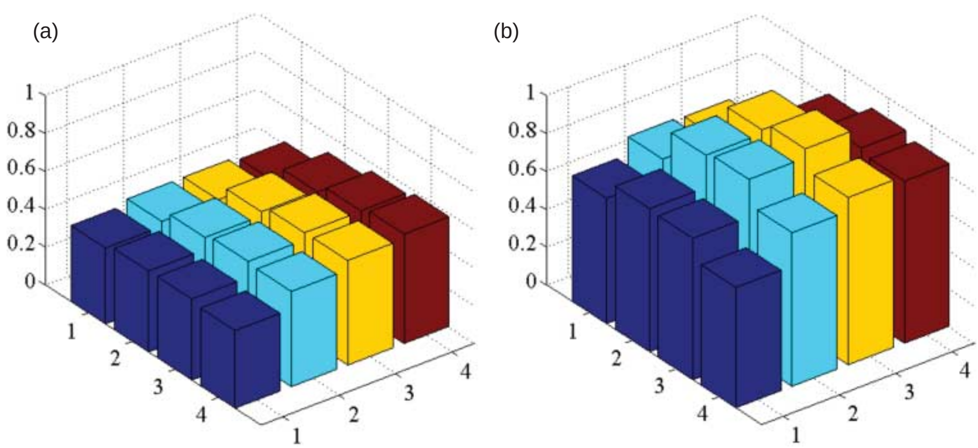

Figure 14. Geometric covariance matrices of plates A (left) and B (right).

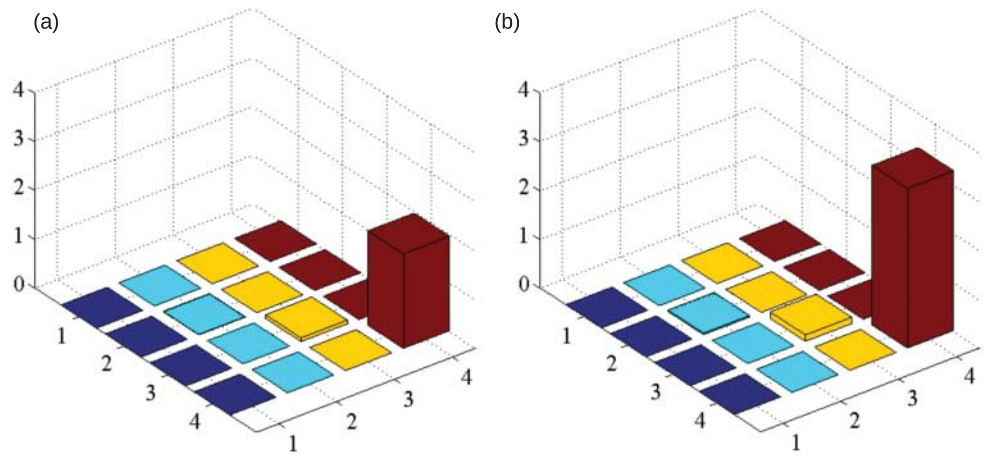

Figure 15. Principal variance matrices of plates A (left) and B (right).

In this case study, data obtained from measurements of the initial deviation of sheet metal parts are considered as the input variables in the analysis. In order to investigate the efficiency of the presented methodology, a set of 10,000 assemblies were also simulated and the results are compared and evaluated with experimental data as presented in Table 5. The table shows a good accuracy in the results of the presented approach compared with experimental data and MCS. The table also shows how the analysis of the covariance structure can affect the final prediction of assembly variation as neglecting the geometric covariance will result in a considerable error in the analysis. Another important point is the type of the output distribution of the assembly's KC. Although all input variables have normal distribution, the output is not normal (in a normal distribution $\gamma_1 = 0$ and $\beta_2 = 3$).

As stated earlier, the Pearson system can predict the output distribution of the assembly's KC. The statistical specifications of the KC indicate that it has a *Type I* or beta distribution in accordance with Pearson classification. Generally, a beta distribution in the interval [a,b] with shape factors p and q is defined as follows where B is the beta function (Johnson, Kotz, and Balakrishnan 1995):

$$p(u) = \frac{1}{B(p,q)} \frac{(u-a)^{p-1}(b-u)^{q-1}}{(b-a)^{p+q-1}} \quad a < u < b, \quad p,q > 0. \tag{15}$$

The beta PDF that was predicted by the proposed approach is presented in Figure 16. The figure shows that the improved SFPA which incorporates PCA into EDR method is in a good accordance with MCS.

Finally, it should be noted that the computational effort of the variation analysis procedure

Table 5. Statistical specifications of the assembly's KC.

	Mean (μ)	Std. dev. (σ)	Skewness (γ_1)	Kurtosis (β_2)
Experimental data	– 0.7398	0.2955	– 0.4217	2.3164
MCSs (% error)	– 0.7429 (0.42%)	0.3091 (4.60%)	– 0.4299 (1.94%)	2.3332 (0.73%)
Proposed methodology (improved SFPA) (% error)	– 0.7427 (0.39%)	0.3080 (4.23%)	– 0.4270 (1.26%)	2.3061 (0.44%)
Analysis regardless of covariance (% error)	– 0.6980 (5.65%)	0.1907 (35.47%)	– 0.0453 (89.26%)	1.7111 (26.13%)

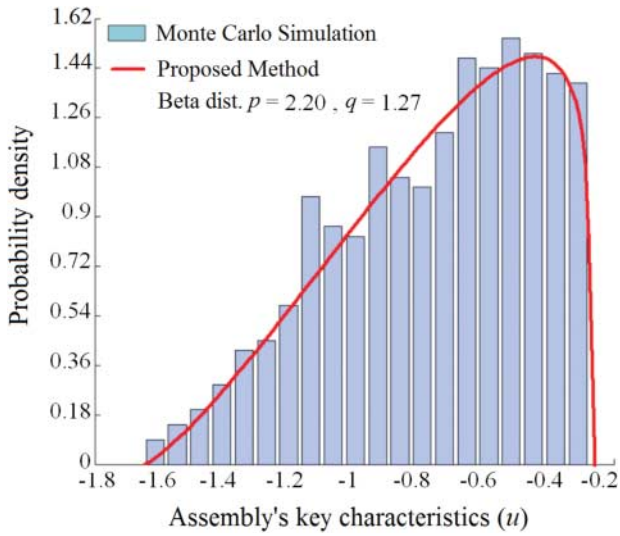

Figure 16. Assembly's KC distributions vs. simulation result.

Table 6. Comparison of number of FE simulations and CPU time.

	Number of FE simulations	CPU time (minutes)
Improved SFPA (two independent variables involved)	9	14
Ordinary SPFA (all variables involved)	33	50
MCSs	10,000	15,000

of sheet metal assemblies is mainly based on the number of FE simulations. Assuming that all necessary input data for the FE analysis are ready, for the current case study, it takes around 90 seconds to simulate the assembly process in ANSYS/LS-DYNA using a 3.0 GHz Intel® Core™2 Duo processor. As stated in Section 3.1.2, SPFA requires $4N + 1$ FE simulations to determine KC's statistical specifications and construct the PDF. Therefore, as reported in Table 6, the improved SPFA which predicts the assembly variation with two independent variables requires 9 simulations, whereas the ordinary SFPA with all involved variables needs 33 simulations. On the other hand, when it comes to MCS which calculates the assembly response for the entire population of random input variables, the number of FE simulations and CPU time will drastically increase.

5. Conclusions

This paper presents a comprehensive methodology for tolerance analysis and variation assessment of compliant sheet metal assemblies. In comparison with previous research, the effect of geometric covariance is included in the nonlinear sheet metal variation analysis where the nonlinearity arises in consequence of contact interactions of mating parts and tools. The methodology integrates two main modules: (1) a nonlinear finite element analysis which includes contact interactions during the assembly process using a sequential explicit–implicit analysis; and (2) an improved SFPA which incorporates PCA into EDR method in order to include the effect of geometric covariance in the final assembly variation. The approach is well applicable to sheet metal variation analysis. The improved SFPA lessens the computational effort of the variation analysis procedure by reducing the number of involved random variables, so that time efficiency of the new approach promisingly increases. The efficiency of the developed approach is evaluated by an experimental case study as well as MCS. Results show that proposed methodology produces an accurate estimation of the assembly's KCs in contrast to the case in which the effect of geometric covariance is overlooked in the analysis.

Disclosure statement

No potential conflict of interest was reported by the authors.

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