Transverse spin structure function $g_2(x, Q^2)$ in the valon model

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The spin dependent structure function, g_2^{ww}, is calculated in the valon model. A simple approach is given for the determination of the twist-3 part of $\bar{g}_2(x, Q^2)$ in Mellin space, thus enabling us to obtain the full transverse structure function $g_2(x, Q^2)$ for the proton, neutron, and deuteron. In light of the new data, we have further calculated the transversely polarized structure function of $g_2^{^3\mathrm{He}}(x, Q^2)$. Our results are checked against the experimental data and nice agreements are observed.

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I. INTRODUCTION

The nucleon polarized structure functions $g_{1,2}(x, Q^2)$ are important tools in understanding the nucleon substructure. In particular, they are indispensable elements for the understanding of the spin dependent parton distributions and their correlations. The $g_2(x, Q^2)$ structure function is important because it probes transversely and also longitudinally polarized parton distributions inside the nucleon. The $g_2(x, Q^2)$ structure function is also sensitive to higher-twist effects, such as quark gluon correlations. They do not disappear even at large Q^2 values and are not easily interpreted in pQCD [1,2]. Since $g_2(x, Q^2)$ is the only function related to the quark-gluon interaction, learning about its behavior will yield further insight into the spin structure of the nucleon beyond the simple quark parton model.

Thus, the main purpose of this paper is to calculate the transverse spin structure function, $g_2(x, Q^2)$. This requires considering both the twist-2 and the twist-3 contributions. Here we will present a simple method to extract the twist-3 part. The twist-2 part is well understood, and requires knowledge about the $g_1(x, Q^2)$ structure function. Therefore, first we will briefly review $g_1(x, Q^2)$ in the context of the so called valon model representation of hadrons.

Finally, the outcome of our results is checked against the experimental data from [3–7], and compared with other phenomenological models.

The layout of the paper is as follows: In Sec. II, we briefly present a review of the polarized nucleon structure function in the valon model. Section II deals with the calculation of $g_2(x, Q^2)$ spin structure function and discusses the numerical results. We also provide some discussion on the effect of higher twists. Section IV is devoted to the sum rules. Our conclusions are given in Sec. V.

II. A BRIEF REVIEW OF SPIN STRUCTURE FUNCTIONS IN THE VALON MODEL

The valon model is a phenomenological model, originally proposed by R. C. Hwa [8] in the early 1980s to provide a bridge between the naive quark model and the partonic structure of the hadrons. The model had many successes. It was improved later by Hwa [9] and others [10–15]. It was further extended to include the polarized cases [16–19]. The model views a hadron as three (two) constituent quark-like objects called valons. Each valon is defined to be a dressed valence quark with its own cloud of sea quarks and gluons. The dressing processes are described by QCD. At high enough Q^2 values the structure of a valon can be resolved, but at low Q^2 values the internal structure of the valon cannot be resolved and it behaves as constituent quarks of the hadron.

In valon model the polarized parton distributions of a polarized hadron are given by the following convolution integral:

$$\delta q_i^h(x, Q^2) = \sum \int_x^1 \frac{dy}{y} \delta G_{\text{valon}}^h(y) \delta q_i^{\text{valon}}\left(\frac{x}{y}, Q^2\right), \quad (1)$$

where $\delta G_{\text{valon}}^h(y)$ is the valon helicity distribution in the hosting hadron; that is, it is the probability of finding a polarized valon inside the polarized hadron. In the next-to-leading order, $\delta G_{\text{valon}}^h(y)$ marginally depends on Q^2. These distributions are shown in Fig. 1.

The term $\delta q_i^{\text{valon}}(x/y, Q^2)$ in Eq. (1) is the polarized parton distribution function (PPDF) inside a valon and is obtained from the solutions of Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations in the valon. Now, using the convolution integral, one can obtain the polarized hadron structure functions as follows:

$$g_1^h(x, Q^2) = \sum_{\text{valon}} \int_x^1 \frac{dy}{y} \delta G_{\text{valon}}^h(y) g_1^{\text{valon}}\left(\frac{x}{y}, Q^2\right), \quad (2)$$

where $g_1^{\text{valon}}(\frac{x}{y}, Q^2)$ is the polarized structure function of the valon. The details of actual calculations are given in [16,18]. In short, the following two steps lead us to both the polarized PPDFs and polarized nucleon structure functions:

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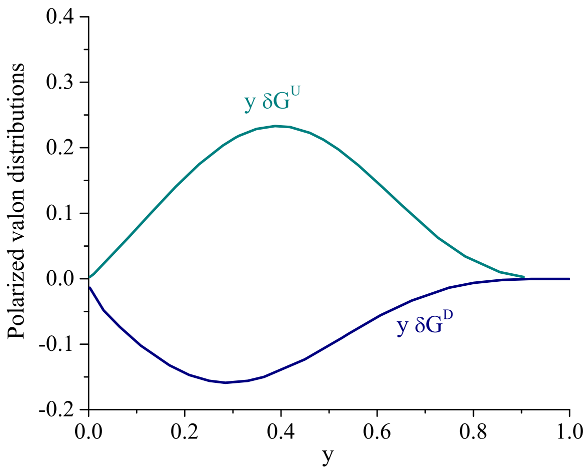

FIG. 1. (Color online) Polarized valon distribution functions for U and D valon types (the helicity distributions for the valons) inside the proton.

(1) Calculate the PPDFs in the valon using DGLAP equations.
(2) With a phenomenological approach, the helicity distributions of the valons in a nucleon are obtained. Then, they are used in Eqs. (1) and (2) to get the polarized parton distribution functions (PPDFs) and the polarized nucleon structure up to $Q^2 = 10^7$ GeV^2.

It should be noted that the valon model is only a phenomenological model. As such, initial conditions, as inputs to the DGLAP equations, are chosen based on phenomenological arguments. The results obtained for the proton structure function $g_1^p(x,Q^2)$ from this model are in excellent agreement with all available experimental data [20–25]. In Fig. 2 we only present a sample of the results along with the existing data. The parton distributions so obtained will be used here to calculate $g_2(x,Q^2)$.

III. TRANSVERSE SPIN-DEPENDENT STRUCTURE FUNCTION $g_2(x,Q^2)$

Polarized deep inelastic scattering (DIS), mediated by a photon exchange, probes two spin structure functions: $g_1(x,Q^2)$ and $g_2(x,Q^2)$. If the target is transversely polarized, the total cross section is a combination of these two structure functions. The transverse spin structure function, $g_2(x,Q^2)$, is made up of two components: a twist-2 part, g_2^{ww}, and a mixed-twist part, $\bar{g}_2(x,Q^2)$. Therefore, it can be written as [26]

$$g_2(x,Q^2) = g_2^{ww}(x,Q^2) + \bar{g}_2(x,Q^2), \tag{3}$$

where

$$\bar{g}_2(x,Q^2) = -\int_x^1 \frac{\partial}{\partial y}\left(\frac{m}{M}h_T(y,Q^2) + \xi(y,Q^2)\right)\frac{dy}{y}. \tag{4}$$

The twist-2 part, g_2^{ww}, comes from operator product expansion (OPE). The $\bar{g}_2(x,Q^2)$ receives a contribution from the transversely polarized quark distributions $h_T(x,Q^2)$ plus a contribution that comes from a twist-3 component, an indication of qgq correlations, given by the $\xi(y,Q^2)$ term in Eq. (4). These higher-twist corrections arise from the nonperturbative multiparton interactions. Their contributions

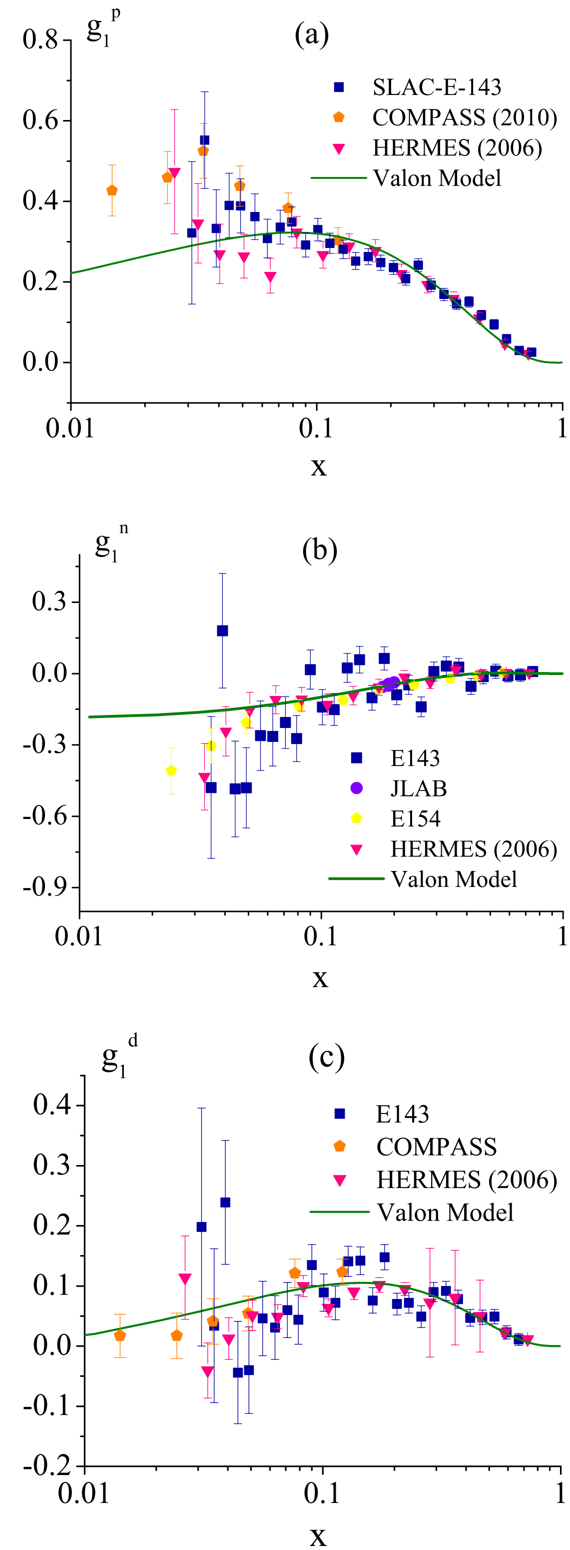

FIG. 2. (Color online) Polarized proton (a), neutron (b), and deuteron (c) structure functions $g_1(x,Q^2)$ at $Q^2 = 5$ GeV^2. The results from the valon model are compared with the experimental data [20–25].

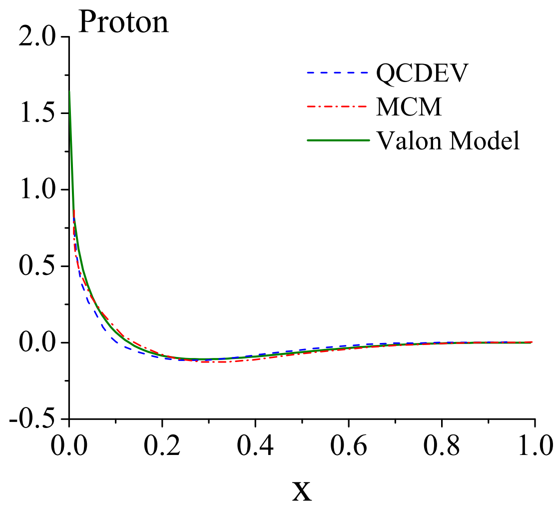

FIG. 3. (Color online) Transverse polarized proton structure functions $g_2^{ww}(x,Q^2)$ at $Q^2 = 5\ \mathrm{GeV}^2$. We compare our results with the results from other phenomenological models [29,30].

at low energy increase as $\frac{1}{Q^\tau}$, reflecting the confinement. Any nonzero result for this term at a given Q^2 will reflect a departure from the noninteracting partonic regime [27].

g_2^{ww} is related to the g_1 structure function by the Wandzura-Wilczek relation [28] as follows:

$$g_2^{ww}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 g_1(y,Q^2)\frac{dy}{y}. \tag{5}$$

A. Calculation of the twist-2 term, $g_2^{ww}(x,Q^2)$

We begin with Eq. (5). Since $g_1(x,Q^2)$ is known in the valon model [16,18], we utilize it without any additional free parameter and evaluate the twist-2 part of $g_2(x,Q^2)$; namely $g_2^{ww}(x,Q^2)$, according to the Eq.(5). The results are shown in Fig. 3 for the proton. We have also included the findings of [29,30] for the purpose of comparison.

B. Calculating the twist-3 term, $\bar{g}_2(x,Q^2)$

As mentioned, the function $\bar{g}_2(x,Q^2)$ has two terms. The first term is a twist-2 contribution related to the transverse polarization of quarks in the nucleon. It is suppressed by the quark-to-nucleon mass ratio and will be ignored here due to its negligibility. The second part is a twist-3 contribution, reflecting the quark-gluon correlations. In the following we will focus on this part.

In the large-N_c limit, Ali, Braun, and Hiller found that the Q^2 evolution of $\bar{g}_2(x,Q^2)$ is qualified by a simple DGLAP type equation with a difference between the anomalous dimensions and the twist-2 distribution [31,32]. It implies that $\bar{g}_2(x,Q^2)$ obeys the following simple equation:

$$\bar{g}_2(n,Q^2) = L^{\frac{\gamma_n^g}{2b_0}}\bar{g}_2\big(n,Q_0^2\big), \tag{6}$$

where

$$\bar{g}_2(n,Q^2) = \int_0^1 x^{n-1}\bar{g}_2(x,Q^2)dx, \tag{7}$$

$$L \equiv \frac{\alpha_s(Q^2)}{\alpha_s\big(Q_0^2\big)}, \tag{8}$$

and

$$b_0 = \frac{11}{3}N_c - \frac{2}{3}N_f, \tag{9}$$

$$\gamma_n^g = 2N_c\left(S_{n-1} - \frac{1}{4} + \frac{1}{2n}\right), \tag{10}$$

$$S_{n-1} = \sum\frac{1}{j}. \tag{11}$$

α_s is the strong coupling constant, N_f is the number of flavors, and S_{n-1} are the harmonic functions. Our purpose is to find the Q^2 evolution of $\bar{g}_2$ with some appropriate initial conditions in moment space. Then we can make a transformation to the momentum space and evaluate the twist-3 contribution to the transverse spin structure function. This is done in two steps, as is the case in the valon model. The first step involves finding a solution to Eq. (6) in a valon. The second step is to convolute the results obtained in the first step with the valon distribution in the nucleon. This will give the nucleon structure function.

We take $Q_0^2 = 0.238$ as our initial scale which is also used in our original calculations of various parton distributions. This value of Q_0^2 corresponds to a distance scale of 0.36 fm which is roughly equal to or less than the radius of a valon [11]. The initial input function for $\bar{g}_2^{\mathrm{valon}}(z,Q_0^2)$ is (see the Appendix)

$$\bar{g}_2^{\mathrm{valon}}\big(z,Q_0^2\big) = A\delta(z-1). \tag{12}$$

The justification for this choice is as follows: In the momentum space one can write

$$\bar{g}_2^{\mathrm{valon}}(z,Q^2) = f(Q^2)\bar{g}_2\big(z,Q_0^2\big) = f(Q^2)A\delta(z-1), \tag{13}$$

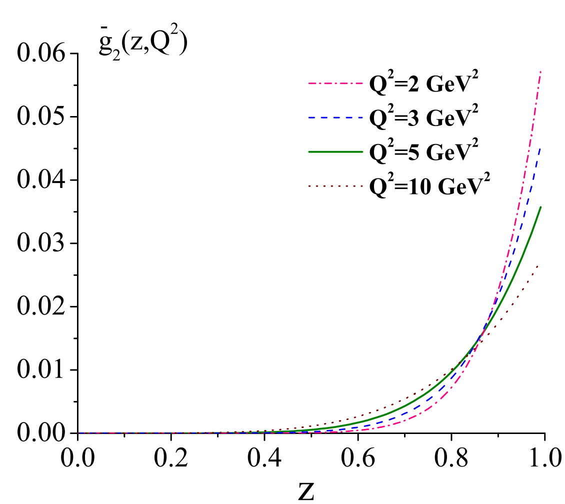

FIG. 4. (Color online) The twist-3 part of proton transverse spin structure function $\bar{g}_2(z,Q^2)$ in the valon, at $Q^2 = 2,3,5,10\ \mathrm{GeV}^2$.

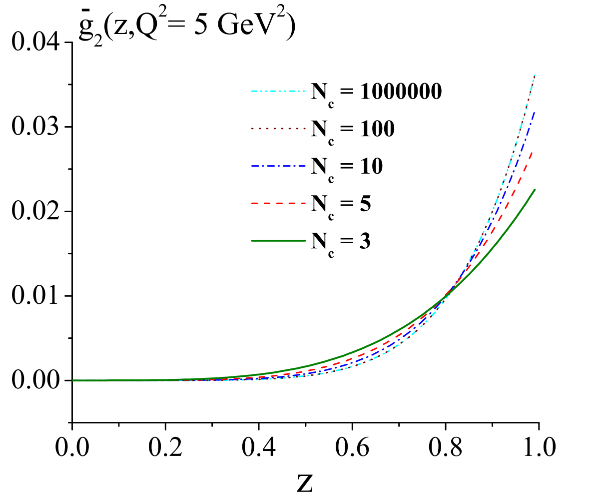

FIG. 5. (Color online) The twist-3 part of the proton transverse spin structure function $\bar{g}_2(z,Q^2)$ in the valon at $Q^2 = 5\ \mathrm{GeV}^2$ for different N_c.

Note that for $Q^2 = Q_0^2$ we get $f(Q^2) \to f(Q_0^2) = 1$, which is apparent from the definition of L in Eq. (8), thus, arriving at Eq. (12). This simple choice for the initial input in $\bar{g}_2^{\mathrm{valon}}(z,Q_0^2)$ stems from the knowledge that it is related to the quark gluon correlations, which in turn are related to the Green function in the momentum space. In the momentum space the correlation function is composed of a Dirac delta term and a function that is related to the momentum. Consequently, at the initial Q_0^2 we can simply assume that $\bar{g}_2^{\mathrm{valon}}(z,Q_0^2)$ is proportional to Dirac delta function which emphasizes the conservation of energy-momentum and the fact that at such a low Q_0^2 a valon behaves as an object without any internal structure. The last point is built into the definition of a valon. So, in the moment space the delta function becomes unity and we can write

$$\bar{g}_2(n,Q_0^2) = A \times 1; \tag{14}$$

all of the QCD effects are summarized in A. This coefficient will be extracted from the experimental data. A fit to E143 data yields a value 0.01 for A. Having specified the initial input values, the moments of $\bar{g}_2^{\mathrm{valon}}(z,Q^2)$ in a valon are readily obtained, with the aid of Eq. (14). An inverse Mellin transformation then takes us to the momentum space, giving the $\bar{g}_2^{\mathrm{valon}}(z,Q^2)$ structure function. For example, at $Q^2 = 10\ \mathrm{GeV}^2$ we have

$$\bar{g}_2^{\mathrm{valon}}(z,Q^2 = 10\ \mathrm{GeV}^2) = 0.028z^{4.645}. \tag{15}$$

This completes the first step described above. In Fig. 4, $\bar{g}_2^{\mathrm{valon}}(z,Q^2)$ is shown for different values of Q^2. According to Fig. 5, as our calculations for the values of $N_c \geqslant 100$ lead to a similar distribution for $\bar{g}_2^{\mathrm{valon}}(z,Q^2)$ for the whole range of z, we are choosing the optimal value of N_c which is equal to 100 [remember that Eq. (6) is valid only for large N_C].

The second step involves the convolution process which takes us to the hadronic level. This is similar to the earlier procedure that we used to extract $g_1^{p,n}(x,Q^2)$. Similarly to g_1, we can write

$$g_2^h(x,Q^2) = \sum_{\mathrm{valon}} \int_x^1 \frac{dy}{y} \Delta_T G_{\mathrm{valon}}^p(y) g_2^{\mathrm{valon}}\left(\frac{x}{y},Q^2\right), \tag{16}$$

where $\Delta_T G_{\mathrm{valon}}^p(y)$ is the transverse valon distribution function describing the probability of finding a valon with spin aligned or antialigned with the transversely polarized proton. In fact, $\Delta_T G_{\mathrm{valon}}^p(y)$ is identical to $\delta G_{\mathrm{valon}}^p(y)$ in the longitudinal case. This is so because we know that in the nonrelativistic limit of the quark motion, the PPDFs and transversity distribution would be identical, since the rotations and Euclidean boosts commute and a series of boosts and rotations can convert a longitudinal polarized proton into a transversely polarized one with an infinite momentum [33–35]. Finally, substituting the valon helicity distribution $\delta G^{U(D)}(y)$ in the corresponding hadron leads us to $\bar{g}_2(x,Q^2)$. In Fig. 6 we present $x^2\bar{g}_2(x,Q^2)$ for both the proton and deuteron along with the E143 data [3]. We have compared our results with those obtained in the bag model [29] and that of Wakamatsu [32].

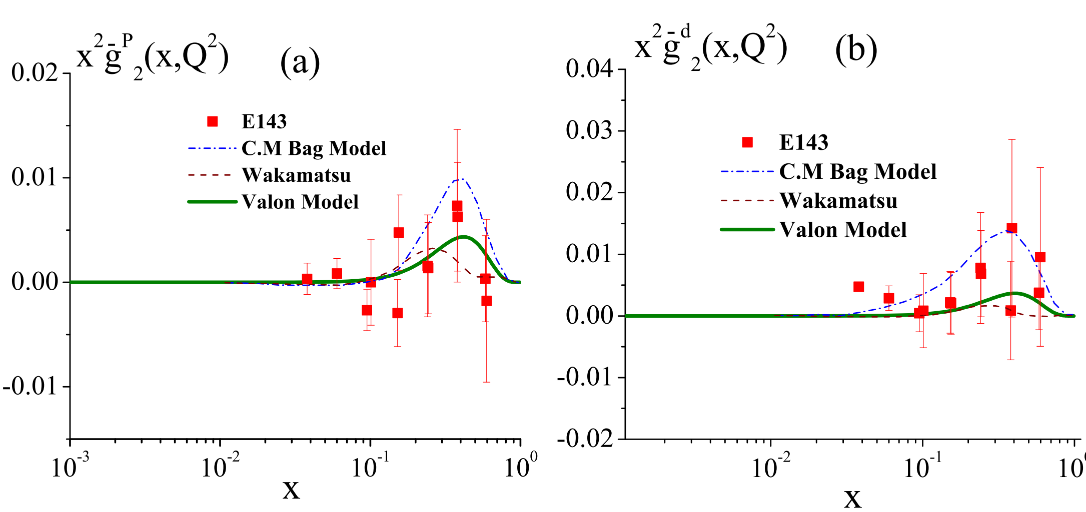

FIG. 6. (Color online) The twist-3 part of proton (a) and deuteron (b) transverse spin structure functions, $x^2\bar{g}_2(x,Q^2)$ at $Q^2 = 5\ \mathrm{GeV}^2$. We compared our results with E143 data [3] and other phenomenological groups [29,32].

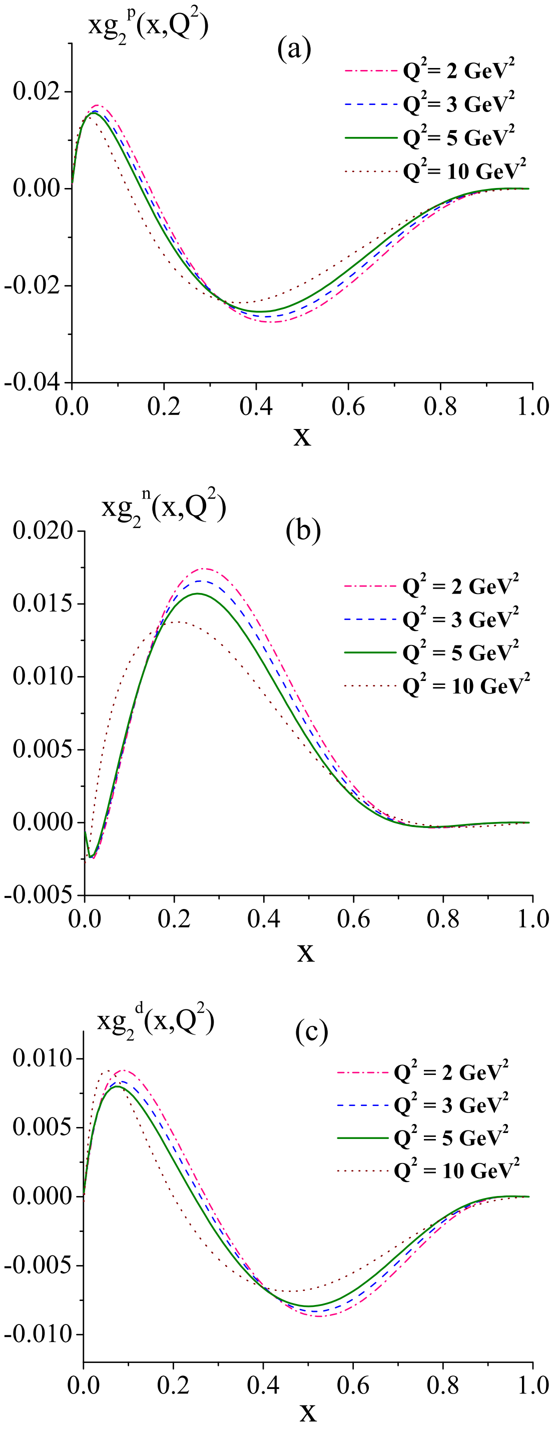

FIG. 7. (Color online) $xg_2(x,Q^2)$ for the proton (a), neutron (b), and deuteron (c) at $Q^2 = 2,3,5,10\ \mathrm{GeV}^2$.

Finally, adding $\bar{g}_2(x,Q^2)$ and $g_2^{ww}(x,Q^2)$ gives the full $g_2(x,Q^2)$. The final results for $xg_2(x,Q^2)$ are presented in Fig. 7 for the proton, neutron, and deuteron at different values of Q^2.

Confronting with the experimental data, in Fig. 8 we show our results for the full transversely polarized structure function $g_2(x,Q^2)$ for the proton, neutron, and deuteron and the experimental findings of [3,4,6].

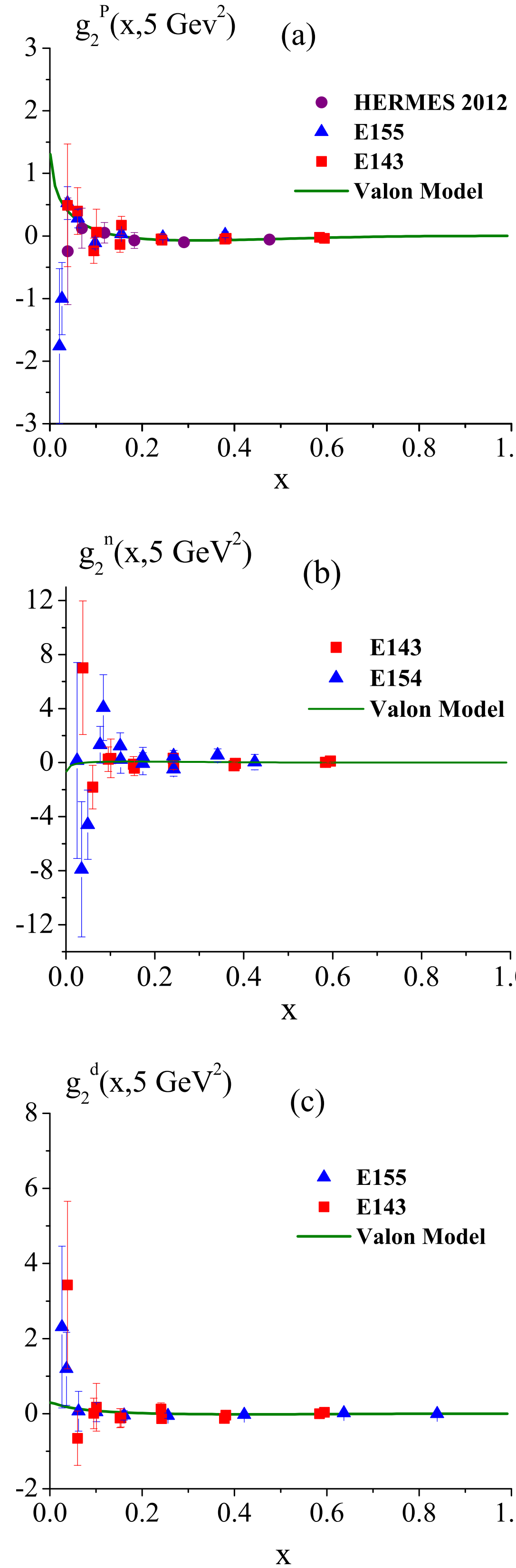

FIG. 8. (Color online) Full transverse polarized structure functions $g_2(x,Q^2)$ for the proton (a), neutron (b), and deuteron (c) at $Q^2 = 5\ \mathrm{GeV}^2$ from the valon model, compared with experimental data [3,4,6].

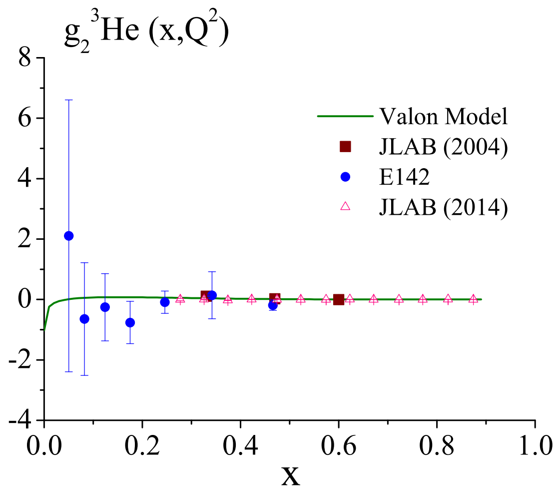

FIG. 9. (Color online) Full transversely polarized ^{3}He structure function $g_2(x,Q^2)$ at $Q^2 = 5\,\mathrm{GeV}^2$. The data points are from [38–40].

C. The case of $g_2^{^3\mathrm{He}}(x,Q^2)$

It is intriguing to investigate $g_2^{^3\mathrm{He}}(x,Q^2)$ as a special case, since there are some newly released data on $g_2^{^3\mathrm{He}}(x,Q^2)$, and the conformity of our result with the experiment will lend further justification to the approach adopted here.

The $g_2^{^3\mathrm{He}}(x,Q^2)$ structure function can be viewed as the sum of $g_2^n(x,Q^2)$ and $g_2^p(x,Q^2)$, each convoluted with the spin dependent nucleon light-cone momentum distributions, $\Delta f^N_{^3\mathrm{He}}(y)$, where y is the ratio of "+" components of the light-cone momenta of struck nucleon to nucleus. One will have

$$g_2^{^3\mathrm{He}}(x,Q^2) = \int_x^3 \frac{dy}{y}\Delta f^n_{^3\mathrm{He}}(y) g_2^n(x/y,Q^2) + 2\int_x^3 \frac{dy}{y}\Delta f^p_{^3\mathrm{He}}(y) g_2^p(x/y,Q^2). \quad (17)$$

So, using Eq. (17), this should be straightforward. We need two functions, namely, $\Delta f^p_{^3\mathrm{He}}(y)$ and $\Delta f^n_{^3\mathrm{He}}(y)$. They can be extracted from the numerical results of [36,37]. In Fig. 9 we have plotted $g_2^{^3\mathrm{He}}(x,Q^2)$ together with the experimental data from [38–40]. As can be seen from the figure, our approach is in fair agreement with the experimental data.

IV. THE SUM RULES

There are two important and well known sum rules regarding $g_1(x,Q^2)$ and $g_2(x,Q^2)$. The first one is the OPE sum rule:

$$\Gamma_1^n = \int_0^1 x^n g_1(x,Q^2)dx = \frac{a_n}{2}, \quad n = 0,2,4,\ldots, \quad (18)$$

$$\Gamma_2^n = \int_0^1 x^n g_2(x,Q^2)dx = \frac{1}{2}\frac{n}{n+1}(d_n - a_n), \quad n = 2,4,\ldots, \quad (19)$$

where a_n and d_n are the twist-2 and the twist-3 matrix element operators, respectively. The study of these sum rules is easy for the simplest case ($n = 2$) where the twist-3 effects exist [1,2].

The second one is the Burkhardt-Cottingham sum rule [41]. It states that the first moment of $g_2(x,Q^2)$ structure function vanishes:

$$\int_0^1 g_2(x,Q^2)dx = 0. \quad (20)$$

Since $g_2 = g_2^{ww} + \bar{g}_2$, upon combining Eqs. (18) and (19) with Eq. (5), provides a third sum rule. It is listed below:

$$\int_0^1 g_2^{ww}(x,Q^2)dx = 0, \quad (21)$$

$$\int_0^1 x^2 g_2^{ww}(x,Q^2)dx = -\frac{1}{3}a_2, \quad (22)$$

$$\int_0^1 x^2 \bar{g}_2(x,Q^2)dx = \frac{1}{3}d_2. \quad (23)$$

We have evaluated d_2^p, d_2^n in the valon model for a number of Q^2 values; the results are shown in Fig. 10 and are compared with the available data, the bag model, and the QCD sum rule results. All results are summarized in Table I

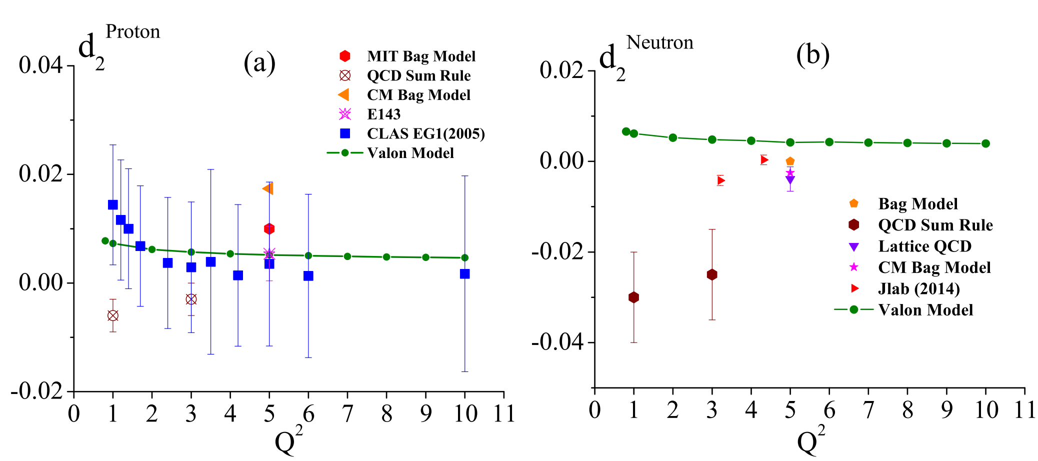

FIG. 10. (Color online) d_2^p (a) and d_2^n (b) are calculated in the valon model. Our results are compared with experimental data as well as with results obtained from other phenomenological groups [3,29,40,42–46].

TABLE I. The twist-2 matrix element operators, a_2, for the proton, neutron, and deuteron, calculated in the valon model. Also included are the experimental data and the results from other theoretical investigations.

	a_2^p	a_2^n	a_2^d
Valon model	0.01956	−0.00004	0.00874
Lattice QCD [45]	$(3 \pm 0.64) \times 10^{-2}$	$-(2.4 \pm 4.0) \times 10^{-3}$	$(13.8 \pm 5.2) \times 10^{-3}$
CM bag model by Song [29]	0.0210	-1.8×10^{-3}	0.0087
E143 [3]	$(2.42 \pm 0.20) \times 10^{-2}$		$(8.0 \pm 0.16) \times 10^{-3}$

and Table II for more clarification. Table III shows our results for the Burkhardt-Cottingham sum rule in the region $0.023 < x < 0.9$ at $Q^2 = 5\ \text{GeV}^2$. They are checked against the data from HERMES in the same region and also with the findings of E143 and E155 in the range $0.02 < x < 0.8$. For the purpose of comparison, results from other sources are also included. While d_2^p is in excellent agreement with the experiment, d_2^n is less so. However, we also notice that there are fewer data for d_2^n, thus making it difficult to arrive at a firm conclusion.

V. CONCLUSION

We have used the so-called valon model to calculate the transverse spin structure functions of the nucleon and the deuteron. To do so, we provide a simple approach for calculating the twist-3 part of the transverse spin structure function $\bar{g}_2(x,Q^2)$ in Mellin space. Furthermore, as a separate check on the validity of our approach, we have considered $g_2^{3He}(x,Q^2)$, where we have utilized some light-cone momentum distributions and compared with the new data from [40]. Evidently our findings are in agreement with the experiment, rendering the conclusion that hadronic structure functions, both polarized and unpolarized, are nicely described in the valon representation.

APPENDIX

Here we attempt to justify our choice of initial input value in $g_2^{\text{valon}}(x,Q_0^2)$:

$$\bar{g}_2^{\text{valon}}\left(z,Q_0^2\right) = A\delta(z-1).$$

As we know, $g_2(x,Q^2)$ is related to the quark-gluon-quark correlation. Since, by definition, at initial scale Q_0^2 the valon behaves as an object with no internal structure, it is reasonable to assume that, at such an initial scale, this object is related to the quark-quark correlations (two-point Green function), because at such a low Q_0^2 gluons have very small and negligible momentum.

The general form of the two-point Green function in momentum space is given here (Eq. (2.102) in [47]):

$$\begin{aligned} &G^{(n)}(x_1,x_2,\ldots,x_n) \\ &\cong \frac{1}{2}\left(\frac{-i\lambda}{\hbar}\right)\prod_{i=1}^{4}\hbar\frac{i}{p_i^2-m^2+i\epsilon} \\ &\quad\times\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta(p_1+p_2+k_1+k_2)(2\pi)^4 \\ &\quad\times\delta(p_3+p_4-k_1-k_2)\hbar\frac{i}{k_1^2-m^2+i\epsilon}\hbar\frac{i}{k_2^2-m^2+i\epsilon}. \end{aligned}$$

The $g_2(x,Q^2)$ structure function is related to the integral over this two-point Green function. We do not know the exact relation between them, but at least we can say that it has two terms: the first one is a function of Q^2 and the second one is a Dirac delta function which implies conservation of energy-momentum. To establish this relation, we resort to the phenomenological arguments. Obviously, the simplest choice for $\bar{g}_2(x,Q^2)$ is

$$\bar{g}_2^{\text{valon}}(z,Q^2) = f(Q^2)\delta(z-1);$$

at the initial scale of Q_0^2, the photon probe detects only three valence quarks inside the proton. Hence, we can assume that $f(Q^2) \to A$ and we have

$$\bar{g}_2^{\text{valon}}\left(z,Q_0^2\right) = A\delta(z-1)$$

The constant A can be determined from experimental data. Our motivation for this value comes from the phenomenological consideration which requires us to choose the initial input

TABLE II. The twist-3 matrix element operators, d_2, for the proton, neutron, and deuteron, calculated in the valon model. Also included are the experimental data and the results from other theoretical investigations.

	d_2^p	d_2^n	d_2^d
Valon model	0.00519	0.0042	0.00437
MIT bag model [29,42]	0.01	0	0.005
QCD sum rule [43]	$-(0.6 \pm 0.3) \times 10^{-2}$	$-(30 \pm 10) \times 10^{-3}$	−0.017
QCD sum rule [44]	$-(0.3 \pm 0.3) \times 10^{-2}$	$-(25 \pm 10) \times 10^{-3}$	−0.013
Lattice QCD [45]	$-(4.8 \pm 0.5) \times 10^{-2}$	$-(3.9 \pm 2.7) \times 10^{-3}$	−0.022
CM bag model by Song [29]	0.0174	-2.53×10^{-3}	0.0067
E143 [3]	$(0.54 \pm 0.5) \times 10^{-2}$		$(3.9 \pm 9.2)10^{-3}$

TABLE III. The results for the Burkhardt-Cottingham sum rule.

	Bag model by Song [29]	E143	E155	HERMES 2012	Valon model
$\int_{0.02}^{0.85} g_2^p(x,Q^2)dx$	−0.0016	-0.013 ± 0.028	-0.022 ± 0.071	$0.006 \pm 0.024 \pm 0.017$	−0.0016
$\int_{0.02}^{0.85} g_2^d(x,Q^2)dx$	−0.00287		0.023 ± 0.044		0.0092

densities as $\delta(z-1)$ at Q_0^2. This mathematical boundary condition means that the internal structure of the valon cannot be resolved at Q_0^2. At this scale of Q_0^2, the nucleon can be considered as a bound state of three valence quarks that carry all the momentum and the spin of the nucleon. As Q^2 is increased, other partons can be resolved at the nucleon.

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