



## PRESENTATION OF EMPIRICAL EQUATIONS FOR ESTIMATING INTERNAL FRICTION ANGLE OF SP AND SC SOILS IN MASHHAD, IRAN USING STANDARD PENETRATION AND DIRECT SHEAR TESTS AND COMPARISON WITH PREVIOUS EQUATIONS

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### ABSTRACT

*Presentation of empirical equations for estimating engineering properties of soils is a simple, low cost and widely-used method. One of the major concerns in using these equations is evaluating their accuracy in different conditions and regions which often leads to doubts about obtained results. Most of these equations were derived in special laboratories, different climate conditions and in soils with different geotechnical and geological engineering properties and were generalized to other conditions. The main question is that whether these methods are also applicable to other conditions. Using local equations and narrowing the usage range of various methods based on each region properties are appropriate methods to solve these problems. This leads to simplified and faster analysis and high reliability in the obtained results. In this paper, empirical equations were derived to estimate internal friction angle, based on SPT numbers of Mashhad City's soils in Iran, using SPT and direct shear tests results from 50 samples (25 SP and 25 SC soils samples). The results showed similar values for predicted  $\phi$  values by SPT test and  $\phi$  values determined by direct shear tests.*

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**Keywords:** Internal friction angle, SP soil, SC soil, Direct shear test, SPT test, Local equations.

### Contribution/ Originality

This study originates new formulas about finding internal friction angle based on SPT number and soil type. Finding local formulas to estimate reliable  $\phi$  values based on SPT number and soil type is a new method that can widely use in all regions.

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## 1. INTRODUCTION

Internal friction angle is one of the most important parameters in analyzing soil geotechnical properties and earthwork calculations. It has a wide range of applications such as calculating retaining walls, foundations, friction and end-bearing piles and so on (Shioi and Fukui, 1982; McGregor and Duncan, 1998).

Based on properties of a given soil profile such as fine or coarse grained, various tests such as direct shear and triaxial tests are recommended for obtaining internal friction angle parameter. Although due to the soil disturbance during sampling as well as special laboratory conditions, these results may not completely represent true properties of soils and even in case of special care in doing the tests, they are still highly time-consuming and requires using simpler empirical equations. This research aims to obtain internal friction angle of soils using standard penetration test for different types of soils in Mashhad. For this purpose, several equations have already been presented (Silva *et al.*, 2010). Internal friction angle for Mashhad city can be estimated using appropriate equations for the city soil conditions, soil types, samples depth and specific unit weight.

SPT number was defined in various equations based on specific weight, grading, relative density, internal friction angle and undrained compressive strength (Hettiarachchi and Brown, 2009; Jianguo, 2012). This number is also used for estimating bearing capacity of soil for foundation and elastic modulus calculations (Kaliniski, 2011). These equations and their approaches are often doubtful due to having a small range of gathered data, lack of focus on special aspects or equations incorrect generalization (Bowles, 1988; Budhu, 2011; Mohammad, 2013).

Equations obtained by Shioi and Fukui (1982) are presented below (Equations 1 to 3). Equation 1 is for roads and bridges, Equation 2 for buildings and Equation 3 is general.

- (1)  $\phi = \sqrt{N'_{70}} + 15$
- (2)  $\phi = 0.36 N_{70} + 27$
- (3)  $\phi = 4.5 N_{70} + 20$  (in general)

## 2. FHWA RECOMMENDED TABLES

Federal Highway Administration recommend using Table 1 for correlating approximate SPT number, relative density and internal friction angle parameters with each other (FHWA, 2003). Following information is necessary to use Table 1.

- 1. Measured SPT numbers were obtained without any correction factors in field tests.
- 2. (Pa) is free sea level pressure.
- 3. Ranges in column (a) is based on Peck *et al.* (1974) study.
- 4. Ranges in column (b) is based on Meyerhof (1956) study.

**Table-1.** correlation between SPT and CPT results and friction angle of cohesionless soils (FHWA, 2003)

	In-Situ Test Results	Relative Density	$\phi$ (degrees)	
			(a) <sup>3</sup>	(b) <sup>4</sup>
SPT N-Value (blows/300 mm or blows/ft)	0 to 4	Very Loose	< 28	< 30
	4 to 10	Loose	28 to 30	30 to 35
	10 to 30	Medium	30 to 36	35 to 40
				<i>Continue</i>

	30 to 50	Dense	36 to 41	40 to 45
	> 50	Very Dense	> 41	> 45
Normalized CPT cone bearing resistance (qc/Pa)	< 20	Very Loose	< 30	
	20 to 40	Loose	30 to 35	
	40 to 120	Medium	35 to 40	
	120 to 200	Dense	40 to 45	
	> 200	Very Dense	> 45	

(3) Ranges in column (a) from Peck *et al.* (1974).

(4) Ranges in column (b) and for CPT are from Meyerhof (1956).

Since the values are from field SPT tests, the table can be very useful and widely applicable (Aggour and Radding, 2001).

### 3. METHODS OF STUDY

First, SPT tests were carried out on 50 samples (25 SC and 25 SP samples in various depths). Results of direct shear tests ( $\phi$  values) and also depth and dry unit weight of samples are shown in Tables 2 and 3 (Soil and Structure Consulting Engineers Company, 2014). These tables show in situ (SPT) and laboratory (direct shear) tests results. Locations of samplings in the city are shown in Figure 1.

Then, based on statistical validations, two equations were derived to estimate internal friction angle, based on SPT number for two soil types (SC, SP).

In order to use Tables 2 and 3, some points must be considered.

1. Narrowing application range was done for special types of soils in order to achieve higher accuracy.
2. In order to attenuate the effects of some parameters such as weathering, all studied samples were taken from the depths of 4 to 15 meters.
3. In order to obtain better results, samples with special dry unit weight of 19 to 21 KN/m<sup>3</sup> were considered.
4. Internal friction angle in these tables were obtained from direct shear tests.
5. All data were obtained from soil profiles on Vakilabad area located in the western part of Mashhad city.

**Table-2.** SP soils data obtained by laboratory and in-situ tests

Soil Type	Row	KN/m <sup>3</sup> (Dry Weight Unit	m(Depth	SPT Number	Internal Friction Angle
	1	20.4	15	25	28.1
	2	19.3	11	26	29.2
	3	20.3	12	24	28.2
	4	19.4	9	27	31.3
	5	19.6	8	28	31.9
	6	20.2	7	28	32.1
	7	20.1	4	27	30.9
	8	20.3	6	28	32.2
	9	20.8	14	28	31.6
					<i>Continue</i>

SP	10	20.1	13	26	29.9
	11	19.8	10	27	30.8
	12	19.4	8	26	31.1
	13	19.8	10	27	31.4
	14	19.8	12	27	31.9
	15	19.6	13	28	33.2
	16	20.2	7	25	30.2
	17	20.1	5	27	32.8
	18	20.4	9	26	31.6
	19	20.2	5	25	29.8
	20	19.6	11	28	33.9
	21	19.2	12	29	34.6
	22	19.6	14	30	35.6
	23	20.1	8	29	35.1
	24	19.7	5	30	34.6
25	19.5	10	28	33.9	

Table-3. SC soils data obtained by laboratory and in-situ tests

Soil Type	Row	γKN/m <sup>3</sup> /Dry Unit Weight	m/Depth	SPT Number	Internal Friction Angle
SC	1	20.7	14	30	33.1
	2	20.3	13	29	32.6
	3	19.7	11	31	33.1
	4	19.4	5	31	33.9
	5	20.4	7	30	33.2
	6	20.3	6	31	33.9
	7	20.5	15	29	32.5
	8	20.4	12	32	34.5
	9	20.2	10	29	32.8
	10	19.9	8	28	32.9
	11	20.2	11	27	31.7
	12	20.7	15	30	33.4
	13	20.1	7	29	32.2
	14	20.4	13	32	35.8
	15	19.8	6	31	35.1
	16	19.4	8	29	33.1
	17	20.3	10	28	32.4
	18	20.1	11	29	32.3
	19	20.7	12	27	30.5
	20	19.8	6	30	32.7
	21	20.1	9	28	31.2
	22	19.9	7	32	34.9
	23	19.5	4	28	31.8
	24	19.3	3	32	35.4
	25	20.6	11	32	35.3

#### 4. RESULTS AND DISCUSSION

Reliability and accuracy of the obtained equations must be measured by statistical reliability ratings. In order to obtain correlations between data, the following steps were taken:

1. Drawing dispersion diagram; Figures 2 and 3 show dispersion diagrams related to SC and SP soils properties.

2. Model fitting and obtaining coefficients: the aim of a proper model fitting is to determine correlation among control (x) and response (y) variables (Equations 4,5,6 and figures 2, 3) (Isotalo, 2001; Rad, 2008).

$$(4) y_i = \alpha + \beta x_i + e_i$$

$$(5) y_i = \alpha + \beta \log x_i + e_i$$

$$(6) y_i = \alpha + \exp(\beta x_i) + e_i$$

3. Obtaining numerical value of Sig for comparing correlations.

Models were studied on 95% reliability level. Thus, for studying meaningfulness of the model, model making and model coefficients evaluation, following statistical hypotheses were considered (Equations 7, 8).

$$7) \begin{cases} H_0: \beta = 0 & \text{The model is not meaningful} \\ H_1: \beta \neq 0 & \text{The model is meaningful} \end{cases}$$

$$8) \begin{cases} sig. (p - value) > \alpha = 0.05 \rightarrow H_0 \text{ acceptable} \\ sig. (p - value) < \alpha = 0.05 \rightarrow H_1 \text{ acceptable} \end{cases}$$

Sig. (p-values) were obtained from Fisher test (Table 4).

**Table-4.** Anova table for studying the meaningfulness of the model.

ANOVA <sup>a</sup>						
Model1 for SP Soils		Sum of Squares	Df	Mean Square	F	Sig.
1	Regression	82.656	1	82.656	113.555	.000b
	Residual	16.742	23	0.728		
	Total	99.398	24			
Model2 for SC Soils		Sum of Squares	Df	Mean Square	F	Sig.
2	Regression	37.402	1	37.402	120.402	.000b
	Residual	7.145	23	0.311		
	Total	44.546	24			

a. Dependent Variable: PHI

b. Predictors: (Constant), SPT

Based on Sig (p-value) obtained from Fisher test: meaningful

$$sig=0.000 < \alpha=0.05 \rightarrow RH0$$

According to Table 4, both models are meaningful. (Equations 9 and 10).

Presented equation for SP soils:

$$9) \varphi = 1.2004 SPT - 0.7674$$

Presented equation for SC soils:

$$10) \varphi = 0.7732 SPT + 10.201$$

In Figures 4 and 5, observed friction angle ( $\varphi_{obs}$ ) (based on direct shear test) and predicted friction angle ( $\varphi_{pre}$ ) (based on the equations obtained in this research) were compared.

Based on the obtained equations for SC and SP soils, following comparisons with FHWA table values were done:

Table-5. Predicted values of internal friction angle in this research

	SPT number(in-situ test)	$\phi$ (using obtained equation)		SPT number(in situ test)	$\phi$ (using obtained equation)
SC soils	-	-	SP soils	24	28.04
	-	-		25	29.24
	-	-		26	30.44
	-	-		27	31.64
	28	31.85		28	32.84
	29	32.62		29	34.04
	30	33.39		30	35.24
	31	34.17		-	-
	32	34.94		-	-

The results indicate similarity between predicted  $\phi$  values calculated using presented equations in this paper and Peck *et al.* (1974) study; they had also predicted the values of  $\phi$  about 30 to 35 degrees (for this range of SPT numbers). In contrast, the results of this research do not conform with the Meyerhof (1956) study and it seems that he had over-predicted the  $\phi$  values.

## 5. CONCLUSIONS

Deriving empirical equations among various geotechnical parameters such as SPT number and internal friction angle can be very effective for different purposes such as fast and simple approximate evaluations and reliability rating of laboratory results. In this paper, these correlations were presented for coarse grained and low cohesive soils profiles of Mashhad city. In order to present the mentioned correlations, SC and SP soils with special dry unit weight of 19 to 21 KN/m<sup>3</sup> were studied. To avoid weathering effect on results, samples with depths between 4 to 15 m were used. By narrowing soil type range, depth of sampling and dry unit weight for predicting internal friction angle based on SPT number, two equations were presented. Due to the direct correlation between SPT number and density, these equations can be generated to other regions according to similar dry unit weights and because of using SPT numbers without any correction factors these equations can be compared with FHWA tables that are deriving by field tests. Based on Table 5, FHWA values are similar to the values of internal friction angle obtained from presented equations in this paper and conform to the results of Peck *et al.* (1974) study. However, the obtained values are mainly lower than the values obtained by Meyerhof (1956) results.

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